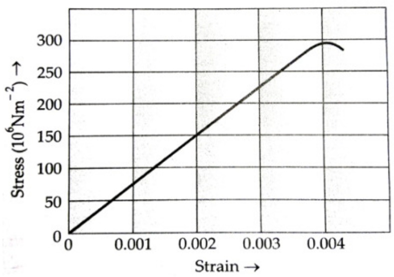
MECHANICAL PROPERTIES OF SOLIDS (Problems)

1. ***A steel wire of length 4.7m and cross section 3.0$\times 10^{-5} m^2$ stretches by the same amount as a copper wire of length 3.5m and cross section 4.0 $\times 10^{-5} m^2$ under a given load. What is the ratio of the Young's modulus of steel to that of copper?***

For steel: l = 4.7 m , A = 3.0$\times 10^{-5}$
For copper: l = 3.5m, A = 4.0 $\times 10^{-5}$
Load (F) and Δl are same for both.
For steel $Y_S = \frac{Fl}{A\,.\Delta l} = \frac{F \times 4.7}{3.0\times 10^{-5} \times \Delta l}$
For copper $Y_C = \frac{Fl}{A\,.\Delta l} = \frac{F \times 3.5}{4.0\times 10^{-5} \times \Delta l}$
$\frac{Y_S}{Y_C}$ = 1.79

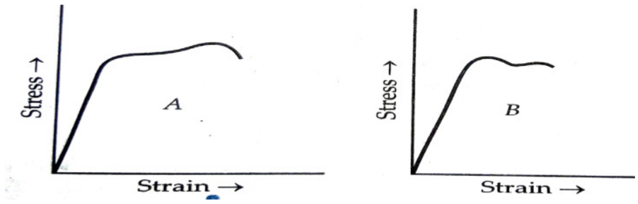
2. ***The stress – strain curve for a given material is given. What are (a) Young's modulus and (b) approximate yield strength for this material?***



From graph, for a stress = 150 $\times 10^6 Nm^{-2}$,
the strain is 0.002.
Y = $\frac{\text{stress}}{\text{strain}} = \frac{150 \times 10^6}{0.002}$ = $7.5 \times 10^{10} Nm^{-2}$
(b) Yield strength of the material
= 300$\times 10^6 Nm^{-2}$ (indicated as P)

3. ***The stress-strain graph for material A and B are shown in figure.***

The graphs are drawn to the same scale. (a) Which of the material has greater Young's modulus?
(b) Which material is more ductile?
(c) Which material is more brittle?
(d) Which of the two is stronger material?



(a) Young's modulus = $\frac{\text{stress}}{\text{strain}}$ = slope of stress-strain graph
Slope for A > Slope for B

∴ Material A has greater Young's modulus than B.

(b) Material A is more ductile than B, as it has larger range of plastic extension between its elastic limit and fracture point.

(c) Material B is more brittle than A, because its plastic range of extension is very small.

(d) Material A is stronger than B , because it can withstand greater stress before breaking.

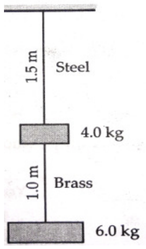
4. ***Read each of the statements below carefully and state , with reason , if it is true or false. (a) The modulus of elasticity of rubber is greater than that of steel***
(b) The stretching of a coil is determined by its shear modulus.

(a) False. When the same deforming force is applied, less extension and hence less strain is produced in steel .

Y = $\frac{\text{stress}}{\text{strain}}$
Hence Y is more for steel than rubber.
(b) True . When the coil is stretched, there is only a change in the shape of the spring, so only shear modulus

5. ***Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as in figure. The length of steel wire is 1.5m and that of brass wire is 1.0m .Young's modulus of steel is 2.0 $\times 10^{11}$ Pa and that of brass is 0.91 $\times$***

10^{11} ***Pa. Compute the elongations of steel and brass wires.***



For steel:

$l_1 = 1.5$ m, $r_1 = \frac{0.25}{2}$ cm

= 0.125×10^{-2} m

F_1 = 6 + 4 = 10 kgwt = 10 × 9.8 N

$Y_1 = 2.0 \times 10^{11}$ pa

Y $= \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{F}{\pi r^2} \cdot \frac{l}{\Delta l}$

