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KINETIC THEORY

• Postulates of Kinetic theory

- ❖ The molecules of a gas are in random motion. They collide with each other and also with the walls of the container.
- ❖ Dimension of a molecule is negligible compared to the average distance between the molecules.
- ❖ The molecules are assumed to be perfect spheres.
- ❖ There is no intermolecular attraction or repulsion among molecules or between the molecules and the walls of the container.
- ❖ The internal energy of gas is the total kinetic energy of all molecules.
- ❖ The collision among molecules or between molecules and the walls of container are elastic.
- ❖ The time of collision is small compared to the average time between two successive collisions.

the mass of the molecule.

$$\frac{1}{2} mN\bar{v}^2 = \frac{3}{2} RT$$

$\frac{1}{2} m\bar{v}^2 = \frac{3}{2} \frac{RT}{N} = \frac{3}{2} k_B T$ where k_B is called Boltzmann constant.

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

• Degree of freedom

It indicates the minimum number of independent co-ordinates required to specify the position of a particle.

Number of degrees of freedom = $3N - k$ where k is the number of independent relations between particles and N is the number of particles in the system.

A rigid body has 6 degrees of freedom – 3 translational and 3 rotational.

For mono atomic molecule $N = 1$ $k = 0$
 $f = 3 \times 1 - 0 = 3$

For diatomic molecule $N = 2$, $k = 1$
 $\therefore f = 3 \times 2 - 1 = 5$

For a non linear tri atomic molecule $N = 3$, $k = 3$
 $\therefore f = 3 \times 3 - 3 = 6$

For a linear tri atomic molecule $N = 3$, $k = 2$
 $\therefore f = 3 \times 3 - 2 = 7$

• Avogadro's hypothesis

Equal volume of all gases under the same conditions of temperature and pressure contains the same number of molecules.

• Root mean square velocity (rms velocity)

Root mean square velocity is the square root of mean of square of velocities of gas molecules.

• Pressure exerted by a gas

$P = \frac{1}{3} \rho \bar{v}^2$ where \bar{v}^2 is the mean square velocity and ρ is density

• Relation connecting velocity and temperature

$$\bar{v} = \sqrt{\frac{3RT}{M}}$$

• Expression for mean kinetic energy of a molecule

$$\frac{1}{2} M\bar{v}^2 = \frac{3}{2} RT$$

Here M (molecular weight) = Nm where m is

- Avogadro number = 6.023×10^{23} .

- **Law of equipartition of energy**

If a system is in equilibrium at absolute temperature (T), the total energy is equally distributed in different energy modes of absorption; the energy in each mode is equal to $\frac{1}{2} k_B T$ where k_B is Boltzmann constant.

Each translational and rotational degree of freedom corresponds to one energy mode of absorption and has energy $\frac{1}{2} k_B T$.

Each vibrational degree of freedom corresponds to two modes of energy.

$$\text{Energy} = k_B T$$

- **Specific heat capacity of mono atomic gas**

$$C_V = \frac{3}{2} R \quad C_P = \frac{5}{2} R \quad \gamma = 1.66$$

- **Specific heat capacity of a diatomic gas**

$$C_V = \frac{5}{2} R \quad C_P = \frac{7}{2} R \quad \gamma = 1.4$$

Note : If the diatomic molecule is not rigid, but has an additional vibrational mode, then

$$C_V = \frac{7}{2} R \quad C_P = \frac{9}{2} R \quad \gamma = \frac{9}{7}$$

- **Specific heats of tri atomic gas**

For a non-linear tri atomic gas (like H_2O , SO_2 etc)

$$C_V = 3R$$

$$C_P = 4R \quad \therefore \gamma = 1.33$$

For a linear tri atomic molecule (like CO_2 , CS_2 etc)

$$C_V = \frac{dU}{dT} = \frac{7}{2} R$$

$$C_P = \frac{9}{2} R$$

$$\therefore \gamma = 1.28$$

- **Specific heats of a polyatomic gas**

A polyatomic molecule has 3 translational, 3 rotational and a certain number (f) of vibrational degrees of freedom.

$$C_V = (3 + f) R \quad C_P = (4 + f) R$$

$$\gamma = \frac{4 + f}{3 + f}$$

- **22.4 litre** of any gas at standard temperature 273 K and pressure 1 atm contains 6.02×10^{23} molecules.

- The mass of 22.4 litre of any gas is equal to its molecular weight in grams at S.T.P. This amount of substance is called a mole.

