9

MECHANICAL PROPERTIES OF SOLIDS

- **Deforming force**
  Deforming force is the force which changes the shape or size of a body.

- **Restoring force**

  Restoring force is the internal force developed inside the body which brings the body back to original shape and size when a deforming force acts on it.

- **Elasticity**

  It is the property of a body by virtue of which it tends to regain its original shape and size when deforming force is removed.

- **Difference between perfectly elastic bodies and plastic bodies**

  If a body immediately regains its original shape and size when the applied force is removed, it is perfectly elastic. eg: quartz
  If a body does not show any tendency to regain its original shape and size when the applied force is removed, it is a plastic body. eg: putty, mud etc.

- **Stress**
  It is the restoring force acting per unit area.
  Stress = $\frac{Restoring\ force}{area}$.
  Its unit is $N/_{m^2}$

- **Strain**
  Strain = $\frac{Change\ in\ dimension}{Original\ dimension}$
  Strain has no unit .

- **Longitudinal stress or linear stress**

  It is the stress when there is change in length.
  Tensile stress causes increase in length.
  Compressive stress causes decrease in length.

- **Normal stress or hydraulic stress**
  It is the stress when there is change in volume.

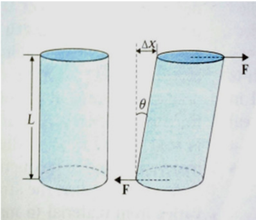
- **Shearing stress or tangential stress**
  Shearing stress or tangential stress is the stress when there is change in shape of the body.

- **Linear strain**

  Linear strain is the ratio of change in length to the original length.

- **Volume strain**
  Volume strain is the ratio of change in the volume to the original volume.
  The strain produced by hydraulic stress is called volume strain.

- **Shearing strain**
  Shearing strain indicates the angle through which deformation take place.



Shearing strain = $\theta \approx \tan\theta = \frac{\Delta x}{L}$ where L is the height and Δx is the tangential displacement.

- **Hooke's law**

For small deformations, the stress is directly proportional to the strain.

Stress α strain

or $\frac{stress}{strain}$ = constant. This constant is called the modulus of elasticity.

- **Modulus of elasticity**

There are three moduli of elasticity. They are Young's modulus, Bulk modulus and Shear modulus (rigidity modulus). Unit of modulus of elasticity: $N/_{m^2}$

- **Young's modulus**

It is the ratio of linear stress to linear strain.

Young' s modulus = $\frac{linear\ stress}{linear\ strain}$

- **Bulk modulus**

It is the ratio of normal stress to volume strain.

Bulk modulus = $\frac{normal\ stress}{volume\ strain}$

Bulk modulus = $\frac{PV}{\Delta V}$

- **Shear modulus**

Shear modulus = $\frac{Shearing\ stress}{Shearing\ strain}$

Shear modulus = $\frac{FL}{A\Delta x}$

- **Young's modulus of the material of a wire**

Young's modulus = $\frac{MgL}{\pi r^2 \Delta L}$ where M is the mass attached to wire, L is the length of wire ,r is its radius and ΔL is the extension in the wire.

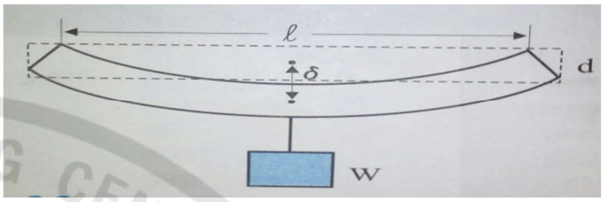
- **Compressibility**

Reciprocal of bulk modulus is called compressibility.

- **The area of ropes used in cranes**

Area of cross – section of the rope ≥ $\frac{W}{Sy}$ where W is weight attached and S_y is the yield strength (For steel, yield strength = $300 \times 10^6\ N/_{m^2}$) (Here W = mg).

- **Ideas to be kept during the construction of a beam for building**



The sag (δ) produced in the beam when weight is placed over it, $\delta = \frac{Wl^3}{4bd^3Y}$ where W is the weight, l is the length of the beam, b is the breadth of the beam, d is the depth of the beam and Y is the young's modulus of the material of the beam.

- **Cross – sectional shape is ideal for load bearing bars**

This shape provides maximum load bearing surface.
As the depth of it is more, the sag produced in it will be less.
As it requires less material, cost can be reduced.
Weight of beam is minimum as the material required is minimum.
At the same time it provides maximum strength.

