10

MECHANICAL PROPERTIES OF FLUIDS

- Liquids and gases can flow and are generally called as fluids.

- **Pressure**
  Pressure is the thrust (normal force) acting per unit area.

  Its unit is pascal (N/m^2).
  Pressure = Thrust/ Area.
  Pressure is inversely proportional to area.

- **Atmospheric pressure**

  Atmospheric pressure is the weight of air column supported by unit area on the surface of the earth(at sea level).

  1 atm=1.013×10^5 Pa.

  It is equal to the pressure exerted by 76 cm of mercury column at sea level.

- **Density**
  Density is the mass per unit volume. Its unit is kg/m^3.

  Density of water at 4^0C is 1000 kg /m^3.

- **Relative density(specific gravity)**

  Relative density(specific gravity) is the ratio of the density of the substance to the density of water.
  It has no unit.

  Relative density = $\frac{\text{Density of substance}}{\text{Density of water}}$

- Liquids are largely incompressible .Therefore the density is almost constant at all pressures.

- **Pascal's law**

  The pressure exerted at any part of a fluid in equilibrium is equally transmitted to the other parts.

  Hydraulic brake and hydraulic jack are the applications of Pascal's law.

- **Pressure at a depth in a fluid**

  $P = P_a + h \rho g$ where P_a is the atmospheric pressure and h is depth. ($P - P_a$ is called gauge pressure.)

- **Mercury barometer**

  It is used to measure atmospheric pressure. The first barometer was devised by Torricelli.
  The pressure equivalent to 1mm mercury is one torr.
  1 torr = 133Pa

- **Open tube manometer**

  It consists of a U-shaped tube. The tube is filled with some fluids. One end of it opens to atmosphere.
  The other end is connected to the system whose pressure is to be measured.

  If h is the difference of fluid levels in the tube, P is the pressure of the system, P_a is the atmospheric pressure, then the gauge pressure, $P - P_a = h \rho g$ where ρ is the density of the manometer fluid.

  If the pressure difference is small, a fluid of low density is used. If the pressure difference is more, high density fluids like mercury are used.

- **Archimedes' principle**

When a body is completely or partially immersed in a fluid, the buoyant force(loss of weight) on it is equal to the weight of fluid displaced by the body.

- **Buoyant force**

The upward force exerted by a fluid on a body immersed in it is called buoyant force .The body immersed in a fluid experiences loss of weight due to this.

- **Principle of floatation**

The weight of a floating body in a fluid is equal to the weight of fluid displaced by it.

- For a denser body , the weight of the body is greater than the weight of fluid displaced by it.

- **Streamline flow**

It is a steady flow of the fluid in which the velocity of each fluid particle passing through a point is the same.

Each particle follows a smooth path and the paths do not cross each other.

The path followed by a fluid particle under steady flow is called stream line.
It is a curve whose tangent at any point gives the direction of flow at that point.

- **Turbulent flow**

Streamline flow is possible at small velocities. Beyond a limit called critical speed the flow becomes unsteady and thus it becomes turbulent.

When a fast moving stream encounters a rock, whirlpool like shapes are formed. This is example for turbulent flow.

- **Laminar flow**

In a laminar flow, the velocity at different points of the fluid is different in magnitude but the directions are parallel.

- **Reynold's number**

It indicates whether the flow is streamline or turbulent.

Reynold's number, $R_e = \frac{\rho v d}{\eta}$

where ρ is the density of the fluid, v is the velocity of the fluid, d is the dimension of the part through which the fluid flows (For a stream, 'd' is its diameter and for a tube,'d' is its diameter)and η is the coefficient of viscosity of the fluid.

If $R_e <$ 1000, the flow is streamline. If $R_e >$ 2000, the flow is turbulent and if R_e is 1000 to 2000 ,it is unsteady.

- **Equation of continuity**

$a_1v_1 = a_2v_2 = a_3v_3$ Or av =constant .This is called the equation of continuity.
Here a is the area of cross-section and v is the velocity. **Here mass is conserved**.

- Gardener closes the tube partially when he waters the plants. While closing, the available area decreases. By the equation of continuity, the velocity of flow increases.

- **Bernoulli's principle**

For the streamline flow of a non –viscous fluid, the sum of the pressure, kinetic energy per unit volume and the potential energy per unit volume is constant. **It is actually the law of conservation of energy.**

$P + \frac{1}{2}\rho v^2 + \rho g h$ is a constant.

- **Application of Bernoulli's theorem**

Spin motion of cricket ball(Magnus effect), Aerofoil (or special shape of aircraft wings), Atomizer

Venturimeter

It is used to find the velocity of flow of a fluid. The area of cross-section of the bigger tube is A and that of smaller tube is a. The velocities at these regions are v_1 and v_2 respectively. The tubes are connected by means of a manometer tube. ρ_m is the density of manometer fluid.

$$v_1 = \sqrt{\frac{2\,h\rho_m g}{\left[\frac{A^2}{a^2}-1\right]\rho}}$$

- **Blood flow and heart attack**

  The artery may get constricted due to the accumulation of plaque on its inner walls. As the area of that part decreases, by the equation of continuity the velocity of blood flow increases.

  As the velocity increases, by Bernoulli's principle the pressure decreases. The artery may be collapsed due to the external pressure. Heart continuously pumps blood and the collapse mechanism repeats. This leads to heart attack.

- **Viscous force**

  It is the frictional force between the layers of a fluid. The tendency of a fluid to exert viscous force is called viscosity.

  A fluid flows as different layers. In the case of a stream, the layer in contact with the ground is practically at rest. The top layer will have the maximum speed.

  In the case of a fluid flowing through a tube, the central layer will have the maximum speed. The layer in contact with the tube will have the minimum speed.

  Coefficient of viscosity, $\eta = \frac{Shearing\ stress}{Rate\ of\ shearing\ strain} = \frac{F}{A} \div \frac{v}{L} = \frac{FL}{Av}$

  The unit of coefficient of viscosity is poiseiulle(PI).

  Its dimensions are $ML^{-1}T^{-1}$

- The viscosity of a liquid decreases with increase of temperature. The viscosity of a gas increases with increase of temperature .

- Fluids of large viscosity are called viscous fluids. eg: coal tar, castor oil,honey,glycerine etc.
  Fluids of small viscosity are called mobile fluids.eg:water,alcohol etc.

- During an electric shock, the body temperature suddenly decreases. Viscosity of blood increases. Heart cannot pump it through arteries and it leads to heart failure.

- **Velocity gradient**

  It is the ratio of the change of velocity between two fluid layers to the distance between them. Its unit is s^{-1}(per second).

- **Newton's law of viscosity**

  The viscous force acting between fluid layers is directly proportional to the area of contact between the layers and the velocity gradient.
  Viscous force, $F = \eta A\frac{dv}{dx}$ where η is the coefficient of viscosity of the fluid.

- **Stokes' formula**

  The viscous force acting on a sphere of radius 'a' falling through a viscous fluid $=6\pi\eta av$ where v is the velocity of the sphere.

- **Terminal velocity**

  Consider a body falling through a viscous fluid. First its velocity increases. Then it attains a constant velocity called terminal velocity.

  $v = \frac{2}{9\eta} a^2 (\rho - \sigma) g$ where η is the coefficient of viscosity of the fluid , a is the radius of sphere, ρ is the density of the body and σ is the density of the fluid.

