SYSTEM OF PARTICLES AND ROTATIONAL MOTION (Problems)

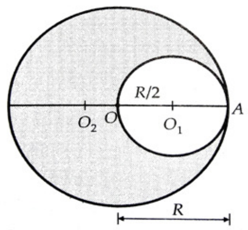
1. *A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.*

M = 3 kg , R = 40 cm = 0.40 m , F = 30N
Torque , τ = F× R = 30 × 0.40 = 12 Nm
$M.I$ .of the hollow cylinder about its own axis,
$I= MR^2$ = 3 × $(0.40)^2$ = 0.48 kg m^2
Angular acceleration,
$\propto = \frac{\tau}{I} = \frac{12}{0.48}$ = 25 rad s^{-2}
Linear acceleration,
a = R∝ = 0.40 × 25 = 10 ms^{-2}.

2. *To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine? Assume that the engine is 100% efficient.*

ω = 200 rad s^{-1} , τ = 180 Nm
∴ Power , P =$\tau\omega$ = 180 ×200 = 36,000 W

3. *From a uniform disc of radius R, a circular hole of radius R / 2 is cut out. The centre of the hole is at R/2 from the centre of the original disc. Locate the centre of gravity of the resulting flat body.*



In figure let O be the CM of circular portion, O_1 that of the circular hole cut out and O_2 that of remaining shaded of portion. Let m be the mass per unit area of the disc.
Mass of the original disc,
M = $\pi R^2 m$
Mass of the circular hole cut,
$m_1 = \pi \left(\frac{R}{2}\right)^2 m = \frac{\pi}{4} R^2 m$
Mass of remaining portion ,
$m_2 = \pi R^2 m - \frac{\pi}{4} R^2 m = \frac{3}{4}\pi R^2 m$
m_1 and m_2 are assumed to be concentrated at O_1 and O_2 respectively and O is their CM.
∴ Moment of m_1 about O = Moment of m_2 about O $m_1 \times O_1O = m_2 \times O_2O$
or $\frac{\pi}{4} R^2 m \times \frac{R}{2} = \frac{3}{4}\pi R^2 m \times O_2O$
or $O_2O = \frac{R}{6}$
Thus the CM of the resulting portion lies at R/6 from the centre of the original disc in the opposite direction to the centre of portion cut out.

4. *A metre stickis balanced on a knife edge at its centre. When two coins, each of mass 5 g are put on the top of other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?*

As it balances at 50.0 cm , its CG is at this mark.
45.0 cm mark is the CG of the metre stick + 2 coins system.
Let m be the mass of the metre stick.
Distance between 50.0 cm mark and new CG
= 50.0 - 45.0 = 5.0 cm
Distance between 12.0 cm mark and new CG
= 45.0 - 12.0 = 33.0 cm
By the principle of moments
mg × 5.0 = (2 × 5) × g × 33.0
or m= $\frac{2 \times 5 \times 33.0}{5.0}$ 66. g.

5. *The oxygen molecule has a mass of 5.30 × 10^{-26} kg and a moment of inertia of 1.94 × 10^{-46} kg m^2 about an axis through its centre perpendicular to the line joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500m/s and that its kinetic energy of rotation is two third of its kinetic energy of translation . Find the average angular velocity of the molecule.*

Rotational K.E = $\frac{2}{3}$ Translational K.E
or $\frac{1}{2} I\omega^2 = \frac{2}{3} . \frac{1}{2} mv^2$

Or $\omega = v \times \sqrt{\frac{2m}{3\,I}}$ = 500 × $\sqrt{\frac{2\times 5.30\times 10^{-26}}{3\times 1.94\times 10^{-46}}}$

= $6.75 \times 16^{12} \text{rad } s^{-1}$

6. ***A solid cylinder rolls up an inclined plane of angle of inclination30^{o}. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5 m/s.***

(a) How far will the cylinder go up the plane?

(b) How long will it take to return to the bottom?

