GRAVITATION (Problems)

1. ***Answer the following***

(a) You can shield a charge from electrical force by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?
(b) An astronaut inside a small spaceship orbiting around the earth cannot detect gravity. If the space station orbiting around the Earth has a large size, can he hope to detect gravity?
(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the sun's pull is greater than the moon's pull. However, the tidal effect of moon's pull is greater than the tidal effect of sun. Why?

(a) Gravitational forces are independent of medium. A body can not be shielded from the gravitational influence of nearby matter.

(b) Yes, with larger mass the value of gravity will be significant.

(c) Tidal effect $\propto \frac{1}{(\text{distance})^3}$gravitational force $\propto \frac{1}{(\text{distance})^2}$.

As moon is nearer to earth tidal effect is more.

2. ***Choosethecorrectalternative:***

(a) Acceleration due to gravity increases/ decreases with increasing altitude.

(b) Acceleration due to gravity increases/ decreases with increasing depth (assume the Earth to be a sphere of uniform density).

(c) The effect of rotation on the effective value of acceleration due to gravity is greatest at the equator/ poles.

(d) Acceleration due to gravity is independent of mass of the Earth/ mass of the body.

(e) The formula – $GMm\,(1/r_2$- $1/r_1)$ is more/ less accurate than the formula $mg(r_2$ - $r_1)$ for the

difference of potential energy between two points r_2 and r_1 distance away from the centre of Earth.

(a) As $g_h = g\left(1 - \frac{2h}{R}\right)$, i.e., g_h decreases with increase in altitude

(b) As $g_d = g\left(1 - \frac{d}{R}\right)$, i.e., g_d decreases with increase in depth.
(c)g_l = g – R $\omega^2 \cos^2 \lambda$
g_l is minimum when λ = 0^0
ie the effect is maximum at equator.

(d) g is independent of the mass of the body.

(e) As –G Mm$\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$is more accurate than the formula mg$(r_2 - r_1)$

3. Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?

T_e = 1 year

$$T_P = \frac{T_e}{2} = \frac{1}{2}\text{year}$$

$r_e = 1$ AU, r_P =?
Using Kepler's third law, $r_P = r_e\left(\frac{T_P}{T_e}\right)^{2/3}$ = 0.63AU

4.One of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22 $\times 10^8$m. Show that the mass of Jupiter is about one thousandth that of the sun.

For a satellite of Jupiter,
orbital period
T_1 = 1.769 days = 1.769 × 24 × 60 × 60s
Radius of the orbit of satellite,
r_1 = 4.22 × 10^8mMass of Jupiter,M_1= $\frac{4\pi^2 {r_1}^3}{G{T_1}^2}$ =

$\frac{4\pi^2\times(4.22\times10^8)^3}{G\times(1.769\times24\times60\times60)^2}$

The orbital period of earth around the sun,

T_2 = 1 year = $365.25 \times 24 \times 60 \times 60$ s

Orbital radius, r_2 = 1 A.U. = 1.496×10^{11} m

Mass of Sun,$M_2 = \frac{4\pi^2 {r_2}^3}{G{T_2}^2} = \frac{4\pi^2\times(1.496\times10^{11})^3}{G\times(365.25\times24\times60\times60)^2}$

$\frac{M_1}{M_2} = \frac{4\pi^2\times(4.22\times10^8)^3}{G\times(1.769\times24\times60\times60)^2} \div \frac{4\pi^2\times(1.496\times10^{11})^3}{G\times(365.25\times24\times60\times60)^2}$

$= \frac{1}{1046} \approx \frac{1}{1000}$

5.Let us assume that our galaxy consist of 2.5× 10^{11}s stars each of one solar mass. How long will a star at distance of 50,000 1y from the galactic centre take to complete one revolution? Take the diameter of the Milky way to be $10^5 ly$.

r = 50,000 ly = 50,000 × 9.46×10^{15}m

= 4.73×10^{20}mM = 2.5× 10^{11} solar mass

= 2.5× 10^{11} × (2×10^{30}) kg

= 5.0×10^{41} kg

$M = \frac{4\pi^2 r^3}{GT^2}$

$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{1/2} = \left[\frac{4\times(22/7)^2\times(4.73\times10^{20})^3}{(6.67\times10^{-11})\times(5.0\times10^{41})}\right]^{1/2}$

$= 1.12 \times 10^{16}$ s

6.***Choose the correct alternatives:***

(a) If the zero of potential energy is at infinity , the total energy of an orbiting satellite is negative energy.

