15

WAVES

- **Wave motion**

  It is the propagation of a disturbance from one point to another point of a medium without the translatory motion of the particles of the medium.

- **Difference between transverse waves and longitudinal waves**

Transverse wave	Longitudinal wave
(a)In a transverse wave ,the particles of medium vibrate perpendicular to the direction of propagation of the wave	(a)In a longitudinal wave ,the particles of medium vibrate parallel or anti-parallel to the direction of propagation of the wave
(b) It is propagated as crests and troughs.	(b)It is propagated as compressions and rarefactions
(c) It may be formed in solids and liquid surfaces.	(c)It is formed in solids ,liquids and gases.
(d) It can be polarized	(d)It cannot be polarized.

- All electromagnetic waves are transverse in nature. They need no medium for propagation.
- Radio waves, infra-red, X-ray, gamma ray, ultra-violet and light are electromagnetic waves.
- Sound is a longitudinal wave. It needs a medium for propagation.
- **Wave length(λ)**

  It is the distance travelled by the wave during the time when the particles of medium complete one oscillation.

- **Frequency**

  It is the number of oscillations in one second. Its unit is hertz(cycle/second).

- **Velocity of a wave**

  It is the distance travelled by the wave in one second.

- **Amplitude**

  It is the maximum displacement of the particles of medium from the mean position.

- Velocity(v) =Frequency(ϑ) X Wave length(λ).
- **Velocity of transverse wave through a string**:

  v = $\sqrt{\frac{T}{m}}$ where T is the tension in the string and m is the linear density(mass/length).

- **Velocity of a longitudinal wave through a medium**

  v = $\sqrt{\frac{E}{\rho}}$ where E is the modulus of elasticity and ρ is the density of the medium.

  For solid,

  E =Y, Young's modulus.

  For liquid, E =B, bulk modulus.

- **Velocity of sound through a gas :**

  v= $\sqrt{\frac{P}{\rho}}$ where P is the pressure and ρ is the density of the medium. This formula is called Newton's formula.

- **Laplace's formula for the velocity of sound through a gas**

  $v=\sqrt{\frac{\gamma P}{\rho}}$ where $\gamma = C_p/C_v$ For air γ =1.4

- **Effect of pressure**
  Pressure has no effect on the velocity of sound.

- **Effect of humidity**
  When humidity in air increases, its density decreases and so velocity of sound increases

- **Relation connecting speed of sound with temperature**

  Velocity is proportional to the square root of absolute temperature.

- **Harmonic wave**

  It is a wave in which the particles of medium vibrate simple harmonically.

- **Displacement of particle in progressive wave**

  Progressive wave is a wave which is propagated through a medium undammed and unobstructed.

  For a progressive harmonic wave propagating along the positive x direction.

  The displacement of a particle y= A Sin $2\pi \frac{vt-x}{\lambda}$
  For a wave propagated in the negative x direction, y= A Sin $2\pi \frac{vt+x}{\lambda}$

- When sound is reflected from a rigid boundary(denser medium) the phase of the wave is reversed.
  When sound is reflected from a free boundary(rarer medium) the phase of the wave is not reversed.

- **Persistence of audibility**

  The sensation of hearing a sound persists for about 1/10 of a second even after the original sound is stopped. This is known as persistence of audibility.

- **Echo**

  A reflected sound reaching the ear after 0.1 second of the original sound is its echo.

- The minimum distance for the production of an echo =17metre

- **Reverberation**

  The lingering of sound due to reflection from nearby obstacles is called reverberation.

- **Standing wave or stationary wave**

  When two harmonic waves of same amplitude and frequency travel in the opposite direction along a line , they mix up and the resultant wave pattern appears to be stationary. This wave is called standing wave.

  The positions of zero displacement are called nodes.
  The positions of maximum displacement are called anti-nodes.

  Distance between two consecutive nodes or anti-nodes is $\lambda/2$.

- **For a stationary wave**

  For a harmonic wave travelling in the positive x direction,

  y_1 = A Sin $2\pi \frac{vt-x}{\lambda}$

  For the reflected wave, y_2 = -A Sin $2\pi \frac{vt+x}{\lambda}$

  Total displacement y = $y_1+y_2 = -2$ ASin$\frac{2\pi x}{\lambda}$Cos$\frac{2\pi vt}{\lambda}$
  which represents a stationary wave.

  Here 2 A Sin$\frac{2\pi x}{\lambda}$ represents the amplitude.

- **Standing waves on a stretched string**

  A string can vibrate in different modes. The first mode is called fundamental.

  It has an anti-node at its centre and nodes at the ends.

  Consider a string of length l. Standing waves of the following modes are formed on it.