(a) Total initial kinetic energy of the cylinder,

$K_i = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$

$= \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}\times\frac{1}{2}MR^2\times\frac{v_{CM}^2}{R^2}$

$= \frac{1}{2}Mv_{CM}^2 + \frac{1}{4}Mv_{CM}^2 = \frac{3}{4}Mv_{CM}^2$

Initial potential energy, U_i = 0

Final kinetic energy, K_f = 0

Final potential energy, $U_f = Mgh$

$= Mgs\sin 30^{o} = \frac{1}{2}Mgs$

where s is the distance travelled up the incline and h is the vertical height covered above the bottom.

Gain in P.E. = Loss in K.E.

$\frac{1}{2}Mgs = \frac{3}{4}Mv_{CM}^2$

s = $\frac{3v_{CM}^2}{2g} = \frac{3\times(5)^2}{2\times 9.8}$ = 3.8 m.

(b) For the motion up the incline,

0 = v_{CM} + at or a = - $\frac{v_{CM}}{t}$

$O^2 - v_{CM}^2$ = 2as

or a = - $\frac{v_{CM}^2}{2s}$

∴ $\frac{v_{CM}}{t} = \frac{v_{CM}^2}{2s}$

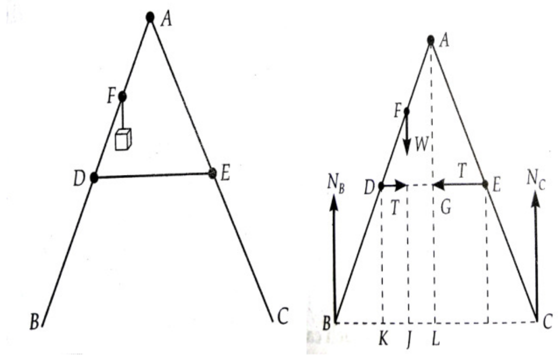
or t = $\frac{2s}{v_{CM}} = \frac{2\times 3.8}{5}$ = 1.5

Total time taken to return to the bottom

= 2 ×1.5 = 3.0 s

7. ***The two sides of a step ladder BA and CA are 1.6 m long and hinged at A. A rope DE, 0.5 m is tied half way up . A weight 40 kg is suspended from a point F, 1.2 m from B along the ladder BA. Assuming the floor to be frictionless and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder.***

(Take g = 9.8 m/s^2).



Let N_B and N_C be the normal reactions of the floor at B and C respectively and T be the tension in the rope DE .

$N_B + N_C$ = W = 40 ×9.8 N

$N_B + N_C$ = 392 N

N_C × LC = T × AG

From figure,

LC = 2GE = DE = 0.5m

or GE = 0.25 m

∴ AG = $\sqrt{AE^2 - GE^2} = \sqrt{(0.8)^2 - (0.25)^2}$

= $\sqrt{0.5775}$ = 0.76 m

N_C × 0.5 = T × AG

For AB, on equating the torques about A

But $JL = \frac{1}{4}$ DE $\frac{1}{4}$× 0.5 m W = 392 N

∴ N_B × 0.5 – 392 × 0.125 = T × 0.76

or N_B × 0.5 – 392 × 0.125 = 0.66 N_C × 0.76

or N_B - 98 = N_C

or N_B - N_C = 98

N_B = 245 N , N_C = 245 - 98 = 147N,

T = 0.66N_C = 0.66 × 147 = 97 N.

8.***A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings***

his arms close to his body with the distance of each weight from the axis changing from 90 cm to 20 cm . The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m^2

(a) what is his new angular speed? (Neglect friction)

(b) Is kinetic energy conserved in the process? If not from where does the change come about?

(a) Total initial moment of inertia,

I_1 = M.I .of man and platform + M.I. of two 5kg weights

= 7.6 + 2 × 5× $(0.90)^2$ = 7.6 + 8.1

= 15.7 kgm^2

Initial angular speed ,

ω_1 = 30 rpm

Total final moment of inertia,

I_2 = 7.6 + 2 × 5× $(0.20)^2$ = 7.6 +0.4

=8.0 kg m^2

Using the principle of conservation of angular momentum,

$I_1\omega_1 = I_2\omega_2$

or 15.7 ×30 = 8.0 × ω_2

or $\omega_2 = \frac{15.7 \times 30}{8.0}$ = 58.875 ≈ 59rpm.