- Number of moles $\mu = \frac{M}{M_0} = \frac{N}{N_A}$ where M is the mass of the gas containing N molecules, M_0 is the molar mass and N_A is Avogadro's number.

- At low pressure or high temperature the molecules are far apart, molecular interactions are negligible. Without interactions a gas behaves like an ideal gas.

- **Dalton's law of partial pressure**

Consider a mixture of non-interacting ideal gases: μ_1 moles of gas 1, μ_2 moles of gas 2 etc. in a vessel of volume V at temperature T and pressure P.

The equation of state of mixture is

$PV = (\mu_1 + \mu_2 + \dots) RT$ OR $P = \frac{\mu_1 RT}{V} + \frac{\mu_2 RT}{V} + \dots = P_1 + P_2 + \dots$ where P_1 is the pressure exerted by gas 1 at the same conditions of volume and temperature in the absence of other gases. It is called partial pressure of the gas.

Total pressure of a mixture of ideal gases is the sum of partial pressures.

- **Expression for mean free path**

Mean free path, $\lambda =$

$$\frac{\text{Distance covered by a molecule in } \Delta t \text{ time}}{\text{Number of collisions in } \Delta t \text{ time}} = \frac{1}{n\pi d^2}$$

where \bar{v} is average velocity, d is the diameter of a molecule and n is the number of molecules per unit volume, P is pressure, k_B is Boltzmann's constant [Here the assumption is that the molecule taken alone is in motion]

If the motion of other molecules are taken into consideration,

$$\lambda = \frac{1}{\sqrt{2}n\pi d^2} = \frac{k_B T}{\sqrt{2}P\pi d^2}$$

• **Gay Lussac's or Regnault's law**

At constant volume the pressure P of a given mass of gas is directly proportional to its absolute temperature T , i.e.

$$P \propto T \quad \text{or} \quad \frac{P}{T} \text{ is constant}$$

Multiple choice Questions:
Class Work

1. According to Gay Lussac's law, when gases combine chemically to yield another gases, their volumes are in the ratios of small

- (a) fractions (b) integers
(c) real numbers (d) irrational numbers

2. If the distance between the molecules of a substance is 10^{-9} m, it is in the

- (a) liquid state (b) solid state
(c) gaseous state (d) solid and gaseous state

3. Relation $pV = KT$, for a sample of a gas is valid at (here, $p \rightarrow$ Pressure $V \rightarrow$ volume, $T \rightarrow$ Temperature and k is a constant for the given sample)

- (a) low pressure and low temperature
(b) low pressure and high temperature
(c) high pressure and low temperature
(d) high pressure and high temperature

4. In perfect gas equation $pV = \mu RT$, R is equal to

- (a) $N_A^2 k_B^2$ (b) $N_A k_B$
(c) $\frac{N_A k_B}{2}$ (d) None of these

5. $pV = \mu RT$ can be rewritten as

- (a) $pV = k_B NT$ ($N \rightarrow$ total number of molecules in the perfect gas)
(b) $p = k_B nT$ ($n \rightarrow$ number density)
(c) $p = \rho RT/M_0$ ($\rho \rightarrow$ mass density and $M_0 \rightarrow$ molar mass)
(d) All of the above

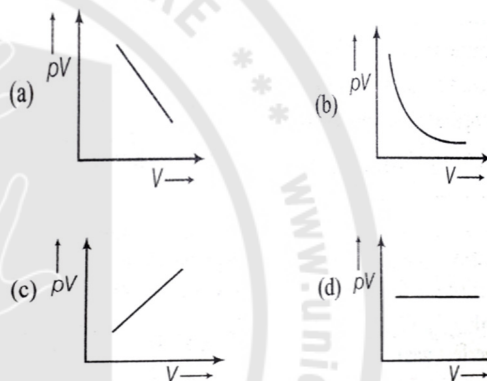
6. A vessel contains 32 g of O_2 at a temperature . The pressure of the gas is p . An identical vessel containing 4 g of H_2 at a temperature $2T$ has a pressure of

- (a) $8p$ (b) $4p$ (c) p (d) $\frac{p}{8}$

7. Boyle's law is good at

- (a) high pressure and low temperature
(b) high temperature and low pressure
(c) only high temperature
(d) only low pressure

8. Which one of the following graphs represents the behavior of an ideal gas?



9. The equation of state for 5 g of oxygen at a pressure p and temperature T , when occupying a volume V , will be

- (a) $pV = (5/32)RT$ (b) $pV = 5RT$
(c) $pV = (5/2)RT$ (d) $pV = (5/16)RT$

10. Temperature remaining constant, the pressure of gas is decreased 20%. The percentage change in volume

- (a) increased by 20%
(b) decreases by 20%
(c) increases by 25%
(d) decreases by 25%

11. The ratio of the molecular masses of two gases is $7/8$. What is the ratio of the number of moles of two gases, if they are two different containers and have same mass for each gas?

