MECHANICAL PROPERTIES OF FLUIDS (Problems)

1. ***Explain why*:**

(a) The blood pressure in humans is grater at the feet than at the brain.
(b) Atmospheric pressure at a height of about 6km decreases to nearly half its value at the seal level , though the height of the atmosphere is more than 100km.
***(c) Hydrostatic pressure is a scalar quantity even through pressure is force divided by area, and force is a vector*.**

(a) The height of the blood column is large at feet than at the brain. So the blood pressure is greater at the feet . (b) The density of air does not decrease linearly with height. The density decreases rapidly to a height of about 6km and above 6km, it decreases rather very slowly. So the atmospheric pressure at a height of about 6km decreases to nearly half its value at the seal level. (c) Hydrostatic pressure is transmitted equally in all directions. It has no particular direction. Hence it is scalar.

2. ***Explainwhy***

(i) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.
(ii) Water on clean glass surface tends to spread out while mercury on the same surface tends to from drops.(Put differently, water wets the glass while mercury does not)
(iii)Surface tension of a liquid is independent of the area of the surface.
***(iv) Detergent should have small angle of contact*.**

(i) The cohesive force between mercury molecules is greater than the adhesive force between mercury and glass molecules .But for water, the adhesive force between water and glass molecules is greater than the cohesive force between water molecules.
(ii) The cohesive force between mercury molecules is greater than the adhesive force between mercury and glass molecules .But for water, the adhesive force between water and glass molecules is greater than the cohesive force between water molecules.

(iii) Surface tension is the force acting per unit length of a line tangential to a liquid surface at rest . It depends on the nature of the liquid and its temperature .It is independent of the area of the liquid surface.

(iv) If the angle of contact is small, then it will have low surface tension and hence greater ability to wet a surface. Capillary rise $\propto cos\theta$.If θ is small, $cos\,\theta$ will be large and hence detergent will penetrate (h = $\frac{2\,\sigma\cos\theta}{r\rho g}$) through pores and removes dirt.

3. ***Fill in the blanks using the work (s) from the list appended with each statement:***
(i) Surface tension of liquids generally………… with temperatures (increases / decreases).
(ii) Viscosity of gases ……….. with temperature , whereas viscosity of liquids …….with temperature(increases/ decreases).
(iii) For solids with elastic modulus of rigidity , the shearing force is proportional to ………while for fluids it is proportional to (Shear strain /rate of shear strain).
(iv) For a fluid in steady flow, the increase in flow speed at a constriction follows from …………while the decrease of pressure there follows from……
(v) For the model of plane in a wind tunnel, turbulence occurs at a ……… speed than the critical speed for turbulence for an actual plane (greater /smaller.).

(i) Decreases
(ii) increases, decreases
(iii) shear strain, rate of shear
(iv) conservation of mass Bernoulli's principle
(v) greater.

4. **Explain why**

(i) To keep a piece of paper horizontal, you should blow over , not under it .

(ii) When we try to close a water tap with our fingers, fast jets of water gush through the opening between our fingers.

(iii)The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administrating an injection.

(iv) A fluid flowing out of a small hole in a vessel result in a backward thrust on the vessel.

(i) When we blow over the paper , the velocity of air increases and hence pressure of air decreases (by Bernoulli's principle).The high pressure below the paper keeps the paper horizontal.

(ii) On closing, the available area decreases. According to the equation of continuity, velocity increases .

(iii) By Bernoulli's principle,
$p + \rho gh + \frac{1}{2}pv^2$ = constant
The total energy of the medicine injected depends upon square of velocity and first power of the pressure .

(iv) As the area of cross-section of the hole is small the fluid flows out of the vessel with large speed .So it has a large linear momentum. In order to conserve the linear momentum, the vessel acquires a backward momentum. Thus a backward thrust acts on the vessel.

5. ***Torricelli's barometer used mercury. Pascal duplicated it using French wine of density 984kg m^{-3}. Determine the height of the wine column for normal atmospheric pressure.***

Pressure exerted by h cm of wine column
= Pressure exerted by 76 cm of Hg column
or $h \times 984 \times 9.8 = 0.76 \times 13.6 \times 10^3 \times 9.8$
$\therefore h = \frac{0.76 \times 13.6 \times 10^3}{984}$ = 10.5 m.