  Fundamental mode(First harmonic):
  Here, frequency $\vartheta_1 = \frac{v}{2l}$

Second mode(First overtone):

Here, frequency $\vartheta_2 = \frac{2v}{2l}$ (Two times fundamental frequency)

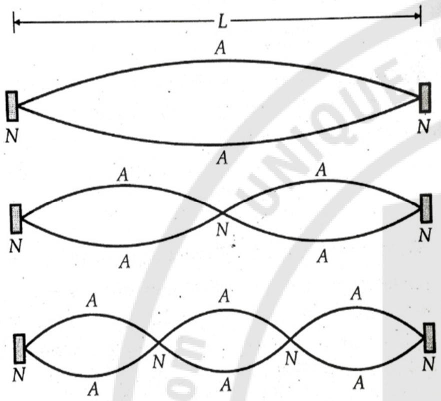
Third mode(Second overtone):

Here, frequency $\vartheta_3 = \frac{3v}{2l}$ (3 times fundamental frequency).

$OR \quad \vartheta_1 : \vartheta_2 : \vartheta_3 ......= 1 : 2 : 3 .....$

ie, All harmonics are present in a string.

First, second and third harmonics are shown below



- **Standing waves in an open pipe**

Air column in an open pipe vibrates in a number of modes. An anti node is formed at the open end in all cases. Consider a pipe of length l.

Fundamental mode(First harmonic):

Here, frequency $\vartheta_1 = \frac{v}{2l}$

Second mode(First overtone):

Here, frequency $\vartheta_2 = \frac{2v}{2l}$ (Two times fundamental frequency)

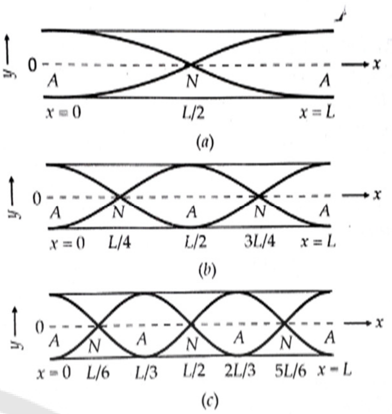
Third mode(Second overtone):

Here, frequency $\vartheta_3 = \frac{3v}{2l}$ (3 times fundamental frequency).

$OR \quad \vartheta_1 : \vartheta_2 : \vartheta_3 ......= 1 : 2 : 3 .....$

ie, All harmonics are present in an open pipe.

First, second and third harmonics are shown below



- **Standing waves in a closed pipe**

Consider a pipe of length l and whose one end is closed. standing waves of the following modes are produced in it. Always a node is produced at the closed end and an anti-node is produced at the open end.

Fundamental mode(First harmonic):

Here, frequency $\vartheta_1 = \frac{v}{4l}$

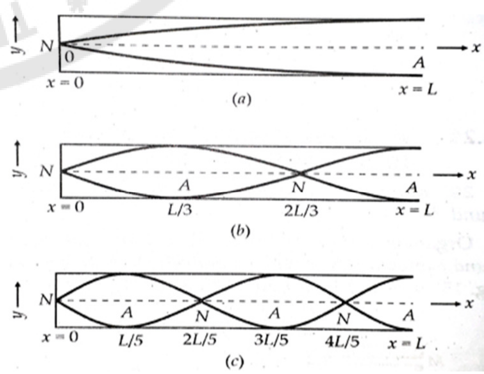
Second mode(First overtone): Here, ϑ_2 frequency $= \frac{3v}{4l}$ (Three times fundamental frequency)

Third mode(Second overtone): Here, frequency $\vartheta_3 = \frac{5v}{4l}$ (5 times fundamental frequency).

$\vartheta_2 : \vartheta_3 ......= 1 : 3 : 5 .....$

ie, Only odd harmonics are present in a closed pipe.

First, third and fifth harmonics are shown below



- **End – correction**

❖ Antinode is not formed exactly at open end. It is slightly displaced away from open end. This distance is called end correction. Let it be represented by x.

- ❖ End – correction = 2x for open pipe. There are two open ends.
- ❖ End – correction = x for closed pipe. There is one open ends.
- ❖ End- correction depends upon internal radius R of organ pipe. Thus x = 0.6R.

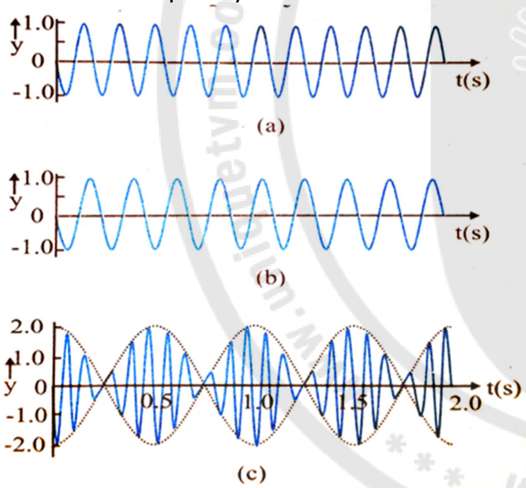
- **Resonance tube**

  End –correction , x = $\frac{l_2 - 3l_1}{2}$ where l_1 and l_2 are the first and second resonance lengths.

- **Beats**

  When two harmonic waves of slightly different frequencies pass through the same region ,they superimpose and the resultant sound intensity increases and decreases at regular intervals and a beat is formed.

  The number of beats produced in one second is called beat frequency.