- (a) $8/7$ (b) $7/8$ (c) $7/5$ (d) $5/7$

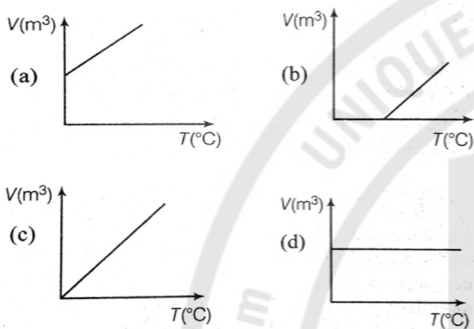
12. A perfect gas at 27°C is heated at constant pressure so as to double its volume. The increase in temperature of the gas will be

- (a) 300°C (b) 54°C (c) 327°C (d) 600°C

13. The tyre of a motorcar contains air at 15°C . If the temperature increases to 35°C , the approximate percentage increase in pressure is (ignore to expansion of tyre)

- (a) 7 (b) 9 (c) 11 (d) 13

14. Volume - temperature graph at atmosphere for a monoatomic gas (V in m^3 , T in $^{\circ}\text{C}$) is



15. A monoatomic gas at a pressure p , having a volume V expands isothermally to a volume $2V$ and then adiabatically to a volume $16V$. The final pressure of the gas is (Take, $\gamma=5/3$)

- (a) $p/64$ (b) $16p$ (c) $64p$ (d) $32p$

16. In collisions of a molecule of an ideal gas with wall of container,

- (a) only total KE remains conserved
 (b) only total momentum remains conserved
 (c) total KE and total momentum remains conserved
 (d) Neither total KE nor total momentum is conserved

17. The internal energy of an ideal gas in the form of

- (a) kinetic energy of molecules
 (b) potential energy of molecules
 (c) Both kinetic and potential energy of molecules
 (d) gravitational potential energy of molecules

18. The internal energy of an ideal gas depends on

- (a) pressure (b) volume
 (c) mass (d) temperature

19. Pressure of an ideal gas is increased by keeping temperature constant. What is the effect on kinetic energy of molecules?

- (a) Increase (b) Decrease
 (c) No change (d) Cannot be determined

20. if 1 mole of a diatomic gas is at a pressure of $8 \times 10^4 \text{Nm}^{-2}$. The density of the gas is 4kgm^{-3} . What is the energy of the gas due to its thermal motion?

- (a) $3 \times 10^4 \text{J}$ (b) $5 \times 10^4 \text{J}$
 (c) $6 \times 10^4 \text{J}$ (d) $7 \times 10^4 \text{J}$

21. The average translation kinetic energy of the molecule of a gas is

- (a) $\frac{3}{2} k_B T$ (b) $k_B T$ (c) $\frac{1}{2} k_B T$ (d) $\frac{2}{3} k_B T$

22. Oxygen and hydrogen are at the same temperature T . The ratio of the mean kinetic energy of oxygen molecules to that of the hydrogen molecules will be

- (a) 16:1 (b) 1:1 (c) 4:1 (d) 1:4

23. At what temperature the kinetic energy of gas molecule is half of the value at 27°C ?

- (a) 13.5°C (b) 150°C (c) 75K (d) -123°C

24. At constant pressure, which of the following is true for an ideal gas? (where, ρ = density of the gas and v = rms speed of the gas)

- (a) $v \propto \sqrt{\rho}$ (b) $v \propto \frac{1}{\rho}$
 (c) $v \propto \rho$ (d) $v \propto \frac{1}{\sqrt{\rho}}$

25. An ideal gas ($\gamma = 1.5$) is expanded adiabatically. How many times the gas has to be expanded to reduce the root mean square velocity of the molecules 2.0 times?