(b)$\frac{final K.E.}{Initial K.E} = \frac{\frac{1}{2}I_1\omega_1^2}{\frac{1}{2}I_2\omega_2^2} = \frac{8.0 \times (59)^2}{15.7 \times (30)^2}$ = 1.97

Thus the final K.E is nearly twice the intial K.E. i.e., K.E. is not conserved in the process.

The increase in K.E. is due to the internal energy the man uses in bringing his arms closer to his body.

9. ***A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the center of the door. The door is 1.0 m wide and weighs 12kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it. (Hint: The moment of inertia of the door about the vertical axis at one end is ML^2 /3)***

Initial angular momentum of the bullet =Final angular momentum of the door

or$pr = I\omega$

or $mvr = \frac{ML^2}{3} \times \omega$ or $\omega = \frac{3mvr}{ML^2}$

m = 10g = 10^{-2} kg , v = 500 ms^{-1}

r = $\frac{1.0}{2}$ = 0.5 m, L = 1.0m, M = 12kg

$\therefore \omega = \frac{3 \times 10^{-2} \times 500 \times 0.5}{12 \times (1.0)^2}$ = 0.625 rad s^{-1}

10. ***Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed ω_1 and ω_2 are brought into contact face with their axes of rotation coincident. (What is the angular speed of the two disc system? (ii)Show the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy / Take$\omega_1 \neq \omega_2$***

(i)Letω be the angular speed of the two –disc system.

By conservation of angular momentum,

$(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2$

or$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

(ii)Initial K.E.of the two discs,

$K_1 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$

Final K.E. of the two –disc system,

$K_2 = \frac{1}{2}(I_1 + I_2)\omega^2$

$= \frac{1}{2}(I_1 + I_2)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$

loss in K.E.= $K_1 - K_2$

$=\frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2) - \frac{1}{2(I_1 + I_2)}(I_1\omega_1 + I_2\omega_2)^2$

$=\frac{1}{2(I_1 + I_2)}[I_1^2\omega_1^2 + I_1I_2\omega_2^2 + I_1I_2\omega_1^2 + I_2^2\omega_2^2$

$- (I_1^2\omega_1^2 + I_2^2\omega_2^2 + 2I_1I_2\omega_1\omega_2]$

$=\frac{1}{2(I_1 + I_2)}[I_1I_2\omega_2^2 + I_1I_2\omega_1^2 - 2I_1I_2\omega_1\omega_2]$

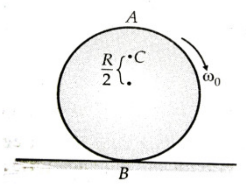
$=\frac{I_1I_2}{2(I_1 + I_2}(\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2)$

$=\frac{I_1 I_2}{2(I_1\ +I_2)}(\omega_1-\omega_2)^2$ = a positive quantity

[$\therefore\ \omega_1\ \neq\omega_2$]

There is a loss of rotation K.E. which appears as heat. When the two discs are brought together, work is done against friction between the two discs.

11. ***A disc rotating about its axis with angular speed $\boldsymbol{\omega_0}$ is placed lightly (without any translational push)on a perfectly frictionless table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the disc shown in figure? Will the disc roll in the direction indicated***?



$$v\ =\ R\omega$$

For A $v_A\ = R\omega_0$
(in the direction of the arrow)

For B : v_B = $R\omega_0$
(in the direction opposite arrow)

For C : v_C = $\left(\frac{R}{2}\right)\omega_0$
(in the direction of the arrow)

12. ***A solid disc and a ring both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10 $\boldsymbol{\pi rad s^{-1}}$Which of the two will start to roll earlier? The co-efficient of kinetic friction is $\boldsymbol{\mu_k}$ = 0.2. For pure rotation without translation, the velocity of centre of mass is zero. The equation of motion for CM is***

$\boldsymbol{\mu_k mg}$ **=ma**

The torque due to force of friction is $\mu_k mg$ ×R.