(b) The energy required to rocket an orbiting satellite out of Earth's gravitational influence is more/ less than the energy required to project a stationary object at the same height (as the satellite) out of the Earth's influence.

(a) Kinetic energy (b) Less

7. Does the escape speed of a body from the Earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched? Explain your answer.

(a) Escape speed of a body is independent of mass of a body.

(b) Escape speed of a body depends upon the location.

(c) Escape speed of a body is independent of the direction of projection.

(d) Escape speed of a body depends upon the height of the location from where the body projected, because the escape velocity depends upon the gravitational potential at the point from which it is projected and this potential depends upon height also.

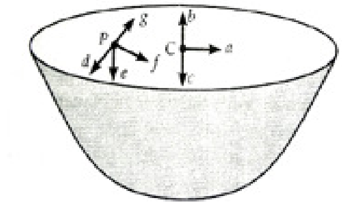
8. A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the sun.

Angular momentum and total energy do not change throughout the orbit whereas all the quantities vary in the orbit.

9. Which of the following symptom is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientationalproblem.

The astronaut in space will suffer from swollen face, headache and orientational problem.

10. The gravitation intensity at the centre of the hemispherical shell has the direction indicated by the arrow $(i)\ a$ $(ii)\ b, (iii)\ c, (iv)$ zero.



Intensity inside a shell is zero. It willbe along c at the centre.

11. For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow $(i) d$ $(ii)\ e$, $(iii)\ f$, $(iv)\ g$.

As in figure, upper portion of the shell is missing, so, gravitational intensity at P and Q should act along e and c respectively.

12. A rocket is fired from the earth towards the sun . At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of sun= 2× $10^{30}kg$, mass of the earth = 6× $10^{24}kg$. Neglect the effect of other planets etc. Orbital radius = 1.5× $10^{11}m$.

Mass of Sun, M = 2 × 10^{30} kg
Mass of Earth, m = 6× 10^{24} kg
Distance between Sun and Earth, r = 1.5 × 10^{11} m
Let at the point P, the gravitational force on the rocket due to earth = gravitational force on the rocket due to sun
Let x = distance of the point P from the earth.

$\frac{Gm}{x^2} = \frac{GM}{(r-x)^2} \rightarrow \frac{(r-x)^2}{x^2} = \frac{M}{m} = \frac{2\times10^{30}}{6\times10^{24}} = \frac{10^6}{3}$

Or

$$\frac{r-x}{x} = \frac{10^3}{\sqrt{3}}$$

Or

$\frac{r}{x} = \frac{10^3}{\sqrt{3}} + 1 \approx \frac{10^3}{\sqrt{3}} \rightarrow x = \frac{\sqrt{3}\,r}{10^3} = \frac{1.732\times1.5\times10^{11}}{10^3}$
$= 2.6 \times 10^8$ m

13. How will you' weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5× $10^8 km$.

Radius of earth's orbit

(R + h) = 1.5× $10^8 \times 10^3$ = 1.5× 10^{11} m

Time period of Sun,

T= 365 days = 365× 24 × 60 × 60 sT =

$2\pi\left(\frac{(R+x)^3}{GM}\right)^{1/2}$

$M = \frac{4\pi^2(R+h)^3}{GT^2} = \frac{4\times(3.14)^2(1.5\times10^{11})^3}{(6.67\times10^{-11})(365\times24\times60\times60)}$

= 2.01× 10^{30} kg

14. A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.5× $10^8 km$ away from the sun?

By Kepler's third law

$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \rightarrow R_2 = R_1\left(\frac{T_2}{T_1}\right)^{2/3}$ =

$(1.5\times10^8)\left(\frac{29.5}{1}\right)^{2/3}$

= $(1.5\times10^8) \times 9.547 = 14.32 \times 10^8$ km

15. A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

$$g_h = g\frac{R^2}{(R+h)^2}$$

$mg_h = mg\frac{R^2}{(R+h)^2} = \frac{63\times R^2}{\left(R+\frac{R}{2}\right)^2} = \frac{63\times R^2}{9R^2/4}$ =28 N.