$\Delta l = \frac{F}{\pi r^2} \cdot \frac{l}{Y}$

$\therefore \Delta l_1 = \frac{F_1}{\pi r^2} \cdot \frac{l_1}{Y_1}$

$= \frac{10 \times 9.8 \times 1.5}{3.14 \times (0.125 \times 10^{-2} \times)^2 \times 2.0 \times 10^{11}}$

$= 1.5 \times 10^{-4} m.$

For brass:

$l_2 = 1.0$ m, $r_2 = 0.125 \times 10^{-2}$ m

F_1 = 6kg wt = 6 × 9.8 N, Y_2

$= 0.91 \times 10^{11}$ pa

$\therefore \Delta l_2 = \frac{F_2}{\pi {r_2}^2} \cdot \frac{l_2}{Y_2}$

$= \frac{6 \times 9.8 \times 1.0}{3.14 \times (0.125 \times 10^{-2} \times)^2 \times 0.91 \times 10^{11}}$

$= 1.3 \times 10^{-4} m.$

6. ***The edge of an aluminum cube is 10cm long . One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminum is 25 G Pa. What is the vertical deflection of this face ?(1Pa = 1 Nm^{-2}).***

$Shear\ modulus \eta = \frac{F}{A} \cdot \frac{L}{X} \therefore x = \frac{F}{A} \cdot \frac{L}{\eta}$

A= 10cm ×10cm = 100cm^2=100 x 10^{-4} m^2= 10^{-2} m^2

F = mg = 100 ×10 = 1000N

$\eta = 25 \times 10^9$ Pa

L = 10cm =10× 10^{-2} m= 10^{-1} m

$x = \frac{F}{A} \cdot \frac{L}{\eta} = \frac{1000 \times 10^{-1}}{10^{-2} \times 25 \times 10^9} = 4 \times 10^{-7}$ m

7. ***Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000kg. The inner and outer radii of each column are 30 cm and 40cm respectively assuming the load distribution to uniform, calculate the compression strain of each column. The Young's modulus of steel is 2.0× 10^{11} pa***

r_1 = 30cm = 0.3m,

r_2 = 40cm = 0.4m , Y = 2.0 × 10^{11} pa

The load on each column $= \frac{50,000}{4}$ kg = 12500kg

F = 12500 × 9.8N

A = Area of cross-section of each column

$= \pi {r_2}^2 - \pi {r_1}^2 = \pi({r_2}^2 - {r_1}^2)$

$= \frac{22}{7} [(0.4)^2 - (0.3)^2] = 0.22 m^2$

Y $= \frac{Stress}{strain}$

$\therefore$ Compressional strain $= \frac{Stress}{Young's\ modulus}$

$= \frac{\frac{F}{A}}{Y} = \frac{F}{AY}$

$= \frac{12500 \times 9.8}{0.22 \times 2.0 \times 10^{11}} = 2.8 \times 10^6$

8. ***A piece of copper having a rectangular cross – section of 15.2 mm ×19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain.***

F = 44500N

A = 15.2 mm ×19.1mm

= 15.2 ×19.1× 10^{-6} m^2

For copper

Y = 1.2 × 10^{11} Nm^{-2}

$Y = \frac{stress}{stran} = \frac{F}{A \times strain}$

$\therefore$ Strain $= \frac{F}{A\,Y} = \frac{44500}{15.2\ \times 19.1 \times 10^{-6} \times 1.2 \times 10^{11}}$

= 0.001277

9. ***A steel cable with a radius of 1.5cm supports a chairlift at a ski area. If the maximum stress is not to exceed $10^8\,Nm^{-2}$, What is the maximum load the cable can support.***

Maximum stress $= \frac{F}{A} = \frac{Maximum\ load}{\pi r^2}$

$\therefore$ Maximum load $= \pi r^2 \times$ Maximum stress

$= 3.14 \times (1.5\ \times 10^{-2})^2 \times 10^8$N

$= 7.07 \times 10^4$ N

10. ***A rigid bar of mass 15kg is supported symmetrically by three wire each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension. Let T be the tension. The bar is supported symmetrically by three wires. The increase in length Δl of each wire is the same.***

$Y = \frac{T}{A}\frac{l}{\Delta l}$

Here $l, \Delta l$ and T are the same.