  For fluids like air, σ is taken as zero.

- **Speed of efflux**

  Efflux means fluid outflow. Consider a tank containing a fluid of density ρ .
  The pressure at the top surface of the fluid is P and the area is there is A_2.It is at a height y_2.There is a small hole at its side near the bottom.It is at a height y_1.The area there is A_1 and the speed of outflow is v_1.The pressure is P_a.The velocity of flow at the top is v_2.

  Here $y_2 - y_1 = h$ and $P_2 = P$

  OR

  $$v_1 = \sqrt{\frac{2(P - Pa))}{\rho} + 2gh}$$

  When $P \gg P_a$, 2gh is neglected(as in the case of rocket propulsion).
  If $P= P_a$(vessel is open to atmosphere),then $v=\sqrt{2gh}$ which is the speed of a freely falling body.This equation is called Torricelli's law.

- **Surface tension**

  It is the force acting on unit length of an imaginary line on liquid surface. It acts tangential to the liquid surface. Its tendency is to reduce surface area. Surface tension = $\frac{Force}{length}$
  Its unit is N/m.

- **Hairs of a shaving brush stick together when it is wet** due to surface tension exerted by water.

- **Liquid drops attain spherical shape** due to surface tension exerted by liquids.

- A liquid surface can support a needle or a shaving blade due to the property of surface tension.

- Surface tension is numerically equal to surface energy.

- **Excess pressure inside a bubble and drop**

  Consider a drop of radius r.
  Excess pressure P = $\frac{2S}{r}$

  For a bubble the excess pressure = $\frac{4S}{r}$

- **Angle of contact**

  It is the angle between the tangent to liquid surface and the body inside the liquid. For water –glass surface, it is acute. For mercury-glass surface, it is obtuse. For water-silver surface, it is right angle.

- **Cohesion and adhesion**

  The force of attraction between molecules of same kind is called cohesion.eg: force between two water molecules.

  The force of attraction between molecules of different kind is called adhesion.eg: force between water molecules and glass.

- Mercury shows **capillary depression.**

- Mercury does not stick to glass. In this case, the force between mercury molecules (cohesion) is more than the adhesion between glass and mercury.

- **Expression for capillary rise**

  The tendency of a liquid to travel up through tiny pores is called capillary rise. If the area of pore is more, capillary rise is small.

  Capillary rise ,h = $\frac{2SCos\theta}{\rho\, gr}$ where S is surface tension, ρ is the density of fluid , r is radius of tube immersed.

  For liquids like water, $\theta \approx 0^o$

- **The tip of nib of a pen is split:** As the tip is split, the ink can reach there by capillary action.

- **In summer , we till the lands:** If it is tilled, the distance between soil particles increases, capillary rise decreases. Thus soil can retain water.

- Oil rises up through the wicks of a lamp by capillary action.

- **Detergents and surface tension:** When detergent is added to water, its surface tension decreases.

- On spilling kerosene over water it spreads over the surface of water. The surface tension decreases.

- When a jet of air at high velocity is passed between two parallel plates, by Bernoulli's theorem, the pressure is decreases. The surrounding pressure presses the plates and they come closer.

- As the velocity of air is high, **light roofs are blown off during storm**: As the velocity of storm is more, by Bernoulli's principle the pressure above the roof is less. The high pressure under the roof pushes it upwards.

- When a number of smaller drops are combined into a bigger drop, volume remains constant but area decreases. Energy is released and temperature of bigger drop increases.

- When a bigger drop of radius R sprayed in to a number of similar smaller drops, each of radius r, volume remains constant but surface area of drops increases. Energy is absorbed. Temperature of system decreases.

- Radius of new bubble when two bubbles coalesce
  Let r_1 and r_2 be the radii of two bubbles. Let them coalesce into a bubble of radius r, under isothermal conditions.
  $r^2 = {r_1}^2 + {r_2}^2$

- If T is the observed weight of a body of density σ when it is fully immersed in a liquid of density ρ, then real weight of the body $w = \frac{T}{1-\frac{\rho}{\sigma}}$

- Work Done in Blowing a Liquid Drop
  W = S . $4\pi(r_2^2 - r_1^2)$
  Work Done in Blowing a Soap Bubble
  W = S. $8\pi(r_2^2 - r_1^2)$
  Work done in splitting a bigger drop into n smaller droplets
  Work done, W = $4\pi\, SR^2(n^{1/3} - 1)$

- Work done in spraying a liquid drop of radius R into n droplets of radius r = S×increase in surface area
  $4\pi\, SR^3\left(\frac{1}{r} - \frac{1}{R}\right)$
  Fall in temperature $\Delta\theta = \frac{3S}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$
- When n small drops are combined into a bigger drop , then work done

  W = $4\pi\, R^2 S\,(n^{1/3} - 1)$
  Increase of temperature

  $\Delta\theta = \frac{3S}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$ where J = 4.2J/cal
- When two soap bubbles of radii r_1 and r_2 are in contact with each other , then radius of common interface,
  r= $\frac{r_1\, r_2}{r_1 - r_2}$

- At boiling point, surface tension of a liquid becomes zero and becomes maximum at freezing point.

Multiple choice questions: Class Work

1. Pressure is a ----- quantity.

   (a) scalar (b) vector
   (c) tensor (d) Either (a) or (c)

2. The two thin bones (femurs), each of cross – sectional area 10 cm^2 support the upper part of human body of mass 40 kg. Estimate the average pressure sustained by the femurs.

   (a) $2 \times 10^5 Nm^{-2}$ (b) $3 \times 10^4 Nm^{-2}$
   (c) $2.5 \times 10^3 Nm^{-2}$ (d) $6 \times 10^4 Nm^{-2}$

3. If two liquids of same masses but densities ρ_1 and ρ_2 respectively are mixed , then density of mixture is given by

   (a) $\rho = \frac{\rho_1 + \rho_2}{2}$ (b) $\rho = \frac{\rho_1 + \rho_2}{2\,\rho_1\rho_2}$
   (c) $\rho = \frac{2\,\rho_1\rho_2}{\rho_1 + \rho_2}$ (d) $\rho = \frac{\rho_1\rho_2}{\rho_1 + \rho_2}$

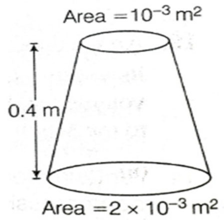
4. If two liquids of same volume but different densities ρ_1 and ρ_2 are mixed , then density of mixture is given by

   (a) $\rho = \frac{\rho_1 + \rho_2}{2}$ (b) $\rho = \frac{\rho_1 + \rho_2}{2\,\rho_1\rho_2}$
   (c) $\rho = \frac{2\,\rho_1\rho_2}{\rho_1 + \rho_2}$ (d) $\rho = \frac{\rho_1\rho_2}{\rho_1 + \rho_2}$

5. Three liquids of densities d, $2d$ and $3d$ are mixed in equal proportion of weights . If density of water is d , then the specific gravity of the mixture is

   (a) $\frac{11}{7}$ (b) $\frac{18}{11}$ (c) $\frac{13}{9}$ (d) $\frac{23}{18}$

6. A uniformly tapering vessel is filled with a liquid of density 900 kgm^{-3} . The force that acts on the base of the vessel due to the liquid is (Take , g = 10 ms^{-2})