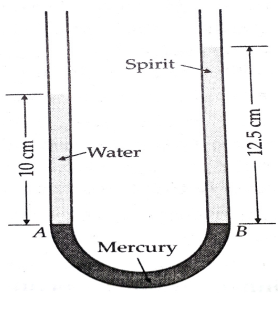
6. **A vertical off – shore structure is built to withstand a maximum stress of 10^9pa..Is the structure suitable for putting up on top of an oil well in Bombay High? Take the depth of the sea to be roughly 3km, and ignore ocean currents.**

h = 3 km = 3000m ρ (water) = 1000 kgms^{-3}
Pressure due to sea water,
P = $h\rho g$ = $3000 \times 1000 \times 9.8 = 2.94 \times 10^7$ Pa $\ll 10^9$ Pa.
Hence the structure is suitable.

7. ***A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000kg. The area of cross-section of the piston carrying the load is 425cm^2. What maximum pressure would the smaller piston have to bear?***

Area of cross section of large piston
A = 425cm^2 = $425 \times 10^{-4} m^2$
Load on larger piston, F = mg = 3000$\times$ 9.8 N
Pressure on large piston,P $= \frac{F}{A} = \frac{3000 \times 9.8}{425 \times 10^{-4}}$
$= 6.92 \times 10^5 Nm^{-2}$
The liquids transmits pressure equally in all directions. So the pressure the smaller piston bears = $6.92 \times 10^5 Nm^{-2}$

8. ***A U –tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0cm of water in one arms and 12.5cm of spirit in the other. What is the specific gravity of spirit ?***



The mercury of columns in the two arms of the U-tube are at the same level,
$\therefore$ Pressure due to water column = Pressure due to spirit column
$h_\omega \rho_\omega g = h_s \rho_s g$ or $h_\omega \rho_\omega = h_s \rho_s$
h_ω = 10cm, ρ_ω = 1g cm^{-3}, h_s = 12.5 cm

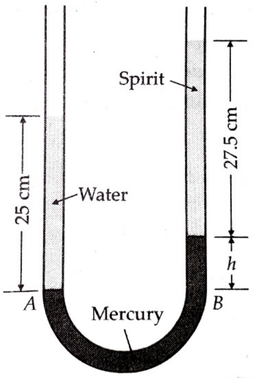
$\therefore$ 10 × 1 = 12.5 × ρ_s or $\rho_s = \frac{10}{12.5}$

= 0.8g cm^{-3}

Specific gravity of spirit $= \frac{\rho_s}{\rho_\omega}$

$= \frac{0.8g\ cm^{-3}}{1g\ cm^{-3}} = 0.8.$

9. ***In previous exercise, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube , what is the difference in the levels of mercury in the two arms?***



Specific gravity of mercury = 13.6 .

Pressure on mercury level in one arms due to water,

$P_1 = h_\omega \rho_\omega$ g = (10 + 15) × 1 × g = 25 g

Pressure on mercury level in another arm due to spirit,

$P_2 = h_s \rho_s$ g = (12.5 + 15) × 0.8 × g = 22g

The pressure in water arm is more ,so the mercury will rise in spirit arm. If this pressure difference corresponds to height difference h in the two arms, then

$P_1 - P_2 = h\ \rho\ g$

25g - 22g = h × 13.6 × g or h = $\frac{3}{13.6}$ = 0.221 cm.

Mercury rises in the arm containing spirit.

The difference in the levels of mercury in the two columns = 0.221cm.

10. ***Can Bernoulli's equation be used to describe the flow of water through rapid in a river? Explain.***

No, The rapid flow of water in a river is turbulent and not streamlined. Bernoulli's equation is applicable to streamline flow of a fluid.

11. ***Glycerin flows steadily through a horizontal tube of length 1.5 m and radius 1.0cm .If the amount of glycerin collected per second at one end is 4.0 × $10^{-3} kgs^{-1}$, what is the pressure difference between the two ends of the tube? Density of glycerin = 1.3 × $10^3 kgm^{-3}$ and viscosity of glycerin = 0.83 Nsm^{-2} .***