Hence $Y \propto \frac{1}{A}$ or $A \propto \frac{1}{Y}$ or $\pi\left(\frac{D}{2}\right)^2 \propto \frac{1}{Y}$

or $\frac{\pi D^2}{4} \propto \frac{1}{Y}$ or $D \propto \frac{1}{\sqrt{Y}} D_{copper} \propto \frac{1}{\sqrt{Y_{copper}}}$

and $D_{iron} \propto \frac{1}{\sqrt{Y_{iron}}}$

$\therefore \frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}} = \sqrt{\frac{9.1 \times 10^{11}}{1.1 \times 10^{11}}} = 1.3$

11. ***A 14.5kg mass, fastened to the end of a steel wire of unstretched length 1.0m,is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle . The cross-sectional area of the wire is 0.005 cm^{-2}. Calculate the elongation of the wire when the mass is at the lowest point of its path.***

Centripetal force= $mr\omega^2$

At the lowest point T -mg = $mr\omega^2$

T = mg + $mr\omega^2$ = m[g + r$(2\pi v)^2$]

=14.5 [9.8 +1.0$\times 4 \times \pi^2 \times (2)^2$] = 2431.94N.

$Y = \frac{stress}{strain} = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{\frac{T}{A}}{\frac{\Delta l}{l}}$

$\therefore \Delta l = \frac{Tl}{AY} = \frac{2431.94 \times 1.0}{0.0065 \times 10^{-4} \times 2 \times 10^{11}} = 1.87\ 10^{-3}$ m

12. ***Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre ,Pressure increase=100.0atm, final volume = 100.5litre (1 atm = 1.013 $\times 10^5$ Pa).***

P = 100atm = $100 \times 1.013 \times 10^5$ pa

V_1= 100.0 litre = $100.0 \times 10^{-3}\,m^3$

V_2 = 100.5 litre = $100.5 \times 10^{-3}\,m^3$

Δ V = (100.5 - 100.0) $\times 10^{-3}$ = $0.5 \times 10^{-3}\,m^3$

$B = \frac{PV}{\Delta V} = \frac{100 \times 1.013 \times 10^5 \times 100.0 \times 10^{-3}}{0.5 \times 10^{-3}}$

= 2.03×10^9 Pa

13. ***What is the density of ocean water at a depth, where the pressure is 8.0atm, given that its density at the surface is 1.03$\times 10^3$ kgm^3? Compressibility of water = 45.8 $\times 10^{-11}\,Pa^{-1}$. Given 1 atm = 1.013$\times 10^5$ pa.***

Ans: Compressibility $= \frac{1}{B}$ = $45.8 \times 10^{-11}\,Pa^{-1}$

Change in pressure, P= 80 -1 = 79atm

= $79 \times 1.013 \times 10^5$ Pa.

Density at the surface, ρ = $1.03 \times 10^3\,kgm^3$

$B = \frac{PV}{\Delta V}$

$\therefore \frac{\Delta V}{V} = P \times \frac{1}{B}$

= $79 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11}$

= 3.665×10^{-3}

$\frac{\Delta V}{V} = \frac{V - V'}{V} = \frac{\frac{M}{\rho} - \frac{M}{\rho'}}{\frac{M}{\rho}} = 1 - \frac{\rho}{\rho'}$

$\frac{\rho}{\rho'} = 1 - \frac{\Delta V}{V}\ \rho' = \rho\left(1 - \frac{\Delta V}{V}\right)$

= 1.034×10^3 kgm^{-3}

14. ***Compute the fractional change in volume of glass slab , when subjected to a hydraulic pressure of 10atm Given: B = 37 × 10^9 Nm^{-2}***

P= 10 × 1.013 × 10^5 Nm^{-2} ,

B = 37 × 10^9 Nm^{-2} , B =$\frac{PV}{\Delta V}$

Fractional change in volume

$\frac{\Delta V}{V}$ = $\frac{P}{k}$ = $\frac{10 \times 1.013 \times 10^5}{37 \times 10^9}$ = 2.74 × 10^{-5}

15. ***Determine the volume contraction of a solid copper cube, 10cm on an edge, when subjected to a hydraulic pressure of 7.0 × 10^6 Pa , For copper B = 140 × 10^9 Pa .***

V = $(10cm)^3$ = 1000cm^3= 10^{-3} m^3

P = 7.0 × 10^6 Pa

For copper B = 140 × 10^9 Pa

Bulk modulus, B= $\frac{PV}{\Delta V}$

Volume contraction of copper cube ,

ΔV = $\frac{PV}{B}$ = $\frac{7.0 \times 10^6 \times 10^{-3}}{140 \times 10^9}$= 0.05 × 10^{-6} m^3

16. ***How much should the pressure on a litre of water be changed to compress it by 0.10%?***