Beat is represented as,

$$S = 2ACos\,\frac{(\omega_1+\omega_2)t}{2}\quad Cos\frac{(\omega_1-\omega_2)t}{2}$$

- Number of beats heard per second = difference of frequencies of two waves

- **Doppler effect in sound**

  The apparent change in the frequency of sound felt by the listener due to the relative motion between the source and listener is called Doppler effect(As it was proposed by Christian Doppler).Consider a source producing sound of frequency ϑ , wave length λ and velocity V.

SOURCE LISTENER

Apparent frequency of sound heard by the listener, $\vartheta' = \vartheta\left[\frac{V - V_L}{V - V_s}\right]$

Case(1):Source and listener at rest : $V_L = 0$ and $V_s = 0$

$\vartheta' = \vartheta\left[\frac{V-0}{V-0}\right] = \vartheta$ ie, the listener hears the original frequency.

Case(2):Source at rest and listener moving towards the source : V_L is -ve and $V_s = 0$.

$$\vartheta' = \vartheta\left[\frac{V - -V_L}{V-0}\right] = \vartheta\left[\frac{V + V_L}{V}\right]$$

Case(3):Source at rest and listener moving away from the source : V_L is +ve and $V_s = 0$.

$$\vartheta' = \vartheta\left[\frac{V - +V_L}{V-0}\right] = \vartheta\left[\frac{V - V_L}{V}\right]$$

Case(4):Listener at rest and source moving towards the listener : $V_L = 0$ and V_s is + ve .

$$\vartheta' = \vartheta\left[\frac{V-0}{V - +V_s}\right] = \vartheta\left[\frac{V}{V - V_s}\right]$$

Case(5):Listener at rest and source moving away from the listener : $V_L = 0$ and V_s is − ve .

$$\vartheta' = \vartheta\left[\frac{V-0}{V - -V_s}\right] = \vartheta\left[\frac{V}{V + V_s}\right]$$

Case(6):Listener and source moving away from each other : V_L is + ve and V_s is − ve.

$$\vartheta' = \vartheta\left[\frac{V - +V_L}{V - -V_s}\right] = \vartheta\left[\frac{V - V_L}{V + V_s}\right]$$

Case(7):Listener and source moving towards each other : V_L is -ve and V_s is + ve .

$$\vartheta' = \vartheta\left[\frac{V - -V_L}{V - +V_s}\right] = \vartheta\left[\frac{V + V_L}{V - V_s}\right]$$

Case(8):Listener moving towards the source and the source moving away : V_L is -ve and V_s is − ve .

$$\vartheta' = \vartheta\left[\frac{V - -V_L}{V - -V_s}\right] = \vartheta\left[\frac{V + V_L}{V + V_s}\right]$$

Case(9):Source moving towards the listener and the listener moving away : V_L is + ve and V_s is + ve .

$$\vartheta' = \vartheta\left[\frac{V - +V_L}{V - +V_s}\right] = \vartheta\left[\frac{V - V_L}{V - V_s}\right]$$

Case(10):If wind blows with a velocity W in the same direction of propagation of sound:

$$\vartheta' = \vartheta\left[\frac{V + W - V_L}{V + W - V_s}\right]$$

Case(11):If wind blows with a velocity W in the opposite direction of propagation of sound:

$$\vartheta' = \vartheta\left[\frac{V - W - V_L}{V - W - V_s}\right]$$

- **Application of Doppler effect**

  - To estimate the speed of submarines.
  - To estimate the speed of automobiles.
  - To estimate the speed of aero planes.
  - To track artificial satellites.

- **Doppler effect in light**

The apparent change in the frequency of light felt by the observer due to the relative motion between the light source and observer is called Doppler effect.

When a star moves away from us , the frequency of light from it decreases and the wave length increases. The spectrum of light from it shifts towards the side of red. This is called red shift.

When a star moves towards us , the frequency of light from it increases and the wave length decreases. The spectrum of light from it shifts towards the side of blue. This is called blue shift.

- **Tuning fork**

Frequency of a tuning fork n = $\frac{kt}{l^2}\sqrt{\frac{Y}{\rho}}$

t = thickness of prong ,l= length of prong ,Y = Young's modulus of material ,ρ = density of material ,k = a constant

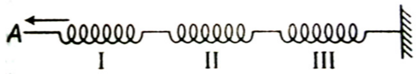
- Frequency of tuning fork **increases** when the prong is **filed**.
  Frequency of tuning fork **increases** when the prong length is decreased.

- Frequency of a tuning fork decreases when the prong is loaded with wax.

- The frequencies of the forks are chosen from notes of musical scale. The frequencies are 256, 288…512 HZ.

Multiple choice Questions:

1. A collection of springs connected to one another is fixe at one end. If the other end is pulled suddenly and released , then which of the following will happen?