(a) 3.6 N (b) 7.2 N (c) 9.0 N (d) 14.4 N

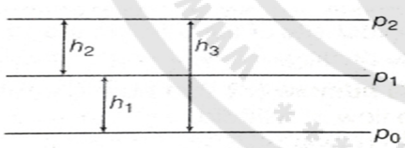
7. The heart of man pumps $5l$ of blood through the arteries per minute at a pressure of 150 mm of mercury. If the density of mercury be $13.6\times10^3 kg\, m^{-3}$ and g = 10 ms^{-2}, then the power of heart in watt is

(a) 1.70 (b) 2.35 (c) 3.0 (d) 1.50

8. Two non - mixing liquids of densities ρ and $n\rho$ $(n > 1)$ are put in a container .The height of each liquid is h. A solid cylinder of length L and density d is put in this container . The cylinder floats with its axis vertical and length pL $(p < 1)$ in the denser liquid . The density d is equal to

(a) {2 + (n + 1) p} ρ (b) {2 + (n - 1) p} ρ
(c) {1 + (n - 1) p} ρ (d) {1 + (n + 1) p} ρ

9. A student measures pressure of gas in a container using a mercury manometer and she also measures atmospheric pressure using a mercury barometer . She gave following representation :



If P_1 = atmospheric pressure , then

(a) Gauge pressure = $h_1 + h_2$
(b) Gauge pressure = h_3 - h_1
(c) Gauge pressure = h_3
(d) Absolute pressure = h_1

10. An ice cube floats on water in a beaker with 9/10 th of its volume submerged under water. What fraction of its volume will be submerged , if the beaker of water is taken to the Moon where the gravity is 1/6th that on the Earth ?

(a) $\frac{9}{10}$ (b) $\frac{27}{50}$ (c) $\frac{2}{3}$ (d) zero

11. A cubical block of steel of each side equal to l is floting on mercury in a vessel. The densities of steel and mercury are ρ_s and ρ_m . The height of the block above the mercury level is given by

(a) $l\left(1+\frac{\rho_s}{\rho_m}\right)$ (b) $l\left(1-\frac{\rho_s}{\rho_m}\right)$

(c) $l\left(1+\frac{\rho_m}{\rho_s}\right)$ (d) $l\left(1-\frac{\rho_m}{\rho_s}\right)$

12. A thin uniform cylindrical sheet , closed at both ends , is partially filled with water. It is floating vertically in water in half - submerged state. If ρ_c is the relative density of the material of the shell with respect to water , then the correct statement is that the shell is

(a) more than half filled if ρ_c is less than 0.5
(b) more than half filled if ρ_c is more than 1.0
(c) half filled if ρ_c is more than 0.5
(d) less than half filled if ρ_c is than 0.5

13. The approximate depth of an ocean is 2700 m. The compressibility of water is $45.4 \times 10^{-11} Pa^{-1}$ and density of water is water $10^3 kg\, m^{-3}$ .What fractional compression of water will be obtained at the bottom of the ocean ?

(a) 0.8×10^{-2} (b) 1.0×10^{-2}
(c) 1.2×10^{-2} (d) 1.4×10^{-2}

14. In a laminar flow , the velocity of the liquid in contact with the walls of the tube is

(a) zero
(b) maximum
(c) in between zero and maximum
(d) equal to critical velocity

15. Two water pipes of diameters 2cm and 4cm are connected with the main supply line . The velocity of flow of water in the pipe of 2cm diameter is

(a) 4 times that in the other pipe
(b) $\frac{1}{4}$ tomes that in the other pipe

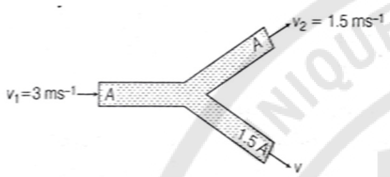
(c) 2 times that in the other pipe

(d) $\frac{1}{2}$ tomes that in the other pipe

16. A liquid flows through a pipe of varying diameter . The velocity of the liquid is $2\,ms^{-1}$ at O pint where the diameter is 6 cm. The velocity of the liquid at a point where the diameter is 3 cm will be

(a) $1\ ms^{-1}$ (b) $4\,ms^{-1}$
(c) $8\,ms^{-1}$ (d) $16\,ms^{-1}$

17. An incompressible liquid flows through a horizontal tube as shown (areas of tubes is marked) then the velocity v of the fluid is



(a) $3.0\,ms^{-1}$ (b) $1.5\ ms^{-1}$
(c) $1.0\,ms^{-1}$ (d) $2.25\ ms^{-1}$

18. The cylindrical tube of a spray pump has radius R , one end of which has n fine holes , each of radius r . If the speed of the liquid in the tube is v , the speed of the ejection of the liquid through the holes is

(a) $\frac{vR^2}{n^2r^2}$ (b) $\frac{vR^2}{n\,r^2}$ (c) $\frac{vR^2}{n^3r^2}$ (d) $\frac{v^2R}{n\,r}$

19. According to Bernoulli's equation ,

$\frac{P}{\rho\,g} + h + \frac{1}{2}\frac{v^2}{g}$ = constant

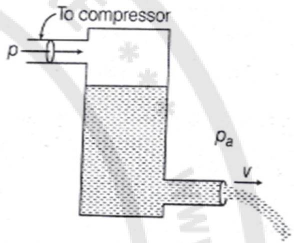
The terms , $\frac{P}{\rho\,g}$, h and $\frac{1}{2}\frac{v^2}{g}$ are generally called respectively :

(a) Gravitational head , pressure head and velocity head
(b) Gravity , gravitational head and velocity head
(c) Pressure head , gravitational head and velocity head
(d) Gravity , pressure and velocity head

20. Air is streaming past a horizontal airplane 's wing such that its speed is 120 ms^{-1} over upper surface and 90 ms^{-1} at the lower surface . If the density of air is 1.3 kgm^{-3} and the wing is 10 m long and has an average width of 2m , then the difference of the pressure on the two sides of the wing is

(a) 4095.0 Pa (b) 409.50 Pa
(c) 40.950 Pa (d) 4.0950 Pa

21. A sealed tank has 2 openings . One at top and other at near bottom . Let height of water filled above the bottom opening is h and compressor producing a pressure P is connected to top opening .
Velocity of water obtained from lower opening is (Take , atmospheric pressure P_a such that $P - P_a = \rho\,gh$



(a) $\sqrt{2\,gh}$ (b) $\sqrt{gh}$ (c) $2\sqrt{gh}$ (d) 0

22. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is

(a) 10 (b) 20 (c) 25.5 (d) 5

23. Water stands at a depth H in a tank whose side walls are vertical . A hole is made in one of the walls at a height h below the water surface . The stream of water emerging from the hole strikes the floor at a distance R from the tank , where R is given by

(a) $R = \sqrt{h\,(H-h)}$ (b) $R = \sqrt{h\,(H+h)}$
(c) $R = 2\sqrt{h\,(H-h)}$ (d) $R = 2\sqrt{h\,(H+h)}$

24. A hole is made at the bottom of the tank filled with water (density 1000 kgm^{-3}) . If the total pressure at the bottom of the tank is 3 atm (1 atm = $10^5 Nm^{-2}$) then the velocity of efflux is

(a) $\sqrt{200}\, ms^{-1}$ (b) $\sqrt{400}\, ms^{-1}$
(c) $\sqrt{500}\, ms^{-1}$ (d) $\sqrt{800}\, ms^{-1}$

25. The applications of venturimeter is /are

(a) carburetor of automobile
(b) sprayers
(c) filter pumps
(d) All of the above

26. The flow of blood in a large artery of an anesthetized dog is diverted through a venturimeter . The wider part of the meter has a cross- sectional area equal to that of the artery , A = 8 mm^2 .