$\mu_k mg$R = - $I\ \propto$

Rolling begins when

$$v\ =\ R\omega$$

v = 0 + a t = $\mu_k\ g$t

and ω =ω_0 + $\propto t = \omega_0$ -$\frac{\mu_k mg\text{R}}{I}$ t

or $\frac{v}{R}$ = ω_0 -$\frac{\mu_k mg\text{R}}{I}$ t

or $\frac{\mu_k g\text{t}}{R}$ = ω_0 -$\frac{\mu_0 mg\text{R}}{I}$ t

or $\frac{\mu_k g\text{t}}{R}$ [1 + $\frac{mR^2}{I}$] = ω_0

or t = $\frac{R\omega_0}{\mu_k g\left(1+\frac{mR^2}{I}\right)}$

For a disc : I = $mR^2/2$

$\therefore$ t = $\frac{R\omega_0}{3\mu_k g}$ = $\frac{0.10\times10\ \pi}{3\ \times0.2\ \times9.8}$ = 0.53 s.

For a ring: I = $mR^2/2$

$\therefore$ t = $\frac{R\omega_0}{2\mu_k g}$ = $\frac{0.10\times10\ \pi}{2\ \times0.2\ \times9.8}$ = 0.80

13. ***A solid cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination$\boldsymbol{30^o}$. The coefficient of static friction, $\boldsymbol{\mu_s}$ = 0.25.(i) Find the force of friction acting on the cylinder .(ii) What is the work done against friction during rolling ? (iii) If the inclination $\boldsymbol{\theta}$ of the plane is increased , at what value of $\boldsymbol{\theta}$ does the cylinder begin to skid, and not roll perfectly***

M = 10 kg , R = 0.15 m , μ_s = 0.25 , θ = 30^o

(i)Force of friction,

f = $\frac{1}{3}$ Mg sin θ = $\frac{1}{3}\times$ 10 × 9.8 × $sin30^o$

= $\frac{1}{3}\times$ 10 × 9.8 ×0.5 = 16.33 N

(ii) Work done against friction during rolling = 0 J

(iii) Condition for skidding is

$$\frac{f}{N}\leq\mu_s$$

or $\frac{\frac{1}{3}\text{Mg}\sin\theta}{\text{Mg}\cos\theta}\leq\mu_s$

or $\frac{1}{3}$ tan $\theta\leq\mu_s$

Thus the cylinder will start skidding .The angle of inclination

θ given by

tan θ = $3\mu_s$ = 3 ×0.25 = 0.75

θ = 36^o52

14. ***Read each statement below carefully , and state, with reasons, if it is true or false:***
(a) During the rolling the force of friction acts in the same direction as the direction of motion of the CM of the body.

(b)The instantaneous speed of the point of contact during rolling is zero.

(d) For perfect rolling motion, work done against friction is zero.

(e)A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling) motion.

(a) True. When a body rolls, friction acts in the same direction as the direction of motion of the CM .

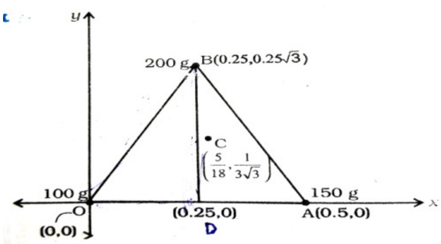
(b) True. A rolling body can be imagined to be rotating about an axis passing through the point of contact of the body and the ground. Hence the instantaneous speed of the point of contact is zero.

(c)False . As the body is rotating ,its instantaneous acceleration cannot be zero.

(d) True. Perfect rolling begins when force of friction is zero .So work done against friction is zero.

(e) True. On a perfectly frictionless inclined plane , there is no tangential force of friction. So the wheel cannot roll. It will simply slip under the effect of its own weight.

15. ***Find the centre of mass of three particles at the vertices of an equilateral triangle. The masses of the particles are 100g, 150g, and 200g respectively. Each side of the equilateral triangle is 0.5 m long.***



With the X and y axes as in figure the coordinates of points O,A and B forming the equilateral triangle are respectively (0,0), (0.5,0) (0.25, 0.25$\sqrt{3}$).

Let the masses 10 g 150g and 200g be located at O,A ands B be respectively.

