WORK,ENERGY& POWER-PROBLEMS

1. ***An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV , and the second with 100 keV. Which is faster the electron or the proton? Obtain the ratio of their speeds.***

Given: Electron mass = 9.11 × 10^{-31}kg proton mass = 1.67 × 10^{-27}kg 1 eV =m 1.60 × 10^{-19} J

v_e = speed of electron,

v_p = speed of proton,

m_e= mass of electron

m_p = mass of proton

$\frac{1}{2}m_e{v_e}^2$ = 10 keV = 10 × 10^3× 1.6 × 10^{-19}

= 1.6 × 10^{-15} J

$\frac{1}{2}m_p{v_p}^2$ = 100 keV = 100 × 10^3× 1.6 × 10^{-19}

= 1.6 × 10^{-14} J

$\therefore \frac{\frac{1}{2}m_e{v_e}^2}{\frac{1}{2}m_p{v_p}^2} = \frac{1.6 \times 10^{-15}}{1.6 \times 10^{-14}} = \frac{1}{10}$

or $\frac{v_e}{v_p} = \sqrt{\frac{m_p}{10 m_e}}$

m_e = 9.11 × 10^{-31} kg and m_p = 1.67 × 10^{-27} kg

$\therefore \frac{v_e}{v_p} = \sqrt{\frac{1.67 \times 10^{-27}}{10 \times 9.1 \times 10^{-31}}}$ = 13.54

For the entire journey,

Work done by gravity = Work done by resistive force = gain in kinetic energy

$2 \times 0.082 + W_r = \frac{1}{2}mv^2$

$$W_r = -0.1623\, J$$

2. ***A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration until at half its original height, it attains its maximum speed, and moves with uniform speed there after. What is the work done by the gravitational force on the drop in the first and the second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 ms^{-1}?***

r = 2 mm = 2 × 10^{-3} ; h = $\frac{500}{2}$ = 250 m

Density of water = ρ = 10^3 kg m^{-3} .

Mass of drop, M = $\frac{4}{3}\pi r^3\rho = \frac{4}{3}\pi \times (2 \times 10^{-3})$

$= \frac{32\pi}{3} \times 10^{-6}$kg

Gravitational force on drop, mg

= $\frac{32\pi}{3} \times 10^{-6} \times 9.8$ N

Work done during each half

= P.E = mgh = $\frac{32\pi}{3} \times 10^{-6} \times 9.8 \times 250$ = 0.082 J

For the entire journey,

The work done by gravity + work done by resistive force = gain in kinetic energy

$2 \times 0.082 + W_r = \frac{1}{2}mv^2$

$W_r = -0.1623$ J

3. ***A molecule in a gas container hits a horizontal wall with speed 200 ms^{-1} and angle 30^0 with the normal, and rebounds with the same speed. Is momentum conversed in the collision? Is the collision elastic or inelastic?***

If m is mass of the gas molecule and M is mass of wall, then total K.E after collision,

$E_2 = \frac{1}{2}m(200)^2 + \frac{1}{2}M(0)^2$ = 2 × 10^4 m J

$E_1 = \frac{1}{2}m(200)^2$ = 2 × 10^4 m J

$E_1 = E_2$

Collision is elastic.

4. ***A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30 %, how much electric power is consumed by the pump?***

Volume of water = 30 m^3 ; t = 15 min = 15 × 60

= 900 s

h = 40 m η = 30 %

Density of water = ρ = 10^3 kg m^{-3}

Mass of water pumped , m = volume × density

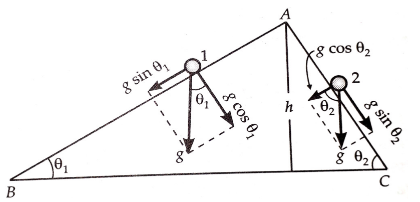
= 30 × 10^3 kg

Power consumed or output power

$P_o = \frac{W}{t} = \frac{mgh}{t} = \frac{30 \times 10^3 \times 9.8 \times 40}{900}$

= 13070 W $\eta = \frac{P_o}{P_i}$ $P_i = \frac{P_o}{\eta} = \frac{13070}{30/100}$ = 43567 × 10^3 W

5. ***Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track. Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given $\theta_1 = 30^0$ $\theta_2 = 60^0$ and h = 10 m, what are the speeds and times taken by the two stones?***



$\frac{1}{2}mv^2 = mgh$ or $v = \sqrt{2gh}$ v = u + at

v = 0 + at t = $\frac{v}{a}$ t ∝ $\frac{1}{a}$ a = g sin θ

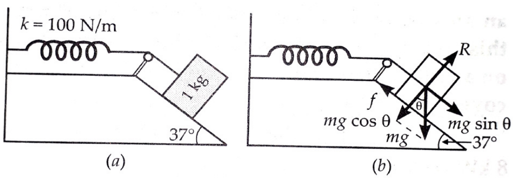
a_1 = g sin θ_1 and

a_2 = g sin θ_2 $a_2 > a_1$

Thus both the stones will take different times.