16. Assuming the earth to be a sphere of uniform mass density, how much would a body weight half way down to the centre of the earth if it weighed 250 N on the surface?

$g_d = g\left(1-\frac{d}{R}\right) \rightarrow mg_d = mg\left(1-\frac{d}{R}\right)$ d = R/2

$\therefore mg_d = (250)\times\left(1-\frac{R/2}{R}\right) = 250 \times \frac{1}{2}$ = 125 N

17. A satellite orbits the earth at a height of 400km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg, mass of the earth = 6.0 × $10^{24}kg$, radius of the earth = 6.4 × $10^6 m$, G = 6.67 × $10^{11} Nm^2 kg^{-2}$.

h=400km = 400×10^3m= 0.4×10^6m

Total energy of satellite in orbit,

$E = -\frac{GMm}{2(R+h)} E = \frac{-(6.67\times10^{-11})\times(6\times10^{24})\times200}{2(6.4\times10^6+0.4\times10^6)}$

$= -5.89 \times 10^9$ J

18. Two stars each of 1 solar mass (= 2× $10^{30}kg$) are approaching each other for a head – on collision. When they are at a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. Use the known value of G.

M = 2 × 10^{30} kg R = 10^4km = 10^7m

$r = 10^9$ km = 10^{12}m

Initial potential energy of the system of the two stars = $-\frac{GMm}{r} = -1.334 \times 10^{43}$ J

∴Decrease in P.E of the system

$= -2.668 \times 10^{38} - (-1.334 \times 10^{43}) = -1.33 \times 10^{43}$ J

By the principle of conservation of energy,

$2 \times 10^{30} v^2 = 1.334 \times 10^{43}$

$v = \sqrt{\frac{1.334 \times 10^{43}}{2 \times 10^{30}}} = 2.588 \times 10^6 \ ms^{-1}$

19. Two heavy spheres each of mass 100kg and radius 0.1m are placed 1.0m apart on a horizontal table. What is the gravitational field at the midpoint of the line joining the centre of the spheres?

$G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$; M = 100 kg ;

R = 0.1 m,

distance between the two spheres, d = 1.0 m

$r = \frac{d}{2} = 0.5$ m.

The resultant gravitational potential at the midpoint

$\left(-\frac{GM}{r}\right) \times 2 = -2.668 \times 10^{-8} \ J\,kg^{-1}$

20. A geostationary satellite orbits the earth at a height of nearly 36,000km from the surface of the earth. What is the potential due to earth's gravity at the side of the satellite? Mass of the earth = 6 $\times$ $\mathbf{10^{24}}$ kg and radius = 6400 km.

Distance of satellite from the centre of earth

= R + h = 6400 + 36000 = 42400 km = 4.24×10^7 m

Potential, $V = \frac{-GM}{r} = -\frac{(6.67 \times 10^{-11}) \times (6 \times 10^{24})}{(4.24 \times 10^7)}$

$= -\ 9.44 \times 10^6 J\ kg^{-1}$

21.A star 2.5times the mass of the sun and collapsed to a size of 12km rotates with a speed of 1.5 rev/s. Will an object placed on its equator remain stuck to its surface due to gravity? Mass of the sun = 2 $\times$ $\mathbf{10^{30}}$ kg.

$F = \frac{GMm}{R^2} = \frac{(6.67 \times 10^{-11}) \times (5 \times 10^{30}) m}{(12 \times 10^3)^2}$

$= 2.316 \times 10^{12}$Nm

$\omega = 2\pi\vartheta = 2 \times \pi \times 1.5 = 3\pi$

Centrifugal pull = $mR\omega^2$

$= m \times (12 \times 10^3) \times (3\pi)^2 = 1.066 \times 10^6$NmCentrifugal force is very small.

Hence the object will remain stuck to the surface due to gravity.