V = 1 litre = 10^{-3} m^3 ,

$\frac{\Delta V}{V}$ = 0.10 % = $\frac{0.10}{100}$ = 0.001

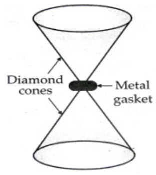
For water , B = 2.2 × 10^9 Nm^{-2}

B = $\frac{PV}{\Delta V}$

$\therefore P = \frac{B\Delta V}{V}$ = 2.2 × 10^9 × 0.001

= 2.2 × 10^6 Nm^{-2}

17. ***Anvils made of single crystals of diamond, with the shape as shown in figure are used to investigate behavior of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.5mm, and the wide ends are subjected to a compressional force of 50,000N. What is the pressure at the tip of the anvil?***



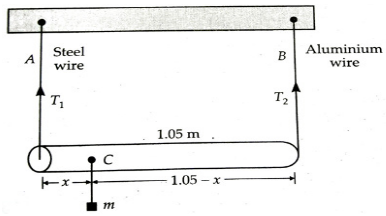
r = 0.25mm = 0.25 × 10^{-3} m F = 50000N

P= $\frac{Force}{Area}$ = $\frac{F}{\pi r^2}$ = $\frac{50,000}{3.14 \times (0.25 \times 10^{-3})^2}$

= 2.55 × 10^{11} Nm^{-2}

18. ***A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminum (wire B) of equal lengths as shown in figure. The cross –sectional areas of wires A and B are 1.0mm^2 and 2.0 mm^2 respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stress and (b) equal strain in both steel and aluminum wires?***

Assume that the mass m be suspended at distance x from the wire.



Let T_1 and T_2 be the tensions in the steel and aluminum wires respectively.

(a) Stress in steel wire = $\frac{T_1}{A_1}$

Stress in aluminum wire = $\frac{T_2}{A_2}$

The stresses are equal

$\therefore \frac{T_1}{A_1} = \frac{T_2}{A_2}$ or $\frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{1.0mm^2}{2.0\ mm^2}$

The system is in equilibrium.

$\therefore T_1 x = T_2(1.05 - x)$

or $\frac{T_1}{T_2} = \frac{1.05 - x}{x}$ or $\frac{1}{2} = \frac{1.05 - x}{x}$

x = 0.7m

(b) Strain = $\frac{Stress}{Young's\ modulus}$

∴ Strain in steel wire $= \frac{\frac{T_1}{A_1}}{Y_1} = \frac{T_1}{A_1 Y_1}$

Strain in aluminum wire $= \frac{T_2}{A_2 Y_2}$

Strain are equal, $\frac{T_1}{A_1 Y_1} = \frac{T_2}{A_2 Y_2}$

$\frac{T_1}{T_2} = \frac{A_1 Y_1}{A_2 Y_2} = \frac{1.0 mm^2 \times 200 \times 10^9\ pa}{2.0\ mm^2 \times 70 \times 10^9\ pa}$

$T_1 x = T_2(1.05 - x)$

or o $\frac{T_1}{T_2} = \frac{1.05 - x}{x}$ or $\frac{10}{7} = \frac{1.05 - x}{x}$

OR $x = 0.43$m

19. ***Two strips of material are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 2.3 $\times 10^9\ pa$? Assume that each rivet is to carry one quarter of the load.***

Let the tension be F

∴ Shearing force on each rivet = $\frac{F}{4}$

Shearing stress on each rivet $= \frac{F/4}{A} = \frac{F}{4A}$

the maximum shearing stress on each rivet is

$2.3 \times 10^9\ Pa$

$\frac{F_{max}}{4A} = 2.3 \times 10^9$

or $F_{max} = 4A \times 2.3 \times 10^9$

$= 4 \times \pi r^2 \times 2.3 \times 10^9$

$= 260.2 \times 10^3\ N$

20. ***The Marina trench is located in the Pacific Ocean and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1$\times 10^8\ Pa$ .A steel ball of initial volume 0.32m^3 is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the trench. What is the change in the volume of the ball when it reaches to the bottom?***

P = 1.01×10^8 Pa , V = $0.32 m^3$

For steel, K = 160×10^9 pa

$B = \frac{P}{\Delta V/V}$

$\therefore \Delta V = \frac{PV}{B} = \frac{1.01 \times 10^8 \times 0.32}{160 \times 10^9} = 2.02 \times 10^{-4}\ m^3$.