The narrower part has an area a = $4mm^2$ and density of blood , i.e., $\rho = 1.06 \times 10^3\ kg\, m^{-3}$. The pressure drop in the artery is 24 Pa . What is the speed blood in the artery?

(a) 0.5 ms^{-1} (b) 0.125 ms^{-1}
(c) 1.25 ms^{-1} (d) 2.5 ms^{-1}

27. A wind with speed 40 ms^{-1} blows parallel to the roof of a house . The area of the roof is $250m^2$. Assuming that the pressure inside the house is atmospheric pressure , the force exerted by the wind on the roof and the direction of the force will be (P_{air} = 1. 2 $kg\ m^{-3}$)

(a) $2.4 \times 10^5\ N, downwards$
(b) $4.8 \times 10^5\ N, downwards$
(c) $4.8\ \times 10^5\ N, upwards$
(d) $2.4 \times 10^5\ N, upwnwards$

28. A fully loaded boeing aircraft has a mass of 3.3×10^5 kg. Its total wing area is $500m^2$. It is in level flight with a speed of 960 kmh^{-1}.

(i) Estimate the pressure difference between the lower and upper surfaces of the wings
(ii) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface . The density of air is ρ = 1.2 kg m^{-3} .

(a) $6.5 \times\ 10^{+3} Nm^{-2}, 0.01$
(b) $6.5 \times\ 10^{3} Nm^{-2}, 0.09$
(c) $6.5 \times\ 10^{3} Nm^{-2}, 0.08$
(d) $2.5 \times\ 10^{3} Nm^{-2}, 0.02$

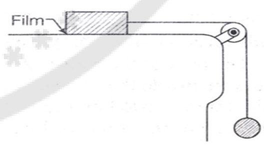
29. The coefficient of viscosity for hot air is

(a) greater than the coefficient of viscosity for cold air
(b) smaller than the coefficient of viscosity for cold air
(c) same as the coefficient of viscosity for cold air
(d) increase or decreases depending on the external pressure

30. We have three breakers A,B and C containing three different liquids. They are stirred vigorously and placed on a table. Then, liquid which is

(a) most viscous comes to rest at the earliest
(b) most viscous comes to rest at the last
(c) most viscous slows down earliest but comes to rest at the last
(d) All of them comes to rest at the same time

31. A metal block of area $0.10m^2$ is connected to a 0.010 kg mass via a string that passes over an ideal pulley (considered massless and frictionless) , as in figure . A liquid with a film thickness of 0.30 mm is placed between the block and the table , When released the block moves to the right with a constant speed of 0.085 ms^{-1} . Find the coefficient of viscosity of the liquid.



(a) $4\ \times 10^{-2}\ pa-s$ (b) $3.45\ \times 10^{-3}\ pa-s$
(c) $5\ \times 10^{-2}\ pa-s$ (d) $7 \times 10^{-7}\ pa-s$

32. A small drop of water falls from through a large height h in air , the final velocity is proportional to

(a) $\sqrt{h}$ (b) h (c) $(1/h)$ (d) h^o

33. For a rain drop of radius a , terminal velocity v_t is given by

(a) $\frac{2a^2}{9\eta}(\rho-\sigma)g$ (b) $\frac{4\pi}{3}\times a^3(\rho+\sigma)g$

(c) $\frac{4\pi}{3}\times a^3(\rho+\sigma)^2 g$ (d) $\frac{5\pi}{3}\times a^3(\rho-\sigma)^2 g$

34. The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20^oC is 6.5 cm^{-1} . Compute the viscosity of the oil at 20^oC . Density of oil is $20^o\,C$. Density of oil is $1.5\times10^3\ kgm^{-3}$, density of copper is $8.9\times10^3\ kgm^{-3}$

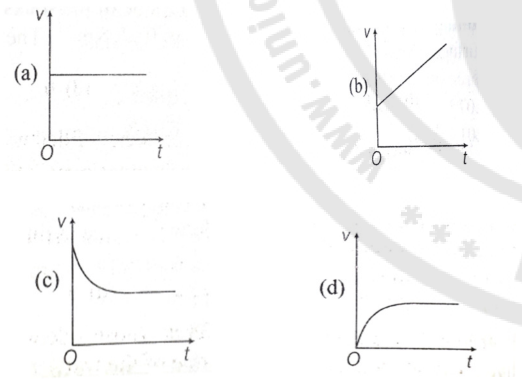
(a) $1\times10^{-1}\ kg\ ms^{-1}$
(b) $9.9\times10^{-1}\ kg\ ms^{-1}$
(c) $24.3\times10^{-2}\ kg\ ms^{-1}$
(d) $2\times10^{-2}\ kg\ ms^{-1}$

35. If a ball of steel (density ρ = 7.8 g cm^{-3}) attains a terminal velocity of 10 cms^{-1} when falling in a tank of water (coefficient of viscosity η_{water} = $8.5\times10^{-14}\ pa-s$) , then its terminal velocity in glycerin (ρ 1.2 g cm^{-3} , η = 13.2 pas) would be nearly

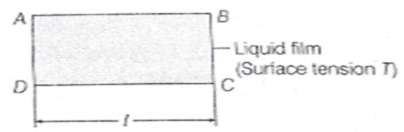
(a) $1.6\times10^{-5}cms^{-1}$ (b) $6.25\times10^{-4}cms^{-1}$

(c) $6.45\times10^{-4}cms^{-1}$ (d) $1.5\times10^{-5}cms^{-1}$

36. Which one shows the variation of the velocity v with time t for a small sized spherical body falling in a column of a viscous liquid .



37. A liquid film is formed over a frame $ABCD$ as shown in figure . Wire CD can slide without friction. Maximum value of mass that can be hanged from CD without breaking the liquid film is



(a) $\frac{T\,l}{g}$ (b) $\frac{2\,T\,l}{g}$ (c) $\frac{g}{2Tl}$ (d) $T\times l$

38. A wooden stick 2 m long is floating on the surface of water . The surface tension of water 0.07 Nm^{-1} . By putting soap solution on one side of the sticks , the surface tension is reduced to 0.06 Nm^{-1} . The net force on the stick will be

(a) 0.07 N (b) 0.06 N (c) 0.01 N (d) 0.02

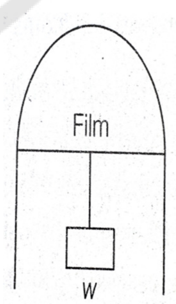
39. The force required to separate two glass plates of $10^{-2}m^2$ with a film of water 0.05 mm thick between them , is (surface tension of water is $70\times10^{-3}Nm^{-1}$)

(a) 28 N (b) 14 N (c) 50 N (d) 38 N

40. A 10 cm long wire is placed horizontally on the surface of water and is gently pulled up with a force of $2\times10^{-2}N$ to keep the wire in equilibrium . The surface tension in Nm^{-1} of water is