- (a) 4 times (b) 16 times
 (c) 8 times (d) 2 times

26. The rms velocity of a gas molecules is 300ms^{-1} . The rms velocity of the molecules of gas with twice the molecular weight and half the absolute temperature is

- (a) 300 ms^{-1} (b) 600 ms^{-1}
(c) 75 ms^{-1} (d) 150 ms^{-1}
27. Two vessels separately contains two ideal gases A and B at the same temperature, the pressure of A being twice that of B . Under such conditions, the density of A is found to be 1.5 times the density of B . The ratio of molecular weight of A and B is
- (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) 2 (d) $\frac{1}{2}$
28. The molecules of a given mass of a gas have rms velocity of 200 ms^{-1} at 27°C and $1.0 \times 10^5 \text{ Nm}^{-2}$ pressure. When the temperature and pressure of the gas are respectively 127°C and $0.05 \times 10^5 \text{ Nm}^{-2}$, the rms velocity of its molecules in ms^{-1} is
- (a) $\frac{400}{\sqrt{3}}$ (b) $\frac{100\sqrt{2}}{3}$ (c) $\frac{100}{3}$ (d) $100\sqrt{2}$
29. The two gases with the ratio of 3 : 2 of their masses in a container are at a temperature T . The ratio of the kinetic energies of the molecule of two gases is
- (a) 3 : 2 (b) 9 : 4 (c) 1 : 1 (d) 4 : 9
30. Molecules of CO at moderate temperature have energy .
- (a) $\frac{7}{2} k_B T$ (b) $\frac{5}{2} k_B T$
(c) $\frac{3}{2} k_B T$ (d) $\frac{1}{2} k_B T$
31. Law of equipartition of energy is used to
- (a) predict the specific heats of gases.
(b) predict the specific heats of solid.
(c) Both (a) and (b)
(d) Neither (a) or (b)
32. N_2 molecule (a rigid rotator) has total
- (a) 3 degree of freedom
(b) 2 degree of freedom
(c) 5 degree of freedom
(d) 1 degree of freedom
33. The amount of heat energy required to raise the temperature of 1 g of helium at NTP from $T_1 \text{ K}$ to $T_2 \text{ K}$ is
- (a) $\frac{3}{8} N_A k_B (T_2 - T_1)$ (b) $\frac{3}{2} N_A k_B (T_2 - T_1)$
(c) $\frac{3}{4} N_A k_B (T_2 - T_1)$ (d) $\frac{3}{4} N_A k_B \left(\frac{T_2}{T_1}\right)$
34. The internal energy of 2 moles of a monoatomic gas is
- (a) $\frac{3}{2} RT$ (b) $3 RT$ (c) $2 RT$ (d) $5 RT$
35. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its temperature. The ratio of $\frac{C_p}{C_v}$ for the gas is
- (a) $\frac{4}{3}$ (b) 2 (c) $\frac{5}{3}$ (d) $\frac{3}{2}$
36. The total energy for one mole of solid is
- (a) $2 RT$ (b) $3 RT$ (c) $4 RT$ (d) $3/2 RT$
37. The ratio of the molar heat capacities of a diatomic gas at constant pressure to that at constant volume is
- (a) $\frac{7}{2}$ (b) $\frac{3}{2}$ (c) $\frac{3}{5}$ (d) $\frac{7}{5}$
38. Consider an ideal gas confined in an isolated closed chamber. As gas undergoes an adiabatic expansion , the average time of collision between molecules increases as V^q , where V is the volume of the gas .The value of q is $\left(\gamma = \frac{C_p}{C_v}\right)$
- (a) $\frac{3\gamma+5}{6}$ (b) $\frac{3\gamma-5}{6}$ (c) $\frac{\gamma+1}{6}$ (d) $\frac{\gamma-1}{6}$
39. Six moles of O_2 gas heated from 20°C to 35°C at constant volume . If specific heat capacity at constant pressure is $8 \text{ cal mol}^{-1}\text{K}^{-1}$ and $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$, then what is change in internal energy of gas ?
- (a) 180 cal (b) 300 cal
(c) 360 cal (d) 540 cal
40. A gas mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature T . Neglecting all vibrational modes, calculate the total internal energy of the system(oxygen has two rational modes)
- (a) $10 RT$ (b) $11 RT$ (c) $5 RT$ (d) $(3/2)RT$
41. If γ is the ratio of specific heat and R is the universal gas constant , then the molar specific heat at constant volume C_v is given by
- (a) γR (b) $\frac{(\gamma-1)R}{\gamma}$ (c) $\frac{R}{\gamma-1}$ (d) $\frac{\gamma R}{\gamma-1}$