$X = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$= \frac{(100(0) + 150\,(0.5) + 200(0.25))gm}{(100+150+200)g}$

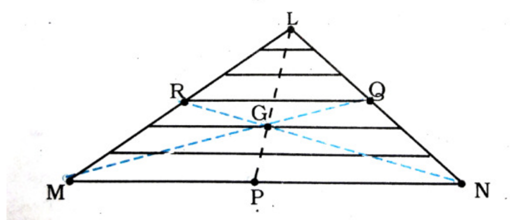
$\frac{75+50}{450}$m $= \frac{125}{450}$m $= \frac{5}{18}$m

y $= \frac{(100(0) + 150\,(0.5) + 200(0.25)\sqrt{3})gm}{450g}$

$= \frac{50\sqrt{3}}{450}$ m $= \frac{\sqrt{3}}{9}m = \frac{1}{3\sqrt{3}}m$

The centre of mass C is shown in the figure.

16. ***Find the centre of mass of a triangular lamina.***

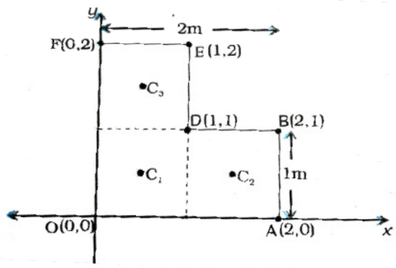


The lamina (ΔLMN) may be subdivided into narrow strips each parallel to the base (MN)

By symmetry each strip has its centre of mass at its midpoint. If we join the midpoint of all the strips we get the median LP. The centre of mass of the triangle lie on the median LP. It lies on the medianMQ and NR .

The centre of mass lies on the point of concurrence of the medians.

17. ***Find the centre of mass of a uniform L – shaped lamina (a thin flat plate)with dimensions as shown. The mass of the lamina is 3 kg.***



Assume that L – shape consists of 3 squares of 1 kg, as the lamina is uniform.

The centre's of massC_1 ,C_2 and $C_3$of the squares are by summery, their geometric centre's and have coordinates (1/2,1/2) (3/2,1/2),(1/2,3/2) respectively. We take the masses of the squares to be concentrated at these points.

The centre of the whole L shape (X, Y) is the centre of mass of these mass points

X = $\frac{[1(1/2)+1(3/2)+1(1/2)]kgm}{(1+1+1)kg} = \frac{5}{6}$ m

y = $\frac{[1(1/2)+1(1/2)+1(3/2)]kgm}{(1+1+1)kg} = \frac{5}{6}$ m

The centre of mass of the L-shape lies on the line OD.

18. ***Find the scalar and vector products of two vectors a= (3$\hat{\imath}$ - 4 $\hat{\jmath}$ + 5$\hat{k}$) and b = (- 2$\hat{\imath}$ + $\hat{\jmath}$ - 3$\hat{k}$)***

a.b = (3$\hat{\imath}$ - 4 $\hat{\jmath}$ + 5$\hat{k}$) . (- 2$\hat{\imath}$ + $\hat{\jmath}$ - 3$\hat{k}$) = -6 -4 – 15
= -25

$a \times b = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -4 & 5 \\ -2 & 1 & -3 \end{vmatrix} = 7\hat{\imath} - \hat{\jmath} - 5\hat{k}$

b× a = -7$\hat{\imath}$ + $\hat{\jmath}$ + 5$\hat{k}$

19. ***Find the torque of a force 7$\hat{\imath}$ + 3 $\hat{\jmath}$ - 5$\hat{k}$ about the origin . The force acts on a particle whose position vector is $\hat{\imath}$ - $\hat{\jmath}$+ $\hat{k}$.***

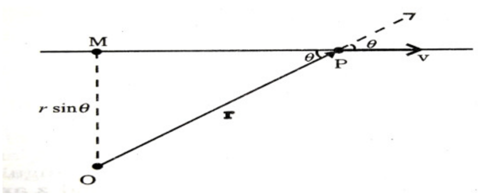
r = î - ĵ + k̂
F = 7$\hat{\imath}$ + 3 $\hat{\jmath}$ - 5$\hat{k}$

$\tau = r \times F$

$\tau = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix}$ = (5- 3) $\hat{\imath}$ - (-5 – 7)$\hat{\jmath}$ + (3-(-7))$\hat{k}$

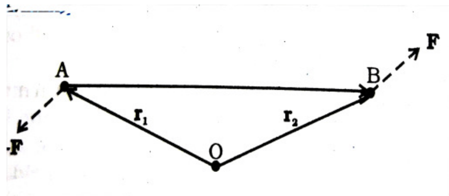
or τ =2 $\hat{\imath}$ + 12 $\hat{\jmath}$ + 10$\hat{k}$

20. ***Show that the angular momentum about any point of a single particle moving with constant velocity remains constant throughout the motion.***
Take a particle with velocity v be at point P at some instant t .