(a) 0. 1 (b) 0.2 (c) 0.001 (d) 0.002

41. A thin liquid film formed between a U shaped wire and a light slider supports a weight of 1.5$\times$ $10^{-2}N$ as shown in the figure . The length of the slider is 30 cm and its weight is negligible . The surface tension of the liquid film is



(a) 0.0125 Nm^{-1} (b) 0.1Nm^{-1}

(c) 0.05 Nm^{-1} (d) 0.025 Nm^{-1}

42. Water rise to a height h in capillary tube. If the length of capillary tube above the surface of water is made less than h, then

(a) water rises up to the tip of capillary tube and then starts overflowing like a fountain
(b) water rises up to the top of capillary tube and stays there without overflowing
(c) water rises up to a point a little below the top and stays there
(d) water does not rise at all

43. The surface tension of water at temperature of the experiment is $7.30 \times 10^{-2} Nm^{-1}$. 1 atm pressure = $1.01 \times 10^5\ pa$, density of water = 1000 kgm^{-3} , g = 9.80 ms^{-2} . Calculate the pressure inside a double of radius 1 mm .

(a) $3 \times 10^2 Pa$ (b) $8 \times 10^4 Pa$
(c) $1.01 \times 10^5 Pa$ (c) $7 \times 10^3 Pa$

44. If the drop is in equilibrium , then energy gain due to pressure difference between the inside and outside the bubble is

(a) $(P_i - P_o)\ 4\pi r^2\ \Delta r$ (b) $P_o\ .\ 4\pi r^2\ \Delta r$
(c) $P_i\ .\ 4\pi r^2\ \Delta r$ (d) $(P_i + P_o)\ 4\pi r^2\ \Delta r$

45. In a soap bubble , pressure difference is

(a) $\frac{2\,S_{la}}{r}$ (b) $\frac{4\,S_{la}}{r}$ (c) $\frac{S_{la}}{r}$ (d) $\frac{8\,S_{la}}{r}$

46. Two small drop of mercury , each of radius R , coalesce to from a single large drop . The ratio of the total surface energies before and after the change is

(a) 1 : $2^{1/3}$ (b) $2^{1/3}$: 1
(c) 2 : 1 (d) 1 : 2

47. Radius of a soap bubble is increased from R to $2R$. Work done in this process in terms of surface tension is

(a) $24\,\pi R^2 S$ (b) $48\,\pi R^2 S$
(c) $12\,\pi R^2 S$ (d) $36\,\pi R^2 S$

48. A soap bubble of radius r is blown up to from a bubble of radius $2r$ under isothermal conditions. If σ is the surface tension of soap solution , the energy spent in doing so is

(a) $3\,\pi\sigma\,r^2$ (b) $6\,\pi\sigma\,r^2$
(c) $12\,\pi\sigma\,r^2$ (d) $24\,\pi\sigma\,r^2$

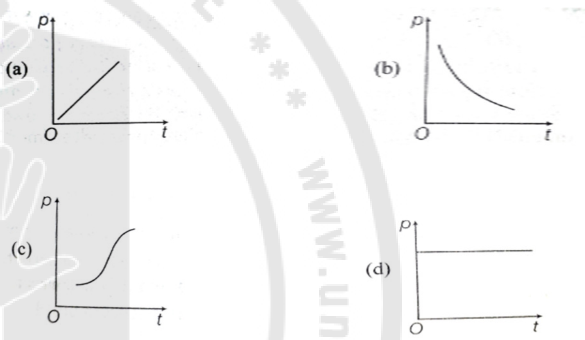
49. In the above question , if the air bubble is formed at a depth h inside the container of soap solution of density ρ , the total pressure inside the bubble is (here , P_o denotes the atmospheric pressure.)

(a) $\frac{2\,\sigma}{r} + hg\rho$ (b) $\frac{2\,\sigma}{r} - hg\rho$
(c) $P_o + \frac{2\,\sigma}{r} + hg\rho$ (d) $P_o + \frac{2\,\sigma}{r} - hg\rho$

50. The excess pressure inside an air bubble of radius r just below the surface of water is P_1. The excess pressure inside a drop of the same radius just outside the surface is P_2 . If T is surface tension , then

(a) $P_1 = 2\,P_2$ (b) $P_1 = P_2$
(c) $P_2 = 2\,P_1$ (d) $P_2 = 0$, $P_1 \neq 0$

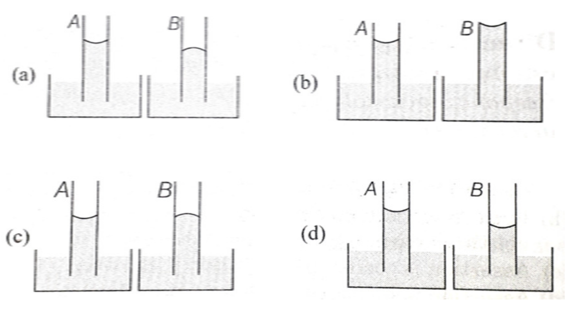
51. A soap bubble formed at the end of the tube is blown very slowly. The graph between excess of pressure inside the bubble with time is



52. The lower end of a capillary tube of diameter 2.00 mm is dipped 8.00 cm below the surface of water in beaker . What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water?

(a) $2 \times 10^5 pa$ (b) $1.01784 \times 10^5 pa$
(c) $3 \times 10^3 pa$ (d) $2.438 \times 10^5 pa$

53. A capillary tube A is dipped in water. Another identical tube B dipped in soap – water solution. Which of the following shows the relative nature of the liquid columns in the two tube?



54. An open glass tube is immersed in mercury in such a way that a length of 8cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now?

(Atmospheric pressure = 76 cm of Hg)

(a)16 cm (b) 22 cm (d) 38 cm (d) 6 cm

55. A certain number of spherical drops of liquid of radius r coalesce to from a single drop of radius R and volume V. If T is surface tension of the liquid, then

(a) Energy = $3VT\left(\frac{1}{r}-\frac{1}{R}\right)$ is released

(b) Energy is neither released nor absorbed

(c) Energy = $4VT\left(\frac{1}{r}-\frac{1}{R}\right)$ is released

(d) Energy = $3VT\left(\frac{1}{r}-\frac{1}{R}\right)$ is absorbed

56. Two capillaries made of same material but of different radii are dipped in a liquid. The rise of liquid in one capillary is 2.2 cm and that in the other is 6.6 cm. The ratio of their radii is

(a) 9 :1 (b) 1 :9 (c) 3 : 1 (d) 1 : 3

57. The lower end of a capillary tube of radius r is placed vertically in water. Then, with the rise of water in the capillary, heat evolved is

(a) $+\frac{\pi^2 r^2 h^2}{2}dg$ (b) $\frac{\pi r^2 h^2 dg}{2}$

(c) $-\frac{\pi^2 h^2 dg}{2}$ (d) $-\frac{\pi r^2 h^2 dg}{2}$

Hints and explanations

1. (a)

2. (a)
Total cross–sectional area of the femurs is
A = 2 × 10 cm^2 = 20 × $10^{-4} m^2$.

The force acting on them is
F = 40 kg wt = 400 N (take, g = 10 cm^2).