(a) The disturbance genearating at end A will propagate to the other end but each spring only executes small oscillations about its equilibrium position
(b) Since , the second spring (II) is connected to the first , it will also be stretched or compressed and so on
(c) Springs(I) and (III) will displace from its equilibrium position while spring (II) will execute small oscillations about its equilibrium position
(d) Both (a) and (b)

2. The displacement as a function of position 'x' and time 't' is given by
$Y(x,t) = a\sin(kx - \omega t + \phi)$
The range of possible value of $y(x,t)$, if 'a' is a positive constant , is
(a) - a ≤ $y(x,t)$ ≤ a (b) -$\frac{a}{2}$ ≤ $y(x,t)$ ≤ $\frac{a}{2}$
(c) - 1 ≤ $y(x,t)$ ≤ + 1 (d) 0 ≤ $y(x,t)$ ≤ 0

3. The displacement of the wave given by equation $y(x,t) = a\sin(kx - \omega t + \phi)$, where ϕ = 0 at point x and t = 0 is same as that at point
(a) $x + 2n\pi$ (b) $x + \frac{2n\pi}{k}$
(c) $kx + 2n\pi$ (d) Both (a) and (b)

4. The angle between particle velocity and wave velocity in a transverse wave is
(a) zero (b) $\pi/4$ (c) $\pi/2$ (d) π

5. A simple wave motion is represented $by\ y = 5(\sin 4\pi t + \sqrt{3}\cos 4\pi t)$. Its amplitude is
(a) 5 (b) $5\sqrt{3}$ (c) $10\sqrt{3}$ (d) 10

6. The equation of a wave is given by $Y = 10 \sin\left(\frac{2\pi}{45}t + \propto\right)$. If the displacement is 5 cm at t = 0, then the total phase at t = 7.5 s is
(a) π (b) $\pi/6$ (c) $\pi/2$ (d) $\pi/3$

7. A wave equation is given by $Y = 4 \sin\left[\pi\left(\frac{t}{5} - \frac{x}{9} + \frac{1}{6}\right)\right]$
where, x is in cm and t is in second. The wavelength of the wave is
(a) 18 cm (b) 9cm (c) 36 cm (d) 6 cm

8. Equation of progressive wave is $Y = a \sin\left(10\pi x + 11\pi t + \frac{\pi}{3}\right)$
The wavelength of the wave is
(a) 0.2 unit (b) 0.1 unit
(c) 0.5 unit (d) 1 unit

9. The equation of a wave is given as $Y = 0.07 \sin(12\pi x - 300\pi t)$
Where, x and y are is in metre and t in second, then the correct statement is
(a) λ = 1/6m (b) a = 0.07 m
(c) Both (a) and (b) (d) None these

10. The equation of a simple harmonic wave is given by $y = 5 \sin\frac{\pi}{2}(100t - x)$ where x and y are in metre and time is in second. The period of the wave (in second) will be
(a) 0.04 (b) 0.01 (c) 1 (d) 5

11. A plane progressive wave is given by $Y = 2\cos 6.284\,(330\,t - x)$. What is period of the wave?
(a) $\frac{1}{330}s$ (b) $2\pi \times 330\, s$
(c) $(2\pi \times 330)^{-2}s$ (d) $\frac{6.284}{330}\, s$

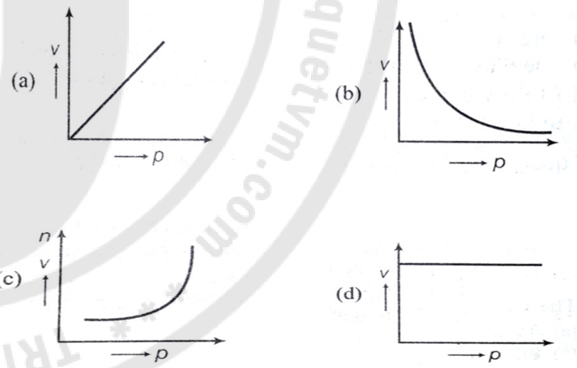
12. In a sinusoidal wave, the time required for a particular point to move from maximum displacement to zero displacement is 0.14 s. The frequency of the wave is
(a) 0.42 Hz (b) 2.75 Hz (c) 1.79 Hz (d) 0.56 Hz

13. The equation of a progressive wave can be given by $y = 15\sin(660\pi t - 0.02\pi x)cm$. The frequency of the wave is
(a) 330 Hz (b) 342 Hz (c) 365 Hz (d) 660 Hz

14. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (Take, g = $10ms^{-2}$)
(a) $2\pi\sqrt{2}$s (b) 2s (c) $2\sqrt{2}$s (d) $\sqrt{2}$s

15. Two strings A and B, made of same material are stretched by same tension. The radius of string A is double of the radius of B. A transverse wave travels on A with speed v_A and on B with speed v_B. The ratio v_A/v_B is
(a) 1/2 (b) 2 (c) 1/4 (d) 4

16. A student plotted the following four graphs representing the variation of velocity of sound in a gas with the pressure p at constant temperature. Which one is correct?