$l = 1.5\,m$, $r = 1.0cm = 1.0 \times 10^{-2}m$,

$\rho = 1.3 \times 10^3 kgm^{-3}$, $\eta = 0.83 Nsm^{-2}$

Mass collected per second = $1.3 \times 10^{-3} kg/s$

$\therefore$ Volume of liquid flowing per second, Q

$= \frac{Mass\ collected\ per\ second}{Density}$

$= \frac{4.0 \times 10^{-3}}{1.3 \times 10^3} m^{-3}s^{-1}$

$Q = \frac{\pi P r^4}{8\,l\,\eta}$ $\therefore P = \frac{8 l \eta Q}{\pi r^4} = \frac{8 \times 1.5 \times 0.83}{3.14 \times (1.0 \times 10^{-2})^4} \times \frac{4.0 \times 10^{-3}}{1.3 \times 10^3}$

= 9.8 × 10^2Pa.

12. ***In a test experiment on a model aero plane in a wind tunnel, the flow speed on the upper and lower surface of the wing are 70 ms^{-1} and 63ms^{-1} respectively. What is the lift of the wing if its area is 25 m^2 ? Density of air = 1.3 kgm^{-3}.***

ρ = 1.3 kgm^{-3} , v_1 = 70 ms^{-1}, v_2 = 63ms^{-1}

Let P_1 and P_2 be the pressure on the upper and lower surface of the wing. By Bernoulli's theorem,

$\frac{P_1}{\rho} + \frac{1}{2}{v_1}^2 = \frac{P_2}{\rho} + \frac{1}{2}{v_2}^2$

or $P_2 - P_1 = \frac{1}{2}({v_1}^2 - {v_2}^2)\rho$

$= \frac{1}{2}(70^2 - 63^2) \times 1.3 = 605.15\ Nm^{-2}$

Lift of the wing = Net upward pressure × area of the wing

$=(P_2 - P_1)$ A = 605.15 × 25 N

= 1512.9 N.

13. ***Figure (a) and (b) refer to the steady flow of a (non – viscous) liquid. Which of the two figures is incorrect? Why?***

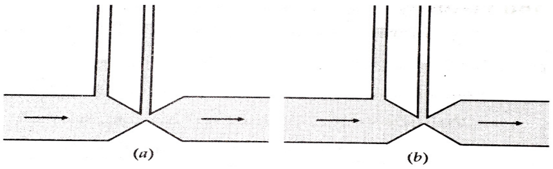


Figure (a) is incorrect. At the constriction, the area of cross-section is small and the liquid velocity is large. So the liquid pressure is small (Bernoulli's principle).

14. ***The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2 , one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1} what is the speed of ejection of liquid through the holes?***

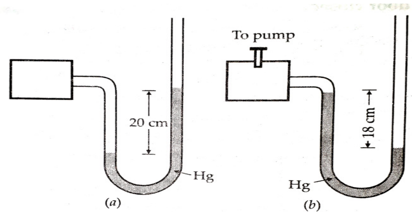
a_1 = 8.0 cm^2 = 8 $\times$ $10^{-4}m^2$
v = 1.5 m min^{-1}= $\frac{1.5}{60}$ ms^{-1}
Radius of a hole = $\frac{d}{2}$ = $\frac{1.0\ mm}{2}$ = 0.5 $\times$ $10^{-3}m$
∴ Cross-section of a hole = π $\times (0.5\ \times\ 10^{-3})^2 m^2$
Total cross-section of 40 holes,
a_2 = π $\times (0.5\ \times\ 10^{-3})^2 \times\ 40m^2$
If v_2 is the speed of ejection of the liquid through the holes, then $a_1 v_1$ = $a_2 v_2$
or v_2 = $\frac{a_1 v_1}{a_2}$ = $\frac{8\ \times 10^{-4} \times 1.5}{\pi \times (0.5 \times 10^{-3})^2 \times 40 \times 60}$
= 0.637 ms^{-1}

15. ***A manometer read the pressure of a gas in an enclosure as shown in figure (a) when some of the gas is removed by a pump, the manometer reads as in figure***

The liquid used in the manometer is mercury and the atmospheric pressure is 76 cm of mercury.

(i) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b) in units of cm of mercury.

(ii) How would the level change in case (b0 if 13.6cm of water (immiscible with mercury) are poured into the right limb of the manometer? (Ignore the small change in volume of the gas)



Atmospheric pressure, P = 76 cm of Hg

(i) Case (a):

Pressure head , h = + 20cm of Hg
Absolute pressure = P + h = 76 +20 = 96cm of Hg
Gauge pressure = P = 20 cm of Hg
Cases (b) : Pressure head , h = - 18cm of Hg
Absolute pressure =P + h= 76+ – 18
= 58 cm of Hg
Gauge Pressure = h = - 18 cm of Hg.