This force is acting vertically downwards and hence, normally on the femurs, Thus, the average pressure is

$P_{av} = \frac{F}{A} = 2 \times 10^5 Nm^{-2}$

3. (c)
$\rho = \frac{Total\ mass}{Total\ volume} = \frac{2m}{V_1+V_2} = \frac{2m}{m\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)}$

$\therefore \rho = \frac{2\rho_1 \rho_2}{\rho_1+\rho_2}$

4. (a)

$\rho = \frac{Total\ mass}{Total\ volume} = \frac{m_1+m_2}{2V} = \frac{V(\rho_1+\rho_2)}{2V} = \frac{\rho_1+\rho_2}{2}$

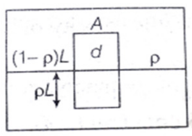
5. (b)
$W_1 = W_2 = W_3 \implies m_1 = m_2 = m_3$ = m (say)
$V_1 = \frac{m}{d}$, $V_2 = \frac{m}{2d}$, $V_3 = \frac{m}{3d}$
$\therefore d_{mix} = \frac{Mass}{Volume} = \frac{3m}{V_1+V_2+V_3} = \frac{18}{11}d$
Specific gravity of mixture = $\frac{d_{mix}}{d_{water}} = \frac{18}{11}d$

6. (b)
Force acting on the base, F = P× $A = hdgA$
= 0.4 × 900 × 10 × 2 × 10^{-3} = 7.2 N

7. (a)
Pressure = 150 mm of Hg
Pumping rate of heart of man,
$\frac{dV}{dt} = \frac{5\times10^{-3}}{60}\ m^3 s^{-1}$

Power of heart = $P.\frac{dV}{dt}$
$= \rho gh.\frac{dV}{dt}$
$= \frac{(13.6\times10^3 kgm^{-3})(10)\times(0.15\times5\times10^{-3})}{60}$ = 1.70 W

8. (c)



Weight of cylinder = (upthrust)$_1$ + (upthrust)$_2$
i.e., $ALdg$ = (1 – P) $LA\rho g$ + $(pLA)n\rho g$
$\implies d = (1-p)\rho + p\,n\rho$
$= \rho - p\rho + nP\rho$
$= \rho + (n-1)P\rho$
$= \rho[1+n-1)p]$

9. (b)
Absolute pressure $= h_3 = h_1 + h_2$
Gauge pressure $= h_2 = h_3 - h_1$

10. (a)

The Buoyant force on the ice cube depends on the value of g , the fraction of the cube submerged under a liquid is independent of the value of g and depends only on the density of the body relative to that of the liquid on which it floats

11. (b)
Volume of block $= l^3$. h be the height of the block above the surface of mercury .
Volume of mercury displaced $= (l-h) \, . \, l^2$
∴ Weight of mercury displaced
$= (l-h) \, . \, l^2 \, . \, \rho_m \, . \, g$

This is equal to weight of the block which is $\rho_s \, . \, l^3 \, . g$
$(l-h) \; l^2 \, . \, \rho_m \, . \, g \;= \rho_s \, . \, l^3 \, . g$
which given h $= l \, . \left(1 - \frac{\rho_s}{\rho_m}\right)$

12. (a)
V_1 = total material volume of shell
V_2 = total inside volume of shell and
x = Fraction of V_2 volume filled with water .
In floating condition ,

Total weight = Upthrust

$$\therefore \; V_1 \; \rho_c g \; + (xV_2)\,(1)g = \left(\frac{V_1 + V_2}{2}\right)(1)g$$

Or $\qquad x = 0.5 + (0.5 - \rho_c)\frac{V_1}{V_2}$

we can see that x > 0.5 if $\rho_c < 0.5$

13. (c)
d = 2700 m and $\rho = 10^3 kgm^{-3}$
Compressibility $= 45.4 \times 10^{-11} per\ pascal$

The pressure at the bottom of ocean
P = $\rho g d = 10^3 \times 10 \times 2700 = 27 \times 10^6 Pa$

Frictional compression = compressibility × pressure
$= 45.4 \times 10^{-11} \times 27 \times 10^6 = 1.2 \times 10^{-2}$

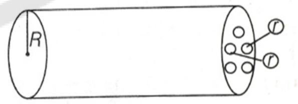
14. (a)
In laminar flow, the velocity of liquid in contact with walls is zero .

15. (a)
From equation of continuity , av = constant
d_A = 2 cm and d_B = 4 cm
∴ r_A = 1 cm and r_B = 2 cm
$$\therefore \; \frac{v_A}{v_B} = \frac{a_B}{a_A} = \frac{\pi(r_B)^2}{\pi(r_A)^2} = \left(\frac{2}{1}\right)^2 \Rightarrow v_A = 4v_B$$

16. (c)
According to equation of continuity of flow,
$a_1 v_1 = a_2 v_2$
$$\therefore \qquad v_2 = \frac{a_1 v_1}{a_2} = \frac{\pi \, r_1^2 \; v_1}{\pi \, r_2^2}$$
$(\because \; a_1 = \pi \, r_1^2$ and $a_2 = \pi \, r_2^2 \;)$
$$= \; v_1 \left(\frac{r_1}{r_2}\right)^2 = 2 \times \left(\frac{6}{3}\right)^2 = 2 \times 2^2 = 8ms^{-1}$$

17. (c)
If the liquid is incompressible , then mass of liquid entering through left end, should be equal to mas of liquid coming out from the right end .
∴ $\quad M = m_1 + m_2 \;\; \Longrightarrow \;\; Av_1 = Av_2 + 1.5Av$
$\Longrightarrow \quad A \times 3 = A \times 1.5 + 1.5 \, Av \;\; \Longrightarrow \;\; v = 1 \, ms^{-1}$

18. (b)
Consider a cylindrical tube of a spray pump of radius R , one end having n fine holes , each of radius r and speed of liquid in the tube is v as shown in figure .



According to equation of continuity , AV = constant where , A is area of the cylindrical tube and v is velocity of liquid in a tube.

Volume in flow rate = Volume in outflow rate
$\pi R^2 v \;= n \, \pi r^2 v' \quad \Longrightarrow v' = \frac{R^2 v}{n \, r^2}$
Speed of the ejection of the liquid through the holes is $\frac{R^2 v}{nr^2}$

19. (c)

Pressure head $= \frac{P}{\rho g}$

h = gravitational head , $\frac{1}{2}v^2/g$ = velocity head

20. (a)

From the Bernoulli's theorem .