42. If universal gas constant is R , the essential heat to increase the temperature from 273 K to 473 K at constant volume for a monoatomic ideal gas of 4 mole is

- (a) $200 R$ (b) $400 R$ (c) $800 R$ (d) $1200 R$

43. The gases carbon monoxide (CO) and Nitrogen (N_2) at the same temperature have kinetic energies E_1 and E_2 , respectively. Then,

- (a) $E_1 = E_2$
 (b) $E_1 > E_2$
 (c) $E_1 < E_2$
 (d) E_1 and E_2 cannot be compared

$$n_2 = \frac{m_2}{M_2} = \frac{4}{2} = 2$$

Volume = V , Temperature = $2T$

$$\therefore p_2 V = 2R(2T) \quad \text{----- (ii)}$$

From Eq (i) and (ii)

$$p_2 = 4p_1 = 4p$$

7. (b)

8. (d)

$$pV = \text{constant}$$

Hence, for a given mass, the graph between pV and V will be straight line parallel to V -axis whatever may be the volume.

9. (a)

$$\text{Number of moles } n = \frac{m}{\text{molecular weight}} = \frac{5}{32}$$

So, from ideal gas equation

$$pV = nRT \Rightarrow pV = \frac{5}{32} RT$$

10. (c)

According to Boyle's law

$$p_1 V_1 = p_2 V_2$$

As the pressure is decreased by 20%

$$p_2 = \frac{80}{100} p_1$$

$$p_1 V_1 = \frac{80}{100} p_1 V_2$$

$$\Rightarrow V_1 = \frac{80}{100} V_2$$

$$\text{Percentage increase in volume} = \frac{V_2 - V_1}{V_1} \times 100$$

$$= \frac{100 - 80}{80} \times 100 = 25\%$$

11. (a)

$$\mu = \frac{M}{M_0} = \frac{\text{Total mass}}{\text{molecular mass}}$$

$$\frac{\mu_1}{\mu_2} = \frac{M/(M_0)_1}{M/(M_0)_2} = \frac{(M_0)_2}{(M_0)_1} = \frac{8}{7}$$

12. (a)

According to Charles law, $\frac{V_1}{V_2} = \frac{T_1}{T_2}$

$$T_1 = 27^\circ C + 273 = 300K$$

$$\therefore \frac{1}{2} = \frac{300K}{T_2}$$

$$\Rightarrow T_2 = 600K = 600 - 273 = 327^\circ C$$

$$\Rightarrow \Delta T = 327 - 27 = 300^\circ C$$

13. (a)

$$\text{Temperature, } T_1 \Rightarrow 15^\circ C \Rightarrow 15 + 273 = 288K$$

$$T_2 \Rightarrow 35^\circ C \Rightarrow 35 + 273 = 308K$$

Volume remains constant.

$$\therefore \frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow \frac{p_1}{p_2} = \frac{T_1}{T_2} \Rightarrow \frac{p_1}{p_2} = \frac{288}{308}$$

$$\therefore \frac{p_2}{p_1} = \frac{308}{288}$$

HINTS AND EXPLANATIONS

1. (b)

2. (c)

For gases the interatomic distance is $10^{-9}m$.

3. (b)

$PV = kT$ for a sample of gas is valid at low pressure and high temperature.

4. (b)

From perfect gas equation

$$pV = k_B N T = k_B (N_A \times \mu) T$$

$$= \mu (k_B N_A) T \quad \text{-----(i)}$$

$$pV = \mu RT \quad \text{-----(ii)}$$

$$R = k_B N_A \rightarrow \text{Universal gas constant}$$

5. (d)

Ideal gas equation can be written in different forms

$$pV = \mu RT \quad \text{----- (i)}$$

$$pV = k_B N T \quad \text{----- (ii)}$$

$$p = k_B n T \quad \text{----- (iii)}$$

$$(\because n = N/V)$$

$$pV = \mu RT \quad \left(\mu = \frac{M}{M_0} = \frac{N}{N_A} \right)$$

$$\Rightarrow p = \frac{M}{M_0} \frac{RT}{V}$$

$$\therefore p = \frac{\rho RT}{M_0}$$

6. (b)

For first vessel, number of moles

$$n_1 = \frac{m_1}{M_1} = \frac{32}{32} = 1$$

Volume V , Temperature = T

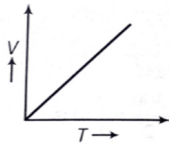
$$\therefore p_1 V = RT \quad \text{----- (i)}$$