17. Two pulses having equal and opposite displacements moving in opposite direction overlap at t= t_1s. The resultant displacement of the wave at t = t_1s is
(a) Twice the displacement of each pulse
(b) Half the displacement of each pulse
(c) Zero
(d) Either (a) or (c)

18. If there are 'n' number of waves moving in the medium represented by the wave functions

$y_i = f_i\,(x - vt)$ where I = 1,2,3 , …. n

Then the resultant wave function describing the disturbance in the medium is

(a) $Y = \sum_{i=1}^{m} f_i\,(x - vt)$
(b) $y = f_1\,(x - vt) + f_2\,(x - vt) +$ --- $+ f_n\,(x - vt)$
(c) $y = f_1\,(x - vt) - f_2\,(x - vt) + f_3\,(x - vt) - f_4\,(x - vt) + \text{---} + (-1)^{n+1}\ f_n\,(x - vt)$
(d) Both (a) and (b)

19. A wave travelling in the positive x- direction having displacement along y- direction 1 m , wavelength $2\pi m$ and frequency of $1/\pi Hz$ represented by

(a) $y = sin\ (x - 2t)$
(b) $y = sin\ (2\pi x - 2\pi t)$
(c) $y = sin\ (10\pi x - 20\pi t)$
(d) $y = sin\ (2\pi x + 2\pi t)$

20. Two sine waves travel in the same direction in a medium. The amplitude of each wave is A and the phase difference between the two waves is 120^o . The resultant amplitude will be

(a) A (b) 2A (c) 4 A (d) $\sqrt{2}$ A

21. Let wave $y(x,t) = a\ sin\ (kx - \omega t)$ is reflected from an open boundary and then the incident and reflected wave overlaps. Then the amplitude of resultant wave

(a) $2a\ cos\ (kx)$ (b) $2a\ sin\ (kx)$

(c) $2a\ cos\ \left(\frac{kx}{2}\right)$ (d) $a\ sin\ (kx)$

22. A person blows into open - end of a long pipe. As a result, a high - pressure pulse of air travels down the pipe . When this pulse reaches the other end of the pipe,

(a) a high - pressure pulse starts travelling up the pipe , if the other end of the pipe is open
(b) a low - pressure pulse starts travelling up the pipe , if the other end of the pipe is open
(c) a low - pressure pulse starts travelling up the pipe , if the other end of the pipe is closed
(d) a high - pressure pulse starts travelling up the pipe , if the other end of the pipe is closed

23. A wire under tension vibrates with a fundamental frequency of 600 Hz. If the length of the wire is doubled , the radius is halved and the wire is made to vibrate under one-ninth the tension. Then the fundamental frequency will become

(a) 400 Hz (b) 600 Hz (c) 300 Hz (d) 200 Hz

24. A string is stretched between two fixed points separated by 75 cm . It is observed to have resonant frequencies of 420 Hz and 315 Hz . There are no other resonant frequencies between these two . The lowest resonant frequency for this string is

(a) 155 Hz (b) 205 Hz (c) 10.5 Hz (d) 105 Hz

25. An air column, closed at one end and open at the other , resonates with a tuning fork when the smallest length of the column is 50 cm . The next larger length of the column resonating with the same tuning fork is

(a) 100 cm (b) 150 cm (c) 200 cm (d) 66.7 cm

26. A pipe of length 85 cm is closed from one end . Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz . The velocity of sound in air is 340 m/s .

(a) 12 (b) 8 (c) 6 (d) 4

27. The systems such as strings and air columns undergoes forced oscillation . If the external frequency of oscillation for forced oscillation is close to one of the natural frequencies , then the system is said to be in

(a) resonance
(b) fundamental mode of oscillation
(c) simple harmonic motion
(d) Both (a) and (b)

28. Two sitar string A and B playing the note 'Dha' are slightly out of tune and produce beats of frequency 5Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease to 3 Hz . The original frequency of B , if the frequency of A is 427 Hz, is

(a) 422 Hz (b) 420 Hz (c) 424 Hz (d) 419 Hz

29. A source of unknown frequency gives 4 beat/s when sounded with a source of known frequency 250 Hz. The second harmonic of the source of unknown frequency gives 5 beat/s when sounded with a source of frequency 513 Hz. The unknown frequency is

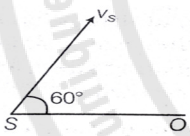
(a) 254 Hz (b) 246 Hz (c) 240 Hz (d) 260 Hz

30. A siren emitting a sound of frequency 800 Hz moves away from an observer towards a cliff at a speed of 15 ms^{-1} . Then the frequency of sound that the observer hears in the echo reflected from the cliff is

(Take , velocity of sound in air = 300ms^{-1})

(a) 800 Hz (b) 838 Hz (c) 885 Hz (d) 765 Hz

31. A source of sound S emitting waves of frequency 100 Hz and an observer O are located at some distance from each other . The source is moving with a speed of 19.4 ms^{-1} at an angle of 60^o with the source observer line as shown in the figure. The observer is at rest. The apparent frequency observed by the observer (velocity of sound in air is 330 ms^{-1}) is



(a) 100 Hz (b) 103 Hz (c) 106 Hz (d) 97 Hz

32. A rocket is moving at a speed of 220 ms^{-1} towards stationary target , emits a sound of frequency 1000 Hz . Some of the sound reaching the target gets reflected back to the rocket as an echo. The frequency of the echo as detected by the rocket is

(Take , velocity of sound = 330 ms^{-1})

(a) 3500Hz (b) 4000Hz

(c) 5000 Hz (d) 3000Hz

HINTS AND EXPLANATIONS

1. (d)
2. (a)

The wave equation ,

$y(x,t)$ = a sin $(kx - \omega t + \phi)$ is a sine function. Since , the function varies between -1 and +1 the displacement y (x,t) varies between a and -a

$\Longrightarrow$ - a $\leq y(x,t) \leq a$

3. (b)

$Y(x,0)$ a sin $kx = a\,sin\ (kx + 2n\pi)$

$= a \sin\ k\left(x + \frac{2n\pi}{k}\right)$

$\Longrightarrow$ The displacement at points x and $\left(x + \frac{2n\pi}{k}\right)$ are the same where n = 1,2,3, …..