Stone 2 will take lesser time.

6. ***A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N m^{-1} as shown in figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the co-efficient of friction between the block and the incline. Assume that the spring has negligible mass and the pulley is frictionless.***



R = mg cosθ F = μR = μ mg cos θ

Net force on the block down along the inclined surface

= mg sin θ - F = mg sin θ - μ mg cos θ

= mg (sin θ - μ cos θ) Distance moved

x = 10 cm = 0.1 m

In equilibrium , Work done = P.E of stretching spring

mg (sin θ - μ cos θ) x = $\frac{1}{2}$ Kx^2

2 mg (sin θ - μ cos θ) = Kx

2 × 1 × 10 (sin 37^0 - μ cos 37^0) = 100 × 0.1

20(0.601 - μ × 0.798) = 10 or μ = 0.126

7. ***A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end tothe other with a speed of 4 ms^{-1} relative to the trolley in a direction opposite to the trolleys motion and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?***

Initial total momentum = P_i = ($m_1 + m_2$) u_1

= (20 + 200) $\left(36 \times \frac{5}{18}\right)$ = 2200 kg ms^{-1}

Final total momentum

= $P_f = m_1v_1 + m_2v_2 = 20(v_2 - 4) + 200\,v_2$

= $(220\,v_2 - 80)$

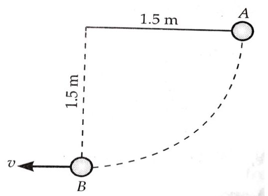
By the law of conservation of total momentum

$P_i = P_f$ 2200 = $(220\,v_2 - 80)$

or

$v_2 = \frac{2280}{220}$ = 10.36 ms^{-1}

8. ***The bob of a pendulum is released from a horizontal position A as shown in fig. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipated 5 % of its initial energy against air resistance?***



P.E = mgh = m × 9.8 × 1.5

5 % of energy is lost,

∴ P.E = m × 9.8 × 1.5 × $\frac{95}{100}$

This energy is converted to K.E ∴ K.E = P.E

$\frac{1}{2}mv^2$ = m × 9.8 × 1.5 × $\frac{95}{100}$

or $v = \sqrt{2 \times 9.8 \times 1.5 \times \frac{95}{100}}$ = 5.29 ms^{-1}

9. ***A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$, where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from x = 0 to x = 2m?***

m = 0.5 kg $v = a x^{3/2}$ $a = 5\ m^{-1/2}\ s^{-1}$

Initial velocity at x = 0 , $v_1 = a \times 0 = 0$

Final velocity at x = 2 , $v_2 = a(2)^{3/2} = 5 \times (2)^{3/2}$

Work done = increase in K.E

$= \frac{1}{2} m(v_2^2 - v_1^2) = \frac{1}{2} \times 0.5\ [(5 \times 2^{3/2})^2 - 0] = 50$ J.

10. ***The blades of a windmill sweep out a circle of are A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t? (b) What is the kinetic energy of the air? (c)Assume that the windmill converts 25 % of the wind's energy into electrical energy, and that $A = 30\ m^2$, v = 36 km/hand the density of air is 1.2 $kg\ m^{-3}$. What is the electrical power produced?***

(a) Volume of wind flowing per second = Av

Mass of wind flowing per second = Avρ

Mass of air passing in time t = Avρt

(b) K.E of air $= \frac{1}{2} mv^2 = \frac{1}{2} (Av\rho t)\ v^2 = \frac{1}{2} Av^3\rho t$

(c) Electrical energy produced = $\left[\frac{25}{100}\right] \times$ K.E of air

$= \frac{1}{4} \times \frac{1}{2} Av^3 \rho t$

Electric power $= \frac{1}{8} \frac{Av^3\rho t}{t}$

$= \frac{1}{8} \times 30 \times (10)^3 \times 1.2 = 4500$ W

11. ***A person trying to lose weight lifts a 10 kg mass 0.5 m, 1000 times. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force? (b) Eat supplies 3.8×10^7 J of energy per kilogram which is converted to mechanical energy with a 20 % efficiency rate. How much fat will be the dieter use up?***

m = 10 kg , h = 0.5 m , n = 1000

(a) Work done against gravitational force

= W = n mgh

= 100 × (10 × 9.8 × 0.5) = 49000 J

(b)Mechanical energy supplied by 1 kg of fat

$= 3.8 \times 10^7$ J $\times \frac{20}{100} = 0.76 \times 10^7$ J/kg

∴Fat used by the dieter $= \frac{1}{0.76 \times 10^7} \times 49000$

$= 6.45 \times 10^{-3}$ kg

12. ***A bullet of mass 0.012 kg and horizontal speed 70 ms^{-1}` strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thinwires. Calculate the height towhich the block rises. Also , estimate the amount of heat produced in the block.***