For second vessel, number of moles

$$\text{Percentage increase} = \frac{p_2 - p_1}{p_1} \times 100 = 7\%$$

14. (c)

At constant pressure, the volume of given mass of gas is directly proportional to its absolute temperature (T).



i.e., $\frac{V}{T} = \text{constant}$

15. (a)

For isothermal expansion $p_2 V_2 = p_1 V_1$

$$\Rightarrow p_2 = \frac{p_1 V_1}{V_2} = \frac{p_1}{2}$$

For adiabatic expansion

$$\Rightarrow p_2 V_2^\gamma = p_3 V_3^\gamma$$

$$\Rightarrow \frac{p_1}{2} (2V)^\gamma = p_3 (16V)^\gamma$$

$$\Rightarrow p_3 = \frac{p_1}{2} \left(\frac{2V}{16V} \right)^\gamma = \frac{p_1}{2} \left(\frac{1}{8} \right)^\gamma = \frac{p_1}{2(32)} = \frac{p_1}{64}$$

16. (c)

17. (a)

18. (d)

Internal energy of an ideal gas depends only on the temperature of the gas.

19. (c)

Kinetic energy of an ideal gas depends only on its temperature. Hence, it remains constant whether its pressure is increased or decreased.

20. (b)

Thermal energy corresponds to internal energy

$$\text{Mass} = 1\text{kg}, \text{Density} = 4\text{kgm}^{-3}$$

$$\therefore \text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{1}{4}\text{m}^3$$

$$\text{Pressure} = 8 \times 10^4 \text{Nm}^{-2}$$

$$\therefore \text{Internal energy} = \frac{5}{2} p \times V = 5 \times 10^4 \text{J}$$

21. (a)

Pressure of gas

$$p = \frac{1}{3} nm \overline{v^2}$$

$$pV = \frac{1}{3} nm V \overline{v^2} = \frac{2}{3} (nV) \frac{1}{2} m \overline{v^2}$$

$$= \frac{2}{3} N \left(\frac{1}{2} m \overline{v^2} \right), (N = nV)$$

Or, $pV = \frac{2}{3} E$ ----- (i)

$$\left(\begin{aligned} E &= \text{total internal energy of the gas} \\ &= N \frac{1}{2} m \overline{v^2} \end{aligned} \right)$$

ideal gas equation

$$pV = k_B NT \quad \text{----- (ii)}$$

$$\begin{aligned} \frac{2}{3} E &= k_B NT \Rightarrow \frac{E}{N} = \frac{3}{2} k_B T \Rightarrow \frac{1}{2} m \overline{v^2} \\ &= \frac{3}{2} k_B T \end{aligned}$$

22. (b)

The mean kinetic energy for gas molecules

$$E = \frac{3}{2} k_B T \Rightarrow E \propto T$$

$$\therefore \frac{E_1}{E_2} = \frac{T_1}{T_2}$$

According to the question, both gases are the same temperature T.

$$\frac{E_1}{E_2} = \frac{T_1}{T_2} = \frac{1}{1} \Rightarrow E_1 : E_2 = 1 : 1$$

23. (d)

Kinetic energy of a gas molecule, $E = \frac{3}{2} kT$ where, k is Boltzmann's constant.

$$\therefore E \propto T$$

$$\text{or } \frac{E_1}{E_2} = \frac{T_1}{T_2}$$

$$\text{or } \frac{E}{(E/2)} = \frac{300}{T_2}$$

$$\therefore T_2 = 150 \text{K} = 150 - 273 = -123^\circ \text{C}$$

24. (d)

Pressure due to an gas is given by

$$pV = \frac{1}{2} mN \overline{v^2} \Rightarrow p = \frac{M}{3V} \overline{v^2}$$

$$\left(\begin{aligned} MN &= M \\ (N &= \text{Total number of gas molecules}) \end{aligned} \right)$$

Putting $\frac{M}{V} = \rho$, the density of gas

$$p = \frac{1}{3} \rho \overline{v^2} \Rightarrow v = \sqrt{\left(\frac{3p}{\rho} \right)}$$

$$\therefore v \propto \frac{1}{\sqrt{\rho}}$$

25. (b)

$$v_{rms} = \sqrt{\frac{3RT}{M}} \quad \text{or } v_{rms} \propto \sqrt{T}$$

v_{rms} is to reduce 2 times,

i.e., The temperature of the gas will have to reduce four times or

$$\frac{T'}{T} = \frac{1}{4}$$

During adiabatic process,

$$TV^{\gamma-1} = T'V'^{\gamma-1} \text{ or } \frac{V'}{V} = \left(\frac{T}{T'}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{V'}{V} = (4)^{\frac{1}{1.5-1}} = 4^2 = 16$$

$$\therefore V' = 16V$$

26. (d)