4. (c)

In transverse wave , the particles of the medium oscillate in the direction perpendicular to the wave propagation .

i.e., angle between particle velocity and wave velocity = $\pi/2$

5. (d)

$Y = 5\,(sin\ 4\,\pi t + \sqrt{3}\cos\ 4\,\pi t)$

$= \left[5\,sin\ 4\,\pi t + 5\sqrt{3}\cos\ 4\,\pi t\right]$ ----- (i)

For the wave travelling towards left,

y = A $sin\ (kx + \omega t) + B\cos(kx + \omega t)$ ------ (ii)

= A $sin\ (kx + \omega t + \phi)$

a = $\sqrt{A^2 + B^2}$; $\phi = tan^{-1}(B/A)$

Represents Eq . (ii) , where x=0

On comparing Eqs. (i) and (ii)

a = $\sqrt{5^2 + \left(5\sqrt{3}\right)^2}$ = $\sqrt{25 + 75}$ = 10

6. (c)

Y = 10 sin $\left(\frac{2\pi}{45}t + \propto\right)$

t = 0 , y = 5 cm , then 5 = 10 (sin $\propto$)

sin $\propto$ = $\frac{1}{2}$ $\Longrightarrow \propto = \frac{\pi}{6}$

t = 7.5 s

The total phase = $\frac{2\,\pi}{45} \times \frac{15}{2} + \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

7. (a)

$y\,(x,t) = 4\,sin\left[\pi\left(\frac{t}{5} - \frac{x}{9} + \frac{1}{6}\right)\right]$ ------ (i)

Wavelength of the wave = $\frac{2\,\pi}{k}$

k = π /9cm^{-1}

$\Longrightarrow$ $\lambda = \frac{2\,\pi}{k} = \frac{2\,\pi}{\pi\,/\,9}$ = 18 cm

8. (a)

wave equation

$y\,(x,t) \;=\; a\,sin\;(10\,\pi x + 11\pi t + \;\pi/3)$ ----(i)

$\lambda \;= \frac{2\,\pi}{k}$

k = $10\,\pi$

$\Longrightarrow \quad \lambda \;=\; \frac{2\,\pi}{k}$ = 0.2 units

9. (c)

wave equation

$y\,(x,t)$ = 0.07 sin $(12\pi x - 300\;\pi t)$

From the equation and comparing with

$y\,(x,t) \;=\; a\,sin\;(kx - \;\omega t)$

a = 0.07 m , k = 12 $\pi m^{-1}, \omega = 300\,\pi\, rads^{-1}$

$\lambda \;= \frac{2\,\pi}{k} = \frac{2\,\pi}{12\,\pi} = \frac{1}{6}\; m$

10. (a)

the given equation is

$Y \;\;=\;\; 5\,sin\;\frac{\pi}{2}\,(\,100\;t - \;x)$

$= 5\,sin\left(50\;\pi t - \;\frac{\pi}{2}\;x\right)$ ----- (i)

Eq . (i) $\;with\;\; y \;\;=\;\; A\;\,sin\;\,(kx - \;\omega t + \phi)$

$\omega \;=\; 50\pi$

$\Longrightarrow\;\; T = \frac{2\,\pi}{\omega} = \frac{2\,\pi}{50\,\pi} = \frac{1}{25}$ = 0.04 s

11. (a)

$y \;=\; 2\,cos\;\;2\,\pi\,(\;330\;t \;\;-\;\; x)$

$y \;=\; 2\,cos\;\;(2\,\pi \times 330\;t \;\;- 2\,\pi\;\;x)$

or $\;\;y \;=\; 2\,sin\;\;\left[\frac{\pi}{2} - \;(\,2\,\pi\;\times 330\;t - 2\pi x)\right]$

$= \;\;2\,sin\;\;\;(2\;\pi x - 2\;\pi\;\times 330\;t - \pi/2)$

On comparing Eq . (i) with

$y \;\;=\;\; A\;\,sin\;\,(kx - \;\omega t + \phi)$

$\omega \;\;=\;\;\; 2\,\pi\;\times 330$

$\Longrightarrow \;\;\; T = \;2\,\pi\;/\;\omega = \left(\frac{1}{330}\right)s$

12. (c)

Time taken by the wave to travel a distance equal to one wavelength = T

$\Longrightarrow \quad T$ = 4 × (time to travel from maximum displacement to zero by a particle)