$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$

$= \frac{1}{2} \times 1.3 \times [(120)^2 - (90)^2]$

$= 4095\ Nm^{-2}\ or\ Pa$

21. (c)

From Bernoulli's theorem,

$P_a + \frac{1}{2}\rho v_1^2 + \rho g Y_1 = P + \rho g Y_2$

$\Rightarrow\ y_2 - y_1 = h,$

$v_1 = \sqrt{2\,gh + \frac{2(P - P_a)}{\rho}}$

$= \sqrt{2\,gh + \frac{2(\rho g h)}{\rho}} = 2\sqrt{gh}$

22. (b)

$\because\ v = \sqrt{2gh} = 20\,ms^{-1}$

23. (c)

h be the depth of the hole below the free surface of water . According to Torricelli's theorem , the velocity of the efflux v of water through the hole

$v = \sqrt{2gh}$ ------------- (i)

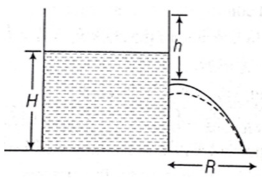
The height through which the water falls is

$s = H - h$

If t is the time taken by water to strike the floor , then

$s = \frac{1}{2}gt^2$ or $(H - h) = \frac{1}{2}gt^2$

$\Rightarrow\ t = \sqrt{\frac{2\,(H-h)}{g}}$ ---------- (ii)



The distance R , where the emerging stream strikes the floor is given by $R = vt$

Substituting for v and t from equs. (i) and (ii)

$R = \sqrt{2g.h} \times \sqrt{\frac{2\,(H-h)}{g}} = 2 \times \sqrt{h\,(H-h)}$

24. (b)

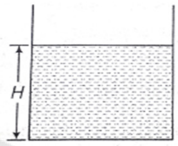
$\rho = 1000\ kgm^{-3}$

$N_1 = 3 \times 10^5 Nm^{-2}$

$P_2 = 1 \times 10^5 Nm^{-2}$

Applying Bernoulli ' s theorem

$p_1 + 0 + \rho g H = p_2 + \frac{1}{2}\rho v^2 + \rho g H$



$p_1 - p_2 = \frac{1}{2}\rho v^2$

$3 \times 10^5 - 1 \times 10^5 = \frac{1}{2}\rho v^2$

$2 \times 10^5 = \frac{1}{2}\rho v^2$

$2 \times 10^5 = \frac{1}{2} \times 10^3 \times v^2$

$v^2 = 400$

$v = \sqrt{400}\ ms^{-1}$

25. (d)

The principle behind this meter has many applications. The carburetor of automobile has a venturi channel (nozzle) through which air flows with a large speed . Filter pumps or aspirators , Bunsen burner , atomiseres and sprayers all work on same principle .

26. (b)

Wider part of the meter cross- sectional area A = $8\,mm^2$ and narrow part , a = $4\,mm^2$ and density of blood $i.e.,\ \rho$ is $1.06 \times 10^3 kgm^{-3}$. The ratio of the areas is $\frac{A}{a} = 2$.

The speed of the blood in the artery ,

$i.e.,\quad v = \sqrt{\frac{2\,\rho_m\,gh}{\rho}}\left[\left(\frac{A}{a}\right)^2 - 1\right]^{1/2}$

$\Rightarrow\quad v = \sqrt{\frac{2 \times 24\,Pa}{1060\,kg - m^{-3} \times (2^2 - 1)}}$

$= 0.125\ ms^{-1}$

27. (d)

From Bernoulli's equation, $p = p_0 + \frac{1}{2}\rho v^2$

Force will act due to pressure difference

$\therefore \quad p - p_0 = \frac{1}{2}\rho v^2 = \frac{1}{2} \times 1.2 \times (40)^2 = 960$

$\therefore$ Force acting upwards,

$F = 960 \times 250 = 2.4 \times 10^5 N$ upwards

28. (c)

The weight of the Boeing aircraft is balanced by the upward force due to the pressure difference .

$\Delta p \times A = 3.3 \times 10^5\ kg \times 9.8 ms^{-2}$

$\Delta p = (3.3 \times 10^5\ kg \times 9.8 ms^{-2})/500 m^2$

$= 6.5 \times 10^3 Nm^{-2}$

We ignore the small height difference between the top and bottom sides

i.e., $p_1 + \left(\frac{1}{2}\right) \rho v_1^2 + \rho g h_1$

$= p_2 + \left(\frac{1}{2}\right) \rho v_2^2 + \rho g h_2$

The pressure difference

$\Delta p = \frac{\rho}{2} [v_2^2 - v_1^2]$

v_2 is the speed of air over the upper surface and v_1 is the speed under the bottom surface .

$v_2 - v_1 = \frac{2 \Delta p}{\rho (v_2 + v_1)}$

Taking the average speed,

$v_{av} = (v_2 + v_1) / 2 = 960\ kmh^{-1}$

$= 267\ ms^{-1}$

$v_2 - v_1 / v_{av} = \frac{\Delta \rho}{\rho v_{av}^2} \approx 0.08$

The air speed above the wing needs to be only 8 % height than that below.

29. (a)

30. (a)

Most viscous fluid comes to rest quickly due to dissipation of energy at a larger rate .

31. (b)

The metal block moves to the right , because of the tension in the string .

The tension T is equal to the magnitude of the weight of the suspended mass m .

Thus the shear force

$F = T = mg = 0.010$ kg $\times\ 9.8\ ms^{-2}$

$= 9.8 \times 10^{-2} N$

Shear stress on the fluid = F /A = $\frac{9.8 \times 10^{-2}}{0.10}$

Strain rate = $\frac{v}{l} = \frac{0.085}{0.30 \times 10^{-3}}$ $\Longrightarrow \eta = \frac{Stress}{Strain\ rate}$

$= \frac{(9.8 \times 10^{-2} N)(0.30 \times 10^{-3}\ m)}{(0.085\ ms^{-1})(0.10\ m^2)}$

$= 3.45 \times 10^{-3} pa - s$

32. (d)

Final velocity is terminal velocity , it does not depend on the height of fall.

33. (a)

In equilibrium , this terminal velocity

$v_t = 6 \pi \eta a\ v_t = (4\pi/3) a^3 (\rho - \sigma) g$

ρ and σ are mass densities of sphere and the fluid, respectively .

$v_t = 2a^2(\rho - \sigma) g / (9\eta)$

34. (b)

$v_t = 6.5 \times 10^{-2} ms^{-1}$, a $= 2 \times 10^{-3} m$

g = $9.8\ ms^{-2}$, $\rho = 8.9 \times 10^3 kgm^{-3}$

$\sigma = 1.5 \times 10^3 kgm^{-3}$

Terminal velocity , $v_t = \frac{2 a^2 (\rho - \sigma) g}{9 \eta}$

$\eta = \frac{2}{9} \times \frac{(2 \times 10^{-3})^2\ m^2 \times 9.8\ ms^{-2}}{6.5 \times 10^{-2}\ ms^{-1}} \times 7.4 \times 10^3 kgm^{-3}$

$= 9.9 \times 10^{-1} kg\ ms^{-1}$

35. (b)

$v_1 = 10\ cms^{-1}$, $\rho_{01} = 1\ gcm^2$

$\rho_{02} = 1.2\ gcm^{-3}$, $\eta_1 = 8.5 \times 10^{-14} Pas$

Terminal velocity , v i.e., v $\propto \frac{\rho - \rho_0}{\eta}$

$\frac{v_2}{v_1} = \frac{\rho - \rho_{02}}{\rho - \rho_{01}} \times \frac{\eta_1}{\eta_2}$

$v_2 = \frac{7.8 - 1.2}{7.8 - 1} \times \frac{8.5 \times 10^4 \times 10}{13.2}$

$= 6.25 \times 10^{-4} cms^{-1}$

36. (d)

Velocity becomes constant as it reaches terminal value.