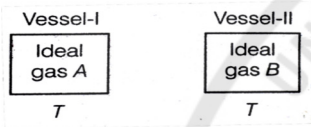
Root means square velocity,

$$v_{rms} = \sqrt{\frac{3pV}{M}} = \sqrt{\frac{3RT}{M}}$$

$$\text{New } v_{rms} = \sqrt{\frac{3R(T/2)}{2M}} = \frac{1}{2} \sqrt{\frac{3RT}{M}}$$

$$= \frac{v_{rms}}{2} = \frac{300}{2} = 150 \text{ ms}^{-1}$$

27. (b)



$$\rho_A = 1.5 \rho_B$$

$$P_A = 2 P_B$$

$$\rho_B$$

$$P_B$$

According to ideal gas equation

Pressure, $p = \frac{\rho RT}{M}$, where M is molecular weight of ideal gas

$$\frac{p}{\rho} = \frac{RT}{M} \Rightarrow M = \frac{\rho RT}{p}$$

R and T are constants.

$$M \propto \frac{\rho}{p} \Rightarrow \frac{M_A}{M_B} = \frac{\rho_B}{\rho_A} \times \frac{p_A}{p_B} = 1.5 \times \frac{1}{2} = 0.75 = \frac{3}{4}$$

28. (a)

$$v_{rms} = 200 \text{ ms}^{-1}, T_1 = 300 \text{ K}, p_1 = 10^5 \text{ Nm}^{-2}$$

$$T_2 = 400 \text{ K}, p_2 = 0.05 \times 10^5 \text{ Nm}^{-2}$$

rms Velocity of gas molecule .

$$\therefore v_{rms} \propto \sqrt{T} \quad \left(\because v_{rms} = \sqrt{\frac{3RT}{m}} \right)$$

For two different cases

$$\Rightarrow \frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{200}{(v_{rms})_2} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$$

$$\Rightarrow (v_{rms})_2 = \frac{2}{\sqrt{3}} \times 200 = \frac{400}{\sqrt{3}} \text{ ms}^{-1}$$

29. (c)

Average KE of a molecule does not depend on the molecular mass and total mass of the gas.

It depends only on the temperature of the gas

$$\frac{(KE)_1}{(KE)_2} = \frac{1}{1}$$

30. (a)

CO at moderate temperature possess vibrational modes as well and oscillate as one dimensional oscillator.

$$\text{Vibrational energy} = \frac{1}{2}mv^2 + \frac{1}{2}kr^2$$

Each vibrational mode contributes two square terms, mode will contribute $2 \left(\frac{1}{2} k_B T \right)$ energy.

$$E = \underbrace{3 \left(\frac{1}{2} k_B T \right)}_{\text{Translational mode}} + \underbrace{2 \left(\frac{1}{2} k_B T \right)}_{\text{Rotational mode}} + \underbrace{2 \left(\frac{1}{2} k_B T \right)}_{\text{Vibrational mode}}$$

$$= \frac{7}{2} k_B T$$

31. (c)

Law of equipartition of energy is used to predict the specific heat of gases and solids.

32. (c)

N_2 molecule has 3 translation and 2 rotational degrees of freedom. It has total 5 degrees of freedom.

33. (a)

$$E = \frac{f}{2} nRT = \frac{f}{2} NkT$$

$$\therefore n = nN_A$$

$$\therefore E = \frac{3}{2} \cdot n \cdot N_A \cdot k_B \cdot T = \frac{3}{8} N_A \cdot k_B \cdot T$$

$$N = \frac{m}{M} = \frac{1}{4}$$

N_A = Avagadro's number and k_B = Boltzmann constant

$$\therefore \Delta E = \frac{3}{8} N_A k_B \Delta T = \frac{3}{8} N_A k_B (T_2 - T_1)$$

34. (b)

$$U = \left(\frac{3}{8} k_B T \right) 2 N_A = 3 (k_B \times N_A) T = 3 RT .$$

35. (d)