(ii)Pressure exerted by mercury = Pressure exerted by water

$h_1\rho_1$g = $h_2\rho_2$g
$h_1 \times 13.6\ \times g = 13.6\ \times 1\ g$ or h_1 = 1 cm
As 13.6 cm of water is poured in right limb, it will displace mercury level by 1 cm in the left limb, so the difference of levels = 19cm.

16. ***Two vessels have the same base area but different shape . The first vessel takes twice the volume of water that the second vessel requires to fill up to a particular common height***

(i)Is the force exerted by the water on the base of the vessel the same in the two cases?

(ii)If so, why do the vessels filled with water to that same height give different readings on a weighing scale?

(i) The vessels are filled to the same height. So the pressure exerted on the bases of the two vessels is equal. The two vessels have the same base area, so force exerted on the bases will be equal.

(ii)Water exerts force on the sides of the vessel also. This force will have a non-zero vertical component when the side of the vessel are not perpendicular to the base. The net vertical component of the force exerted by water on the sides of the vessel is greater for the first vessel than the second. Hence they weigh differently.

17. ***During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? The density of whole blood = 1.06$\times$ $10^3 kg\, m^{-3}$.***

Let h be the height of container. Then

hρg $=P_g$ OR h = $\frac{P_g}{\rho g}$ = $\frac{2000}{1.06 \times 10^3 \times 9.8}$

= 0.1925m

The blood will just enter the vein if the blood container is at height slightly greater than 0.1925m

18. ***In deriving Bernoulli's equation, we equated that work done on the fluid in the tube to its change in the potential and kinetic energy. (a) How does the pressure change as the the fluid moves along the tube if dissipative force are present? (b) Do the dissipative force become more important as the fluid velocity increases? Discuss qualitatively.***

a) If dissipative force are present , some of the pressure energy of the fluid is spent in doing work against these forces, so the fluid pressure decreases with the increase in length of the tube.

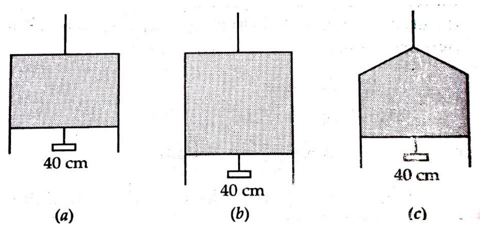
(b) Yes , the dissipative forces become more important when the fluid velocity increases.

19. ***A U – shaped wire is dipped in soap solution, and removed. The thin soap film formed between the wire and a light slider supports a weight of 1.5 $\times$ $10^{-2}N$ (which include the small weight of the slider). The length of the slider is 30cm. What is the surface tension of the film?***

F = 1.5 $\times$ 10^{-2}N, l = 30cm = 0.3 m

S = $\frac{F}{2l}$ = $\frac{1.5 \times 10^{-2}}{2 \times 0.30}$ = 2.5 $\times$ $10^{-2} Nm^{-1}$.

20. ***Figure (a) shows a thin liquid film supporting a small weight = 4.5 $\times$ $10^{-2}N$. What is the weight supported by a film of the same liquid at the same temperature in figure (b) and (c) Explain your answer physically.***



The weight supported both in (b) and (c) is 4.5 $\times$ $10^{-2}N$

The weight supported = 2 s l.

As s and l are same in all cases, the weight supported is same.

21. ***What is the pressure inside a drop of radius 3.00 mm of room temperature (20^oC), S=4.65 $\times$ $10^{-1}Nm^{-1}$. The atmospheric pressure is 1.01$\times$ 10^5 Pa. Also find the excess pressure inside the drop.***

R = 3.00 mm = 3.00 $\times$ 10^{-3}m,

S = 4.65 $\times$ $10^{-1}Nm^{-1}$, P_o = 1.01 $\times$ 10^5 pa

Excess pressure P = $\frac{2S}{R}$ = $\frac{2 \times 4.65 \times 10^{-1}}{3.00 \times 10^{-3}}$