37. (b)

Weight of the body hang from wire (mg) = upward force due to surface tension $(2Tl)$

$\Longrightarrow \quad m = \frac{2\,Tl}{g}$

38. (d)

Force on one side of the stick $F_1 = T_1 \times L$
$= 0.07 \times 2 = 0.14\ N$

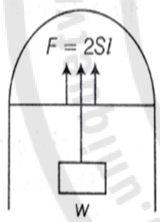
And force on other side of the stick
$F_2 = T_2 \times L = 0.06 \times 2 = 0.12\ N$

Net force on the stick $= F_1 - F_2$
$= 0.14 - 0.12 = 0.02$ N

39. (a)
Force required to separate the plates,
$F = \frac{2\,TA}{t} = \frac{2 \times 70 \times 10^{-3} \times 10^{-2}}{0.05 \times 10^{-3}} = 28\ N$

40. (a)
$T = \frac{F}{2l} = \frac{2 \times 10^{-2}}{2 \times 10 \times 10^{2}} = 0.1\ Nm^{-1}$

41. (d)
$W = 1.5 \times 10^{-2} N$
$l = 30$ cm $= 3 \times 10^{-2} m$
A liquid film has two free surfaces A solider will support the weight when the force of surface tension acting upwards on the slider (=2Sl) balance upward force to weight (W)



$\therefore\ 2Sl = W \implies S = W/2l \implies \frac{1.5 \times 10^{-2}}{2 \times 30 \times 10^{-2}}$

$\therefore$ S = 0.025 Nm^{-1}

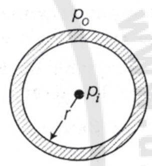
42. (b)
It is given that water rises to a height h in capillary tube .

So, the length of capillary tube above the surface water is made less than h , then height of water column > length of capillary tube .
So , liquid will stay there.

43. (c)
Pressure inside the bubble is
$p_i = p_0 + 2S/r$
$= 1.01000 \times 10^5 Pa + (2 \times 7.3 \times 10^{-2} Pa\, m / 10^{-3} m)$
$= (1.01000 + 0.00146) \times 10^5 Pa$
$= 1.01146 \times 10^5 Pa$

44. (a)
If the drop is in equilibrium , this energy lost is balanced by the energy gain due to expansion under the pressure difference $((p_i - p_0)$ between the inside of the bubble and the outside . The work done is
$W = (p_i - p_0)\, 4\pi r^2 \Delta r$
$(p_i - p_0) = (2\, S_{la}/r)$

45. (b)
A bubble as shown in figure , differs from a drop and a cavity in this it has two interface . For a bubble
$(p_i - p_0) = 4\, S_{la} / r$



46. (b)
$R = n^{1/3}\, r = 2^{1/3} r$
$\implies R^2 = 2^{2/3}\, r^2$
$\implies \frac{r^2}{R^2} = 2^{2/3}$
$\frac{Initial\ surface\ energy}{Final\ surface\ energy} = \frac{2(4\pi r^2 T)}{(4\pi r^2 T)} = 2\left(\frac{r^2}{R^2}\right)$
$= 2 \times 2^{-2/3} = 2^{1/3}$

47. (a)
W $= 8\pi T(R_2^2 - R_1^2) = 8\pi S\,[(2R)^2 - (R^2)]$
$= 24\,\pi R^2 S$

48. (d)
Surface area of bubble of radius , r = $4\pi R^2$
Surface area of bubble of radius
$2\,r = 4\,\pi\,(2r)^2 = 16\,\pi\, r^2$
Increase in surface area
$= (16\,\pi\, r^2 - 4\pi\, r^2)$
A bubble has two surfaces , the total increase in surface area $= 24\pi\, r^2$.
$\therefore$ Energy spent = Work done = $24\pi\, \sigma r^2$

49. (c)

If p_0 is the atmospheric pressure, the pressure outside the air bubble when it is at a depth $h = p_0 + h\rho g$. Therefore, the total pressure inside the air bubble is

$P = p + p_0 + h\rho g$

$= \frac{2\sigma}{r} + p_0 + h\rho g$

50. (b)

Excess pressure inside a bubble just below the surface of water, $p_1 = \frac{2T}{r}$

And excess pressure inside a drop, $p_2 = \frac{2T}{r}$

$\therefore \quad p_1 = p_2$

51. (b)

As excess pressure inside the soap bubble radius r, is

P = $\frac{4S}{r}$ or p $\propto \frac{1}{r}$

Therefore, as r increases with time, p decreases.

52. (b)

The excess pressure in a bubble of gas in a liquid is given by 2S/r,

There is only one liquid surface in this case.

Pressure outside the bubble,

P_0 = Atmospheric pressure + pressure due to 8cm of water column

$\therefore P_0 = (1.01 \times 10^5 Pa + 0.08 \times 1000 \times 9.80)$

$= 1.01784 \times 10^5 Pa$

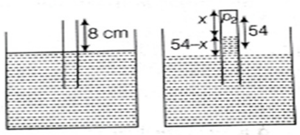
53. (d)

Soap solution has lower surface tension, T as compared to pure water and capillary rise in tube

i.e., h = $\frac{2T\cos\theta}{\rho r g}$, so h is less for soap solution

54. (a)

pressure in the same level will be same.



Air trapped in tube, $p_1V_1 = p_2V_2$

$p_1 = p_{atm} = \rho g\, 76$

$V_1 = A.8$ (where, A = area of cross- section)

$p_2 = p_{atm} - \rho g\,(54 - x) = \rho g\,(22 + x)$

$V_2 = Ax$

$\rho g\, 76 \times 8A = \rho g\ (22 + x)\, Ax$

$x^2 + 22x - 76 \times 8 = 0 \implies x = 16$ cm

55. (a)

$\because \quad V_i = \frac{V}{n}$

$r_i = \frac{R}{n^{\frac{1}{3}}} \implies n^{\frac{1}{3}} = \frac{R}{r}$

$\Delta U = U_f - U_i = T4\pi(R^2 - nr^2)$

T is the surface tension of the liquid.

$= T4\pi R^2\left(1 - \frac{n}{n^{\frac{2}{3}}}\right) = T4\pi R^2\left(1 - \frac{R}{r}\right)$

$= T4\pi R^3\left(\frac{1}{R} - \frac{1}{r}\right) = T\,3\left(\frac{4}{3}\pi R^3\right)\left(\frac{1}{R} - \frac{1}{r}\right)$

$= 3VT\left(\frac{1}{R} - \frac{1}{r}\right)$

ΔU is negative, so energy released.

$= 3VT\left(\frac{1}{r} - \frac{1}{R}\right)$

56. (c)

$h \propto \frac{1}{r}$

$\therefore \frac{h_1}{h_2} = \frac{r_2}{r_1} \quad or \ \frac{r_1}{r_2} = \frac{h_2}{h_1} = \frac{6.6}{2.2} = \frac{3}{1}$

57. (b)

When the tube is placed vertically in water rises through height h given by h = $\frac{2T\cos\theta}{rdg}$

Upward force $= 2\pi r \times T\cos\theta$

Work done by this force in raising water column through height h is

$\Delta W = (2\pi r T\cos\theta)h = (2\pi r h\cos\theta)T$

$(2\pi r h\cos\theta)\left(\frac{rhdg}{2\cos\theta}\right)\pi r^2h^2dg$

The increase in potential energy ΔE_p of the raised water column = mg $\frac{h}{2}$

m is the mass of the raised column of water

$\therefore \quad m = \pi r^2 hd$

So, $\quad \Delta E_p = (\pi r^2 hd)\left(\frac{hg}{2}\right)$

$= \frac{\pi r^2h^2dg}{2}$

$\Delta W = \Delta E_p = \frac{\pi r^2h^2dg}{2}$