- **The maximum height of a mountain on the earth's surface is 10km**

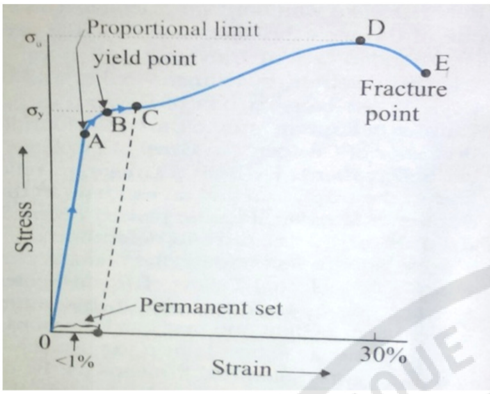
The stress per area must be less than elastic limit of the rocks at the base of the mountain. The elastic limit of rock is $300 \times 10^6\ N/_{m^2}$

h ρg ≤ $300 \times 10^6\ N/_{m^2}$

- **Stress-strain graph for a metal**

In the first section, the stress and strain are proportion up to a limit called proportional limit (point A in figure).

The body behaves as elastic. In the section from A to B the stress and the strain are not in direct proportion. But if force is removed, the body returns to its original position. The point B is called elastic limit or yield point. The corresponding stress is called yield strength.

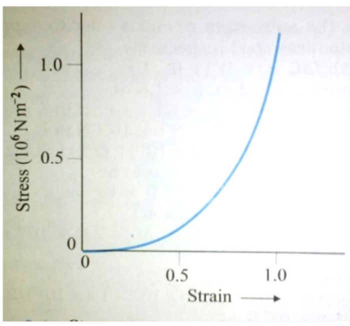


From B onwards if stress is increased, the strain increases much up to the point D. When stress is removed from B to D (at C) the body will not regain its original shape and size. It attains permanent set. The deformation is called plastic deformation. The point D is called ultimate yield point.

Beyond it additional strain is produced even for decreased stress and the body breaks at the point E called fracture point (breaking point). If D and E are close, the material is **brittle.** If they are far, it is **ductile.** Substances which can be extended to produce large values of strain are called **elastomers**. eg: rubber, elastic tissue of aorta etc.

- **The stress- strain graph for the elastic tissue of aorta**

  Here the elastic region is large. But it does not obey Hooke's law.



- **Poisson's Ratio**

  If the length of a wire changes from l to $l + \Delta l$ and diameter changes from D to $D - \Delta D$,

  Poisson's ratio = $\frac{Lateral\ strain}{Longitudinal\ strain} = \frac{-\Delta D/D}{\Delta l\ /l} = \frac{-\Delta D.l}{\Delta l\ .D}$

  Poisson's ratio has no unit or dimension.

- **Elastic fatigue**

  When a material is subjected to repeated stresses over a long period ,it loses its strength. This is known as elastic fatigue.

- **Elastic strain energy**

  For causing deformation on a body, work has to be done. This work is stored in the form of elastic strain

- **Expression for elastic strain energy per unit volume of a wire**

  Elastic potential energy stored in the wire (Strain energy) $=\frac{1}{2}\times$stress$\times$strain$\times$volume

- **Relation among different elastic constants:**

  Y = **3B** (1- 2σ)

  Y = **2η** (1+ σ)

  $\sigma = \frac{3B - 2\eta}{2\eta + 6B}$

  $\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$ where Y= Young's modulus, σ = Poisson's ratio, B= bulk modulus, η = shear modulus.

- **Cantilever**

  A beam clamped at one end and loaded at free end is called cantilever.

  Depression at the free end of a cantilever

  $\delta = \frac{wl^3}{3YI_G}$

  where w = load, l = length of the cantilever,

  Y = Young's modulus of elasticity, and

  I_G = geometrical moment of inertia.

  For a beam of rectangular cross-section having breadth b and thicknes d.