By the law of conservation of momentum

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$

or

$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)} = \frac{0.012 \times 70 \times 0.4 \times 0}{0.012+0.4} = 2.04$ ms^{-1}

When bullet and block rise to height h, using the law of conservation of total energy

$\frac{1}{2}(m_1 + m_2)v^2 = (m_1 + m_2)$ gh

$h = \frac{v^2}{2g} = \frac{(2.04)^2}{2 \times 9.8} = 0.212$ m

Heat produced = loss in kinetic energy

$\frac{1}{2} m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$

$= \frac{1}{2}(0.012)(70)^2 - \frac{1}{2}(0.012+0.4)(2.04)^2$

= 29.4 – 0.86 = 28.54 J

13. ***A body is moving uni directionally under the influence of a source of constant power. Its displacement in time t is proportional to***

(i) $t^{1/2}$ (ii) t (iii)$t^{3/2}$ (iv)t^2

Using work –energy theorem,

$W = P \times t = \frac{1}{2} mv^2$

$v^2 = \frac{2\ Pt}{m}$

∴ $v = \frac{dX}{dt} = \left(\frac{2\ Pt}{m}\right)^{1/2}$

Integrating

$x = \left(\frac{2\ p}{m}\right)^{1/2} \frac{2}{3} t^{3/2}$

∴ $s \propto t^{3/2}$

14. ***A body constrained to move along the Z – axis of a co-ordinate system is subject to a constant force $\vec{F} = -\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ N, where $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are unit vectors along the X, Y , and Z axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the Z-axis?***

$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N

The body moves a distance of 4 m along Z- axis.

$\therefore \vec{s} = 4\hat{k}$ m.

$\therefore W = \vec{F} \cdot \vec{s} = (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (4\hat{k})$

$= -1 \times 0 + 2 \times 0 + 3 \times 4 = 12J.$

15.*A large family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square metre. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8kW?*

(b)Compare this area to that of the roof of a house constructed on a plot of size 20 m × 15 m with a permission to cover up to70%

(a) Let the area needed to supply 8 kW = Am^2

Energy incident per unit area = 200W

Energy incident on area A = 200 × A W

Energy converted into useful electrical energy

= 20% of 200 × A = 40 A W

40 AW = 8 kW = 8000 W

or

A = $\frac{8000}{40}$ = $200\mathrm{m}^2$.

(b) Area of the roof of the given house,

A' = 70% of 20 m× 15m

=$\frac{70\times20\times15}{100}$ = $210\mathrm{m}^2$.

Ratio = $\frac{A}{A'}$ = $\frac{200}{210}$ = 20: 21.

16. *A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform sped of 7 ms^{-1}. It hits the floor of the elevator (length of the elevator 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different, if the elevator were stationary?*

The elevator is moving down with a uniform speed (a=0). Hence the value of g remains the same.

m = 0.3 kg , h = 3 m. g = $9.8\mathrm{ms}^2$

P.E. lost by the bolt = mgh = 0.3 ×9.8 × 3 = 8.82 J

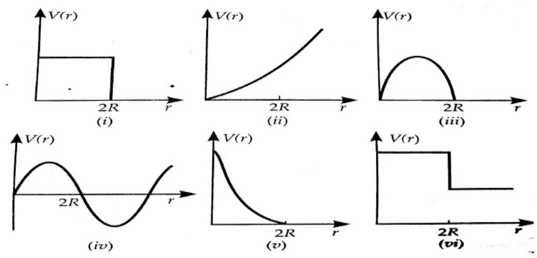
The bolt does not rebound. So the energy is converted into heat.

∴ Heat produced = 8.82J

The value of g is same in all inertial frames of reference.

Hence even if the elevator were stationary, the same amount of heat would have produced .

17. *Which of the following potential energy curves in cannot possibly describe the elastic collision of twobilliard balls? Here r is the distance between centres of the balls.*



18.*A trolley of mass 300kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand start leaking out of a hole on the trolley's floor at the rate of 0.05kgs^{-1} . What is the speed of the trolley after the entire sand bag is empty?*

The trolley carrying the sandbag is moving with uniform speed of 27 km/h.

The external force acting on the trolley + sandbag system = zero

When the sand leaks out, it does not cause any external force to act on the system.

So the speed of the trolley remains unchanged even after the sandbag become empty.