= 310 Nm^{-2}

Total pressure inside the drop ,

P = Atmospheric pressure + Excess pressure

= 1.01 $\times$ 10^5 + 310 = 1.0131 $\times$ 10^5 pa

22. ***What is the excess pressure inside a bubble of soap solution of radius 5.00mm? Given that the surface tension of soap solution at the temperature (20^oC)is 2.50$\times$ $10^{-2}Nm^{-1}$ If an air bubble of the same dimension were formed at a depth of 40.0cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atm = 1.01 $\times$ 10^5 Pa)***

r = 5.00mm = 5.00 $\times$ $10^{-3}m$,

S = 2.50 $\times$ $10^{-2}Nm^{-1}$, h = 40cm = 0.4m ,

P_0 = 1.01 $\times$ 10^5 Pa

Excess pressure inside the soap bubble ,

P = $\frac{4S}{R}$ = $\frac{4 \times 2.50 \times 10^{-2}}{5.00 \times 10^{-3}}$ = 20 Pa .

Excess pressure inside an air bubble under soap solution

P' = $\frac{2S}{R}$ = $\frac{2 \times 2.50 \times 10^{-2}}{5.00 \times 10^{-3}}$ = 10 Pa.

Density of soap solution ,

ρ = Relative density $\times$ 1000 = 1.20 $\times$ 1000

= 1200kgm^{-3}

Total pressure inside the air bubble

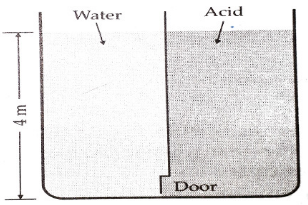
= Atmospheric pressure + Pressure due to 40cm soap solution + Excess pressure

= 1.01 $\times$ 10^5 + $h\,\rho$ g + P'

= 1.01 $\times$ 10^5 + 0.40 $\times$ 1200 $\times$ 9.8 + 10

= 105714 Pa.

23. A tank with a square base of area 1.0m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relativedensity 1.7) in the other, both to a height of 4.0m. Compute the force necessary to keep the door closed.For the compartment containing water:



Height of water column , h = 4.0 m

Density of water , ρ = $\times$ $10^3 kgm^{-3}$

Pressure due to water at the door at the bottom ,

P_ω = $h\rho$g = 4.0 $\times$ 10^3 $\times$ 9.8 = 39.2 $\times$ 10^3 pa

For compartment containing acid :

Height of acid column = 4.0 m

Density of acid ,$\rho = 1.7 \times 10^3 kg\, m^{-3}$

Pressure due to acid at the door at the bottom,

P_a = pρg = 4.0 $\times$ 1.7 $\times$ 10^{-3} $\times$ 9.8

= 66.64 $\times$ 10^3 Pa

$\therefore P_a - P_\omega$ = 66.64 $\times$ 10^3 - 39.2 $\times$ 10^3

=27.44 $\times$ 10^3 Pa

Area of the door , A = 20 cm^2 = 20 $\times$ $10^{-4} m^2$

Force on the door due to difference of pressure on its two sides.

= $(P_a - P_\omega)$$\times$ A

= 54.88 N.

To keep the door closed a force of 54.88N must be applied from the water side.

24. (a)What is the largest average velocity of blood flow in an artery of radius 2$\times$ 10^{-3}m if the flow must remain laminar ? (b) What is the corresponding flow rate? Take viscosity of blood to be 2.084 $\times$ 10^{-3} Pa and density of blood = 106 $\times$ $10^3 kgm^{-3}$

(a) ρ = 1.06 $\times$ $10^3 kgm^{-3}$,

D = 2r = 4 $\times$ 10^{-3}m , η = 2.084 $\times$ 10^{-3} Pa

v_c = $\frac{R_e \eta}{\rho D}$ = $\frac{2000 \times 2.084 \times 10^{-3}}{1.06 \times 10^3 \times 4 \times 10^{-3}}$ = 0.98 ms^{-1}

(b) Volume of blood flowing per second,

Q= av_c = $\pi r^2 v_c$ = $\frac{22}{7}$ $\times$ $(2 \times 10^{-3})^2$ $\times$ 0.98

= 1.23 $\times$ $10^{-5} m^3 s^{-1}$.