In adiabatic process

$$p \propto T^{\gamma/\gamma-1} \text{ and } p \propto T^3$$

$$\Rightarrow \frac{\gamma}{\gamma-1} = 3$$

$$\therefore \gamma = 3\gamma - 3 \Rightarrow \gamma = 3/2$$

36. (b)

In solids, there will be only vibrational modes .
 For the possibilities of three - dimensional
 oscillation the total energy
 $= 3 (k_B T) = 3 k_B T$

For one mole solid , total energy $= 3 k_B N_A T$
 $= 3RT$

37. (d)

$$C_V = \frac{5}{2}R \quad \text{and} \quad C_p = \frac{7}{2}R \Rightarrow \gamma = \frac{C_p}{C_V} = \frac{7}{5}$$

38. (c)

For an adiabatic process , $T^\gamma = \text{constant}$.
 We know that average time of collision
 between molecules

$$\tau = \frac{1}{n\pi\sqrt{2}v_{rms}d^2}$$

n = number of molecules per unit volume

v_{rms} = rms velocity of molecules

$$n \propto \frac{1}{V} \quad \text{and} \quad v_{rms} \propto \sqrt{T}$$

$$\Rightarrow \tau \propto \frac{V}{\sqrt{T}}$$

$$n = K_1 V^{-1} \quad \text{and} \quad v_{rms} = K_2 T^{1/2}$$

K_1 and K_2 are constants .

For adiabatic process , $TV^\gamma = \text{constant}$

$$\tau \propto V T^{-1/2} \propto V (V^{1-\gamma})^{-1/2} \quad \text{or} \quad \tau \propto V^{\frac{\gamma+1}{2}}$$

39. (d)

For isochoric process $V = \text{constant}$
 From first law of thermodynamics ,

$$\Delta Q = \Delta W + \Delta U \quad \text{----- (i)}$$

$\Delta W = 0$ as $V = \text{constant}$

$$\Delta Q = nC_V \Delta T$$

On substituting Eq. (i)

$$\Delta U = nC_V \Delta T \quad \text{----- (ii)}$$

Mayer's relation can be written

$$C_p - C_V = R \Rightarrow C_V = C_p - R \quad \text{----- (iii)}$$

From Eqs. (ii) and (iii)

$$\Delta U = n(C_p - R) \Delta T$$

$$n = 6, \quad C_p = 8 \text{ cal mol}^{-1} \text{K}^{-1}$$

$$R = 8.31 \text{ J mol}^{-1} \text{K}^{-1} = 2 \text{ cal mol}^{-1} \text{K}^{-1}$$

$$\Delta U = 6(8 - 2)(35 - 20) \\ = 6 \times 6 \times 15 = 540 \text{ cal}$$

40. (b)

O_2 is a diatomic gas having 5 degrees of freedom .

Energy(total internal) per mole of the gas
 $= \frac{5}{2}RT$

For 2 moles of the gas total internal energy
 $= 2 \times \frac{5}{2}RT = 5RT \quad \text{----- (i)}$

Neon (Ne) is monoatomic gas having 3 degrees of freedom .

\therefore Energy per mole $= \frac{3}{2}RT$, We have 4 moles of Ne .

$$\text{Energy} = 4 \times \frac{3}{2}RT = 6RT \quad \text{----- (ii)}$$

$$\text{Total energy} = 5RT + 6RT = 11RT$$

41. (c)

$$C_p - C_V = R \quad \text{----- (i)}$$

$$\gamma = \frac{C_p}{C_V} \Rightarrow \gamma C_V = C_p \quad \text{---- (ii)}$$

On substituting Eq. (ii) in Eq (i)

$$\gamma C_V - C_V = R \Rightarrow C_V(\gamma - 1) = R$$

$$\therefore C_V = \frac{R}{\gamma - 1}$$

42. (d)

Specific heat for a monoatomic gas

$$C_V = \frac{3}{2}R$$

$$\left(C_V = \frac{f}{2}R, \text{ where } f = \text{degree of freedom} \right)$$

$$\begin{aligned} \therefore \text{Heat} \quad dQ &= \mu C_V \Delta T \\ &= \mu \times \frac{3}{2} \times R (473 - 273) \\ &= 4 \times \frac{3}{2} \times R \times 200 \\ &= 1200 R \quad (\because \mu = 4) \end{aligned}$$

43. (a)

The gases carbon monoxide (CO) and nitrogen (N_2) are diatomic , so both have equal kinetic energy $5/2 kT$,

$$i.e., \quad E_1 = E_2 .$$