  $I_G = \frac{bd^3}{12}$

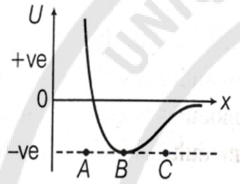
For a beam of circular cross-section area having radius r , $I_G = \frac{\pi r^4}{4}$

Multiple choice questions: Class Work

1. The nature of molecular forces resembles with the nature of the

(a) gravitational force (b) nuclear force
(c) electromagnetic force (d) weak force

2. The potential energy U between two molecules as a function of the distance x between them has been shown in the figure. The two molecules are



(a) Attracted when x lies between A and B and are repelled when x lies between B and C
(b) Attracted when x lies between B and C are repelled when x lies between A and B
(c) Attracted when they reach B
(d) Repelled when they reach B

3. Elasticity is due to

(a) decrease of PE with separation between atoms / molecules
(b) increase of PE with separation between atoms/ molecules
(c) asymmetric nature of PE curve
(d) None of the above

4. A uniform bar of square cross-section is lying along a frictionless horizontal surface . A horizontal force is applied to pull it from one of it ends, then

(a) the bar is under same stress throughout its length
(b) the bar is not under any stress because force has been applied only at one end
(c) the bar simply moves without any stress in it
(d) the stress developed gradually reduces to zero at the end of the bar where no force is applied

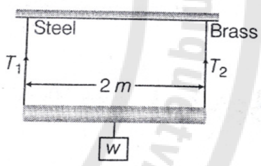
5. A and B are two wires. The radius of A is twice that of B . They are stretched by the same load. Then , the stress on B is

(a) equal to that on A
(b) four times that on A
(c) two times that on A
(d) half that on A

6. On suspending a weight Mg , the length l of elastic wire having area of cross- section A, becomes double the initial length . The instantaneous stress action on the wire is

(a) Mg/A (b) $Mg / 2A$
(c) $2Mg/A$ (d) $4Mg/A$

7. A 2 m long rod is suspended with the help of two wires of equal length. One wire is of steel and its cross-sectional area is 0.1 cm^2 and another wire is of brass and its cross-sectional area is $0.2 cm^2$. If a load W is suspended from the rod and stress produced in both the wires is same , then the ratio of tensions in them will be



(a) depend on the position of W (b) T_1/T_2 = 2
(c) T_1/T_2 = 1 (d) T_1/T_2 = 0.5

8. The length of a wire increases by 1% by a load of $2kgwt$. The linear strain produced in the wire will be

(a) 0.02 (b) 0.001 (c) 0.01 (d) 0.002

9. A uniform cube is subjected to volume compression. If each side is decreased by 1% , then bulk strain is

(a) 0.01 (b) 0.06 (c) 0.02 (d) 0.03

10. A cube of aluminum of side 0.1 m is subjected to a shearing force of $100N$. The top face of the cube is displaced through 0.2 cm with respect to the bottom face . The shearing strain would be

(a) 0.02 (b) 0.1 (c) 0.005 (d) 0.002

11. A copper and a steel wire of the same diameter are connected end to end . A deforming force F is applied to this composite wire which causes a total elongation of 1cm . The two wires will have

(a) the same stress and strain
(b) the same stress but different strain
(c) the same strain but different stress
(d) different strains and stress

12. A steel rod of length 1 m and radius 10 mm is stretched by a force 100 kN along its length . The stress produced in the rod is (Given : Y_{steel} = 2× $10^{11} Nm^{-2}$)

(a) $3.18 \times 10^{6} Nm^{-2}$ (b) $3.18 \times 10^{7} Nm^{-2}$
(b) $3.18 \times 10^{8} Nm^{-2}$ (d) $3.18 \times 10^{9} Nm^{-2}$

13. The graph shows the behavior of a length of wire in the region for which the substance obeys Hooke's law . P and Q represent

(a) P = applied force , Q = extension
(b) P= extension , Q = applied force
(c) P= extension , Q = stored elastic energy
(d) P = stored elastic energy , Q = extension

14. On applying a stress of $20 \times 10^{8} Nm^{-2}$, the length of a perfectly elastic wire is doubled . Its Young's modulus will be

(a) $40 \times 10^{8} Nm^{-2}$ (b) $20 \times 10^{8} Nm^{-2}$
(c) $10 \times 10^{8} Nm^{-2}$ (d) $5 \times 10^{8} Nm^{-2}$

15. A wire of length 2 m is made from 10 cm^3 of copper . A force F is applied so that its length increases by $2mm$. Another wire of length $8m$ is made from the same volume of copper . If the force F is applied to it its length will increase by

(a) 0.8 cm (b) 1.6 cm (c) 2.4 cm (d) 3.2 cm

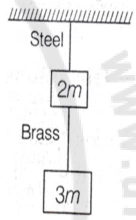
16. The diameter of a brass rod is 4 mm and Young's modulus of brass is $9 \times 10^{10} Nm^{-2}$. The force required to stretch by 0.1 % of its length is

(a) 360 πN (b) 36 N
(c) $144\, \pi \times 10^{3} N$ (d) $36\, \pi \times 10^{5} N$

17. For a perfectly rigid body ,

(a) Young's modulus is infinite and bulk modulus is zero .
(b) Young's modulus is zero and bulk modulus is infinite
(c) Young's modulus is infinite and bulk modulus is also infinite.
(d) Young's modulus is zero and bulk modulus is also zero.