$\Longrightarrow \quad T = 4\times 0.14\;s$ = 0.56 s

Frequency , $f = \frac{1}{T} = \frac{1}{0.56} = \frac{100}{56} = 1.79\;Hz$

13. (a)

$y \;=\; 15\,sin$ $(660\,\pi t - 0.02\,\pi x\,)$

On comparing with general equation of progressive wave

$y \;=\; (x,t)\;\;a\,sin\;\left(\frac{2\,\pi}{T}t - \frac{2\pi}{\lambda}x\right)$

$\therefore\;\; \frac{2\,\pi}{T}$ = 660 π

$\frac{1}{T}$ = 330 $\qquad$ or v = 330 Hz

14. (c)

At distance x from the bottom

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\left(\frac{mgx}{L}\right)}{\frac{m}{L}}} = \sqrt{gx}$

$\therefore \quad \frac{dx}{dt} = \sqrt{x}\,\sqrt{g} \quad \Longrightarrow\; \int_0^L x^{-1/2}\;dx$

$= \sqrt{g}\;\int_0^t dt$

$\Longrightarrow \qquad \left[\frac{x^{1/2}}{(1/2)}\right]_0^L \;=\; \sqrt{g}$ .t

$\Longrightarrow \qquad\qquad$ t = $2\,\frac{\sqrt{L}}{\sqrt{g}}$

$\Longrightarrow \qquad\qquad$ t = $2\,\frac{\sqrt{20}}{\sqrt{10}} = 2\sqrt{2}\;s$

15. (a)

radius of wire A = 2 (radius of wire B)

i.e., $\qquad r_A = 2\,r_B$

Also , tension in the string $\;T_B \;= T_A$

m_A = mass of string off A = volume × density

$= (\pi\; r_A^2) \times l_A\; \times \rho_A$ ----- (i)

$m_B \;=\; (\pi\; r_B^2) \times l_B\; \times \rho_B$ ----- (ii)

$\therefore \qquad \frac{m_A}{m_B} \;=\; \frac{r_A^2\,l_A}{r_B^2\,l_B}$ $\qquad (\;\because\;\; \rho_A \;=\; \rho_B)$

$\mu \;=\; \frac{m}{l}$

$\Longrightarrow \qquad \frac{\mu_A}{\mu_B} \;=\; \frac{m_A\,/l_A}{m_B\;/\;l_B} \;=\; \left(\frac{r_A}{r_B}\right)^2$ ----- (iii)

speed of transverse wave = $\sqrt{\frac{T}{\mu}}$

$\Longrightarrow \qquad \frac{v_A}{v_B} \;=\; \sqrt{\frac{\mu_A}{\mu_B}} \;=\; \frac{r_B}{r_A} = \frac{1}{2}$

$(\;\because\;\; r_A \;=\; 2\,r_B)$

16. (d)

The speed of sound in a gas does not depended upon pressure of the gas , till temperature remains constant i.e., speed remains the same whatever be the pressure . therefore , graph (d) is correct.

17. (c)

The displacement due to two pulses will exactly cancel out each other and there is no displacement throughout .

18. (d)

For n number of waves , the resultant wave function can be written as

$y = \sum_{i=1}^{n} f_i\;\;(x - vt)$

or $\qquad y \;=\; f_1\,(x - vt) + \;f_2\,(x - vt) + \ldots\ldots +$ $f_n\,(x - vt)$

19. (a)

a = 1 m

$y = a\ sin\ (kx\ - \omega t)$

= $\sin\ \left(\frac{2\pi}{2\pi} \times x - 2\pi\ \times \frac{1}{\pi}\right) = \sin(x - 2t)$

20. (a)

Amplitude of the resultant of the two waves

= A $(\phi) = 2Acos\ \left(\frac{\phi}{2}\right)$

$\phi\ = 120^o$

$\Longrightarrow$ A (ϕ)= $2Acos\ \left(\frac{120^o}{2}\right)$ = $2\ Acos\ 60^o$

or A (ϕ)= $(2A) \times \frac{1}{2}$ = A

21. (b)

we have incident wave $y_1 = a\ sin\ (kx\ - \omega t)$

So the reflected wave is $y_2 = a\ sin\ (kx + \omega t)$

From principle of superposition ,

The standing wave equation is obtained as

$y(x,t) = 2a\ sin\ kx\ cos\ \omega t$

On comparing Eqs. (i) and (ii)

$A(x)\ =\ 2a\ sin\ kx$

22. (a,c,d)

23. (d)

Fundamental frequency

= $v_o = \frac{nv}{2L}$; n = 1 $= \frac{v}{2L}$ -----(i)

Wave velocity = $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{(m/L)}}$ ---- (ii)

$\because$ Mass of the string = m

= Volume $\times$ Density

= $(\pi r^2\ \times L) \times\ \rho\ \Longrightarrow \mu\ = \frac{m}{L} = \ \pi r^2 \rho$

On substituting the value of μ in Eq . (ii)

$\Longrightarrow$ $v\ = \sqrt{\frac{T}{\pi r^2 \rho}}$ $= \frac{1}{r}\sqrt{\frac{T}{\pi\ \rho}}$