$L = r \times mv$.
Its magnitude is$mvrsin\theta$.
OM = r $sin\theta$ is a constant.
The direction of 1 is perpendicular to the plane of r and v.
It is into the page of the figure.
These direction does not change with time.
Thus ,L remains the same in magnitude and direction and is therefore conserved.

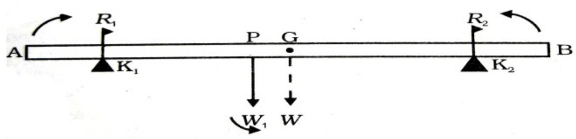
21. ***Show that moment of a couple does not depend on the point about which you take the moments. The force F and F act respectively at point B and A. These points have position vectors r_1 and r_2 with respect to origin O. Take the moments of the forces about the origin.***



The moment of the couple = sum of the moments of the two forces making the couple
= r_1 ×(-F) + r_2 × F
= r_2 × F - r_1 × F
= (r_2- r_1) × F
But r_1 + AB = r_2 , and hence AB = r_2 - r_1The moment of the couple, therefore , is AB × F
This is independent of the origin , the point about which we took the moments of the forces.

22.*A metal bar 70 cm long and 4.00 kg in mass supported on two knife – edges placed 10 cm from each end . A 6.00 kg load is suspended at 30 cm from one end. Find the reactions at the knife –edges. (Assume the bar to be of uniform cross section and homogeneous.)*

A and B are the positions of the knife edges $K_1$and K_2.



The centre of gravity of the rod at G and the suspended load at P.

Weight of the rod W acts at its centre of gravity G.

AB = 70 cm.

AG = 35cm ,AP = 30cm , PG = 5 cm , AK_1 = $\boldsymbol{B}K_2$ = 10cm and

K_1G = K_2G= 25 cm .

W = weight of the rod = 4.00 kg

W_1 = suspended load = 6.00kg:

R_1 and R_2 are the normal reactions of the support at the knife edges.

For translational equilibrium R_1 + R_2 - W_1 = W = 0

W_1 and W and W act vertically down and R_1 and R_2 act vertically up

For rotational equilibrium,

-$R_1(K_1G)$ + W_1(PG) + R_2 (K_2 G) = 0

W = 4.00g N and W_1 = 6.00g

R_1 + R_2 -4.00g – 6.00g = 0

or R_1 + R_2 = 10.00g N

= 98.00N

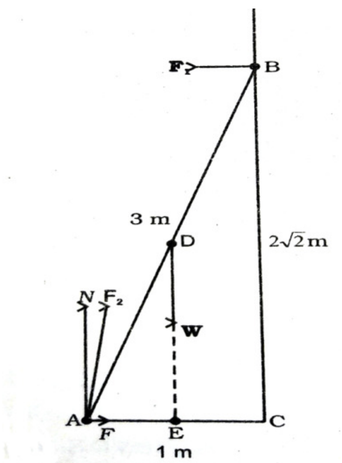
(ii) = 0.25R_1 + 0.05W_1 + 0.25 R_2 = 0

or R_1 - R_2 = 1.2 g N = 11.76 N

R_1 = 54.88 N , R_2 = 43.12N

The reactions of the support are about 55 N at K , 43 N at K_2.

23. ***A 3m long ladder weighing 20 kg leans on a frictionless wall . Its feet rest on the floor 1 m from the wall as shown in fig. Find the reaction forces of the wall and the floor.***



The ladder AB is 3 m long, its foot A is at distance AC = 1 m from the wall.