18. If the ratio of diameters, lengths and Young's moduli of steel and brass wires shown in the figure are p, q and r, respectively . Then the corresponding ratio of increase in their lengths would be



(a) $\frac{3q}{5p^2r}$ (b) $\frac{5q}{3\,p^2r}$ (c) $\frac{3q}{5\,pr}$ (d) $\frac{5\,q}{3\,pr}$

19. Which of the following statements is incorrect?

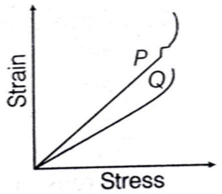
(a) Young's modulus and shear modulus are relevant only for solids .
(b) Bulk modulus is relevant for liquid and gases.
(c) Metals have larger values of Young's modulus than elastomers.
(d) Alloys have larger values of Young's modulus than metals.

20. Identical springs of steel and copper ($Y_{steel} > Y_{copper}$) are equally stretched .

(a) Less work is done on copper spring.
(b) Less work is done on steel spring.
(c) Equal work is done on both the springs.
(d) Data is incomplete.

21. In plotting stress versus strain curves for two materials P and Q , a student by mistake puts strain on the Y-axis and stress on the x- axis

as shown in the figure . Then, the correct statement is /are



(a) P has more tensile strength than Q
(b) P is more ductile than Q
(c) P is more brittle than Q
(d) The Young's modulus of P is more than that of Q

22. The Young's modulus of steel is twice that of brass. Two wires of same length and of same area of cross-section , one of steel and another of brass are suspended from the same roof . If we want the lower ends of the wires to be at the same level , then the weight added to the steel and brass wires must be in the ratio of

(a) 1 : 2 (b) 2 : 1 (c) 4 : 1 (d) 1 : 1

23. Two wires A and B of same material. Their lengths are in the ratio 1 : 2 and diameters are in the ratio 2 : 1 when stretched by force F_A and F_B respectively they get equal increase in their lengths. Then , the ratio F_A/F_B should be

(a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 8 : 1

24. One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R . When the arrangement is stretched by applying forces at two ends , the ratio of the elongation in the thin wire to that in the thick wire is

(a) 0.25 (b) 0.50 (c) 2 (d) 4

25. The upper end of a wire of radius 4 mm and length 100 cm is clamped and its other end is twisted through an angle of 30^o . Then , angle of shear is

(a) 12^o (b) 0.12^o (c) 1.2^o (d) 0.012^o

26. When a pressure of 100 atmosphere is applied on a spherical ball of rubber , then its volume reduces to 0.01% . The bulk modulus of the material of the rubber in dyne cm^{-2} is

(a) 10×10^{12} (b) 100×10^{12}
(d) 1×10^{12} (d) 20×10^{12}

27. A solid sphere of radius R made of a material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, the fractional change in the radius of the sphere is

(a) $\frac{Mg}{KA}$ (b) $\frac{Mg}{2\,KA}$ (c) $\frac{Mg}{3\,KA}$ (d) $\frac{Mg}{4\,KA}$

28. The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube . The vertical deflection of this face is (Shear modulus of aluminium = 25 GPa , g = 10 ms^{-2})

(a) $4 \times 10^{-5} m$ (b) $4 \times 10^{-6} m$
(c) $4 \times 10^{-7} m$ (d) $4 \times 10^{-8} m$

29. The approximate depth of an ocean is $2700m$. The compressibility of water is $45.4 \times 10^{-11} Pa^{-1}$ and density of water is $10^3 kg/m^3$. What fractional compression of water will be obtained at the bottom of the ocean ?