25. A plane in level flight at constant speed and each of its two wings has an area of 25 m^2. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. Take air density to be 1 kgm^{-3} and g = 9.81ms^{-2}

v_1 = 180 km h^{-1} = 180 $\times$ $\frac{5}{18} = 50 ms^{-1}$

v_2 = 234 km h^{-1} = 234 $\times$ $\frac{5}{18} = 65\ ms^{-1}$

Area of the wings, A = 2 $\times$ 25 = 50 m^2 ,

ρ = 1 kgm^{-3}

For a plane in the level flight , Bernoulli's equation is

$P_1 + \frac{1}{2}\rho {v_1}^2 = P_2 + \frac{1}{2}\rho {v_2}^2$

or $P_1 - P_2 = \frac{1}{2}\rho({v_2}^2 - {v_1}^2)$

$= \frac{1}{2} \times 1\,(65^2 - 50^2)$ = 862.5 Nm^{-2}

Upward force on the plane = (P_1 -P_2) $\times$ A

= 862.5 $\times$ 50 = 43125 N

In level flight, the upward force balances the weight of the plane . $\therefore$ mg = 43125 N

Mass of the plane , m = $\frac{43125}{g}$ = $\frac{43125}{9.81}$ = 4396 kg.

26. In Millikan's oil drop experiment, what is the terminal speed of a drop of radius 2.0 $\times$ 10^{-5} m and density

1.2 × $10^3 kgm^{-3}$? Take the viscosity of air at the temperature of the experiment to be 1.8 × $10^{-5} Nm^{-2}$. How much is the viscous force on the drop at that speed ? Neglect buoyancy of the drop due to air.

r = 20 × 10^{-5}m , ρ = 1.2 × $10^3 kgm^{-3}$
η =1.8 × $10^{-5} Nm^{-2}$
If the buoyancy of the drop due to air is neglected
$v = \frac{2}{9}\frac{r^2 \rho g}{\eta} = \frac{2}{9} \times \frac{(2.0 \times 10^{-5})^2 \times 1.2 \times 10^3 \times 9.8}{1.8 \times 10^{-5}}$
= 5.8 × $10^{-2} ms^{-1}$
Viscous force , F= 6 $\pi\eta r v$=6 × $\frac{22}{7}$ × 1.8 × 10^{-5} × 20 × 10^{-5} × 5.8 × 10^{-2} = 3.9 × 10^{-10} N.

27. ***Mercury has an angle of contact equal to 140^o with soda lime glass. A narrow tube of radius. 1.00mm made of thin glass is dipped in through containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465Nm^{-1}. Density of mercury = 13.6 × $10^3 kgm^{-3}$***

θ = 140^o , r = 1.00mm = 1.00 × 10^{-3} m
S = 0.465Nm^{-1} , ρ = 13.6 × $10^3 kgm^{-3}$
$cos 140^o$ = $cos(180^o - 40^o)$ = - $cos 40^o$
= - 0.7660
∴ h = $\frac{2S\cos\theta}{r\rho g}$ = $\frac{2 \times 0.465 \times cos 140^o}{13.6 \times 9.8}$
= - 5.34 × 10^{-3} m
The negative sign shows capillary depression.

28. ***The narrow bores of diameter 3.0 mm and 6.0 mm are joined together contains water, what is the difference in level in the two limbs of the tube? Surface tension of the water at the temperature of experiment is 7.3 × $10^{-2} Nm^{-1}$.Take the angle of contact to be zero and density of water to be 1.0× $10^3 kgm^{-3}$ Take g = 9.8ms^{-2} .***

r_1 = $\frac{3.0}{2}$ = 1.5 mm = 1.5 × 10^{-3}m,
r_2 = = $\frac{6.0}{2}$ = 1.5 mm = 3.0 × 10^{-3}m,
S = 7.3 × $10^{-2} Nm^{-1}$, θ = 0^o
ρ = 1.0 × $10^3 kgm^{-3}$, g = 9.8ms^{-2}
Let h_1 and h_2 be the height to which water rises in the two tubes .
Then, h_1 = $\frac{2S\cos\theta}{r_1 \rho g}$ and h_2 = $\frac{2S\cos\theta}{r_2 \rho g}$
∴ h_1 - h_2 = $\frac{2S\cos\theta}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$
= $\frac{2 \times 7.3 \times 10^{-2} cos 0^o}{10^3 \times 9.8}$ = $\left[\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-2}}\right]$
= 5.290 mm.