By Pythagoras theorem , BC = $2\sqrt{2}$ m.

The forces on the ladder are its weight W acting at its centre of gravity D , reaction forces F_1 and F_2 of the wall and the floor respectively.

$F_1$is perpendicular to the wall.

Force F_2 is resolved into two components, the normal reaction N and the force of friction F.

F prevents the ladder from sliding away from the wall and is towards the wall.

For translational equilibrium N – W = 0

For the horizontal direction , F - F_1 = 0

For rotational equilibrium, $2\sqrt{2}F_1$ - (1/2) W = 0

W = 20g =20 ×9.8 N = 196.0 N

(i) N = 196.0

$(iii)F_1$ W / $4\sqrt{2}$ = 196.0 / $4\sqrt{2}$ = 34.6 N

$(ii)F$ = F_1 = 34.6N

The force F_2 makes an angle ∝ with the horizontal.

$tan\alpha$ N / F = $4\sqrt{2}$, α= tan^{-1} ($4\sqrt{2}$,) ≈ 80^o

24. ***The angular speed of motor wheel is increased from 1200rpm to 3120 rpm in 16 seconds.(i) What is its angular acceleration ,assuming the acceleration to be uniform?***

$(\boldsymbol{i})\omega = \omega_o + \propto$t

ω_o = initial angular speed in rad /s

= 2π × angular speed in rev/s

= $\frac{2\pi\times \text{angular speed in rev/min}}{60\ s/min}$

= $\frac{2\pi\times 1200}{60}$ rad/s

= 40π rad/s

ω = final angular speed in rad/s

= $\frac{2\pi\times 3120}{60}$ rad/s

= 2π ×52 rad /s

= 104π rad/s

Angular acceleration

∝ = $\frac{\omega-\omega_o}{t}$ = 4π rad/s^2

(ii) The angular displacement in time t is

$\theta = \omega_0 t + \frac{1}{2} \propto t^2$

= $\left(40\,\pi \times 16 + \frac{1}{2} \times 4\pi \times 16^2\right)$rad

= $(640\,\pi + 512\pi\,)rad$

= 1152 $\pi\,rad$

Number of revolutions

= $\frac{1152\,\pi}{2\pi}$ = 576

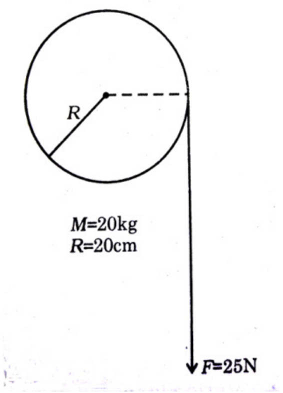
25. ***A cord of negligible mass is wound round the rim of a fly wheel of mass 20kg and radius 20cm. A steady pull of 25N is applied on the cord as shown in figure.The flywheel is mounted on a horizontal axle with frictionless bearings.***

(a) Compute the angular acceleration of the wheel.

(b) Find the work done by the pull when 2m of cord is unwound.

(c)Find also the kinetic energy of the wheel at this point. Assume that the wheels start from rest.

(d) Compare answers to parts (b) and (c)



a)$I \propto \lambda$

λ = FR

= $25 \times 0.20 \ Nm$

= 5.0 Nm

I = $M.I$.of flywheel about its axis = $\frac{MR^2}{2}$

= $\frac{20.0 \times (0.2)^2}{2}$ = 0.4 kg m^2

∝ = angular acceleration

= 5.0 Nm /0.4 kg m^2 = 12.5 s^{-2}

(b) Work done by the pull when unwinding 2m of the cord

=25 N ×2m = 50J

(c) The kinetic energy gained = $\frac{1}{2} I\omega^2$

As the wheel starts from rest .

Now ω^2 = ω_o^2 + 2∝ θ,ω_o = 0

Angular displacement θ= length of unwound string / radius of wheel = 2m/0.2 = 10rad

ω^2 = $2 \times 12.5 \times 10.0$ = 250 $(rad/s)^2$

∴ K.E. gained = $\frac{1}{2} \times 0.4 \times 250$ = 50J

(d) The kinetic energy gained by the wheel = work done by the force .

There is no loss of energy due to friction.