(a) 0.8×10^{-2} (b) 1.0×10^{-2}
(c) 1.2×10^{-2} (d) 1.4×10^{-2}

30. To what depth must a rubber ball be taken in deep sea ,so that its volume is decreased by 0.1 % . (Take , density of sea water as $10^3 kg\, m^{-3}$ bulk modulus of rubber as $9 \times 10^8 Nm^{-2}$, g = $10ms^{-2}$)

(a) 9 m (b) 18 m (c) 90 m (d) 180 m

31. Two wires of the same material and length but diameter in the ratio 1:2 are stretched by the

same load. The ratio of elastic potential energy per unit volume for the two wires is

(a) 1:1 (b) 2:1 (c) 4:1 (d) 16: 1

32. When the load on a wire is increased from 3 $kgwt$ to $5kg\ wt$, the elongation increases from 0.61 mm to 1.02 mm. The required work done during the extension of the wire ,is

(a) $16\ \times 10^{-3} J$ (b) $8\ \times 10^{-2} J$
(c) $20\ \times 10^{-2} J$ (d) $11\ \times 10^{-3} J$

33. If the work done in stretching a wire by 1 mm is $2J$, the work necessary for stretching another wire of same material but with double radius of cross – section and half the length by 1mm is

(a) 16 J (b) 8 J (c) 4J (d) 1/4J

34. A stone of mass m is tied to one end of a wire of length L. The diameter of the wire is D and it is suspended vertically . The stone is now rotated in a horizontal plane and makes an angle θ with the vertical. If young's modulus of the wire is Y , then the increase in the length of the wire is

(a) $\frac{4\,mg\,L}{\pi D^2\,Y}$ (b) $\frac{4\,mg\,L}{\pi D^2\,Y\sin\theta}$
(c) $\frac{4\,mg\,L}{\pi D^2\,Y\cos\theta}$ (d) $\frac{4\,mg\,L}{\pi D^2\,Y\tan\theta}$

35. To break a wire , a force of $10^6 Nm^{-2}$ is required . If the density of the material is $3 \times 10^3 kgm^{-3}$, then the length of the wire which will break by its own weight will be

(a) 34 m (b) 30 m (c) 31 m (d) 29 m

36. A wire of diameter 1 mm breaks under a tension of 1000 N . Another wire of same material as that of the first one, but of diameter 2 mm breaks under a tension of

(a) 500 N (b) 1000 N (c) 10000N (d) 4000N

37. In steel , the Young's modulus and the strain at the breaking point are $2\ \times 10^{11} Nm^{-2}$ and 0.15 , respectively . The stress at the breaking point for steel is therefore

(a) $1.33 \times\ 10^{11} Nm^{-2}$ (b) $1.33 \times\ 10^{12} Nm^{-2}$
(c) $7.5 \times\ 10^{-13} Nm^{-2}$ (d) $3 \times\ 10^{10} Nm^{-2}$

38. A uniform rod of length L and density ρ is being pulled along a smooth floor with a horizontal acceleration α. The magnitude of the stress at the transverse cross- section through the mid-point of the road is

(a) $L\,\rho \propto$ (b) $\frac{L\,\rho\propto}{2}$
(c) $\frac{2}{3}\,L\,\rho \propto$ (d) None of these

39. A metal wire of length L_1 and area of cross-section A is attached to a rigid support . Another metal wire of length L_2 and of the same cross- sectional area is attached to the free end of the first wire. A body of mass M is then suspended from the free end of the second wire . If Y_1 and Y_2 are the Young's moduli of the wires respectively , the effective force constant of the system of two wires is

(a) $\frac{[(Y_1 Y_2)\,A]}{[2\,(Y_1\,L_2+Y_2\,L_1)]}$ (b) $\frac{[(Y_1 Y_2)\,A]}{(L_1 L_2)^{1/2}}$
(c) $\frac{[(Y_1 Y_2)\,A]}{(Y_1\,L_2+Y_2\,L_1)}$ (d) $\frac{[(Y_1 Y_2)^{1/2}\,A]}{(L_1 L_2)^{1/2}}$

40. A uniform rod of mass m , length L , area of cross-section A is rotated about an axis passing through one of its ends perpendicular to its length with constant angular velocity ω in a horizontal plane . If Y is Young's modulus of the material of rod , the increase in its length due to rotation of rod is

(a) $\frac{m\,\omega^2 L^2}{A\,Y}$ (b) $\frac{m\,\omega^2 L^2}{2\,A\,Y}$ (c) $\frac{m\,\omega^2 L^2}{3\,A\,Y}$ (d) $\frac{2\,m\,\omega^2 L^2}{A\,Y}$

41. Two strips of metal are riveted together at their ends by four rivets , each of diameter 6mm. Assume that each rivet is to carry one quarter of the load . If the shearing stress on the rivet is not to exceed $6.9 \times 10^7 pa$, the maximum tension that can be exerted by the riveted strip is