$v_o\ = \frac{1}{2rL}\sqrt{\frac{T}{\pi\ \rho}}$

$\Longrightarrow$ $\frac{v_{01}}{v_{02}} = \frac{L_2}{L_1} \times \frac{r_2}{r_1}\sqrt{\frac{T_1}{T_2}}$

$\Longrightarrow$ $\frac{600}{v_{02}} = \frac{2}{1} \times \frac{1}{2} \times \sqrt{\frac{T}{T/9}} = 3$

$\therefore$ $v_{02}\ = \frac{600}{3}$ = 200 Hz

24. (d)

l = 75 cm , f_1 = 420 Hz and f_2 = 315 Hz

As , two consecutive resonant frequencies for a string fixed at both ends will be

$f_1\ = \frac{nv}{2l}$ and $f_2\ = \frac{(n+1)}{2\ R}$

$\Longrightarrow$ $f_2 - f_1$ = 420 -315

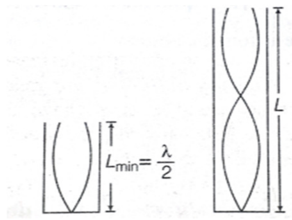
$\Longrightarrow$ $\frac{(n+1)v}{2l} - \frac{nv}{2l}$ = 105 Hz $\Longrightarrow$ $\frac{v}{2\ l}$ = 105 Hz

Thus , lowest resonant frequency of a string is 105 Hz.

25. (b)

The smallest length of the air column is associated with fundamental mode of vibration of the air column as shown in the diagram.

$\because$ $L_{min} = \frac{\lambda}{4}$ $\Longrightarrow$ 50cm $= \frac{\lambda}{4}$ $\Longrightarrow$ λ = 200 cm



The next higher length of the air column is

$L = \frac{\lambda}{4} + \frac{\lambda}{2} = \frac{\lambda + 2\,\lambda}{4} = \frac{3\,\lambda}{4}$

$= \frac{3}{4} \times 200$ = 150cm

26. (c)

For closed organ pipe

$= \frac{(2\,n+1\,)v}{4\,l}$ (where n= 0,1,2, ----)

$\frac{(2\,n+1\,)v}{4\,l}$ < 1250

$\Longrightarrow (2\,n\ +\ 1\,) < 1250 \times\ \frac{4\ \times 0.85}{340}$

$\frac{(2\,n+1\,)v}{4\,l}$ < 12.52 n < 11.50

n < 5.25

n = 0,1,2,3, …. ,5

So , we have 6 possibilities.

27. (a)

The system like strings and air columns, can undergo forced oscillations. If the external frequency is close to one of the natural frequencies, the system shows resonance.

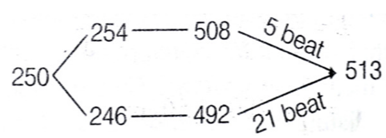
28. (a)

Increase in the tension of a string increase its frequency . If the original frequency of B(v_B) were greater than that of A(v_A) , further increase in v_B should have resulted in an increase in the beat frequency. But the beat frequency is found to decrease . This shows that $V_B < V_A$.

$V_A\ -\ V_B$ = 5 Hz and V_A = 427 Hz

V_B = 422 Hz

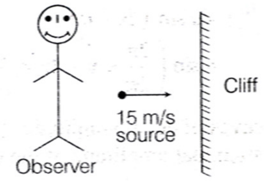
29. (a)



Unknown frequency is 254 Hz.

30. (b)

According to question , situation can drawn as follow.



Frequency of sound that the observer hear in the echo reflected from the cliff is given by

$f' = f\left(\frac{v}{v - v_s}\right)$

where f = original frequency of source ;

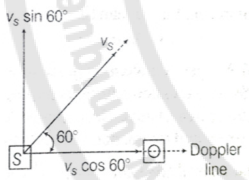
v = velocity of sound

v_s = velocity of source

$f' = \left(\frac{330}{330 - 15}\right) 800 = 838\, Hz$

31. (b)

Given , as a source of sound S emitting waves of frequency 100 Hz and an observer O are located at some distance . Such that , source is moving with a speed of 19.4 m/s at angle 60^o with source - observer line as shown in figure .



The apparent frequency heared by observer

$f_0 = f_s \left[\frac{v}{v - v_s \cos 60^o}\right]$

$= 100 \left[\frac{330}{330 - 19.4 \times 1/2}\right]$

$= 100 \left[\frac{330}{330 - 9.7}\right] = 100 \left[\frac{330}{320.3}\right]$

$= 103.02\, Hz$

32. (c)

As the source (i.e., rocket) is moving towards the stationary target

Frequency of sound detected by the target is

$\nu' = \frac{\nu_0\, \nu}{\nu - \nu_s} = \frac{1000 \times 330}{330 - 220} = \frac{1000 \times 330}{110}$ = 3000 Hz

Now the target is the source (as it is the source of echo) and the rocket's detector is the observer who intercepts the echo of frequency ν' . Hence , the frequency of the echo detected by the rocket is

$\nu'' = \frac{\nu'(\nu + \nu_0)}{\nu} = \frac{3000\,(330 + 220)}{330} = 5000\, Hz$