6

WORK , ENERGY AND POWER

- **Meaning of the term work**

  Work is said to be done if the point of application of it is displaced. Work is the scalar product of force and displacement.

  $W = \vec{F}.\vec{d}$

  $W = Fd\cos\theta$ where θ is the angle between $\vec{F}$and $\vec{d}$.

$W = (F\cos\theta)d$

  Thus work is the product of magnitude of displacement and the component of force in the direction of displacement Work is a scalar quantity. Its unit is joule or Nm (newton metre).

- If a force of one newton displaces an object through 1m in the direction of force, the work done is ***one joule***.

  In the following cases work done is zero
  - When displacement is zero
  - When force is zero
  - When force and displacement are mutually perpendicular.

- Work can be positive, negative or zero.

- The work done by centripetal force in circular motion is zero. The centripetal force is directed towards the centre of the circular path. The instantaneous displacement is tangential to the circular path. The angle between force and displacement is 90^o

∴Work done is zero.

- If force and displacement are in the opposite directions the work is **negative** as the angle between $\vec{F}$ and $\vec{d}$ is 180^0.

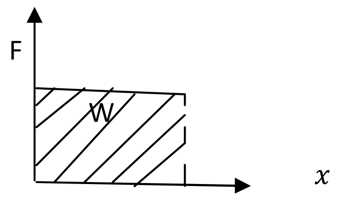
- If force and displacement are at right angles, the work done is zero.

- $1eV = 1.6 \times 10^{-19} J$.

- 1cal = 4.186 J.

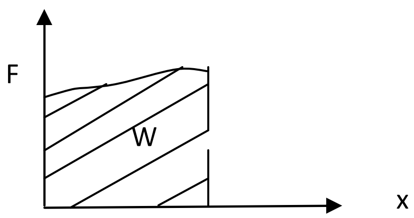
- $1kWh = 3.6\times10^6 J$

- **Graphical representation of the work done by a constant force**



- **Work done by variable force**, W = $\int_{xi}^{xf} \mathbf{F}.\mathbf{d}\boldsymbol{x}$

- **Graphical representation of the work done by a variable force**



- Power is the work done in unit time. OR It is the rate of doing work.
  Power =$\frac{work}{time}$.
  It is a scalar quantity. Its unit is watt OR J/s.

  Power = $\frac{\vec{F}.\vec{d}}{t}$ =$\vec{F}.\vec{v}$

- Power = $\frac{dW}{dt}$ = $\frac{d(\vec{F}.\vec{r})}{dt}$= $\vec{F}.\frac{d\vec{r}}{dt}$ = $\vec{F}.\vec{v}$
- 1 horse power = 746 watt.

- Energy is the capacity to do work. Mechanical energy is classified into two- kinetic energy and potential energy. Energy is scalar quantity.

- Kinetic energy is the energy possessed by a body due to its motion.
  Kinetic energy, K =$\frac{1}{2}$ mv^2
  Kinetic energy = $\frac{1}{2}$ m$\vec{v}.\vec{v}$
  It is always positive.

- Potential energy is the energy possessed by a body due to its higher position or strain. Potential energy of a body at a height h is mgh.

- **Work - energy theorem for a constant force**

  We know that $v^2 - v_o^2 = 2aS$

Multiplying by $\frac{1}{2}$ m

$\frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m2aS$

$\frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = maS$

$\frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = FS$

$K_f - K_i = W$

- ie, Work done is equal to the change of kinetic energy. This is called work-energy theorem.

- **Work -energy theorem for a variable force**

We know that kinetic energy, $K = \frac{1}{2}mv^2$

Differentiating w.r.t. time,

$\frac{dK}{dt} = \frac{d}{dt}(\frac{1}{2}mv^2)$

$\frac{dK}{dt} = \frac{1}{2}m\frac{d}{dt}(v^2)$ OR $\frac{dK}{dt} = \frac{1}{2}mX\,2v\frac{dv}{dt}$

$\frac{dK}{dt} = mva\,\frac{dK}{dt} = Fv$

$\frac{dK}{dt} = F\frac{dx}{dt}\,dK = Fdx$

Integrating, $\int_{Ki}^{Kf} dK = \int_{xi}^{xf} Fdx$

$[K]_{Ki}^{Kf} = \int_{xi}^{xf} Fdx$ OR $K_f - K_i$ =Work done. This is the work –energy theorem for a variable force.

- Work – energy theorem is an **integral form of Newton's second law**.

- When a body is lifted to a height 'h', the work done = mgh.
  This work is stored as gravitational potential energy.
  ∴ Gravitational potential energy V = mgh.

- **Prove that gravitational force is conservative**

To lift a body of mass 'm', to a height 'x' work done= mgx
This work is stored as the gravitational potential energy V = mgx

Differentiating w.r.t.x

$\frac{dV}{dx} = \frac{d}{dx}(mgx)$

$\frac{dV}{dx} = mg\frac{dx}{dx}$

$\frac{dV}{dx} = mg$

Or $-\frac{dV}{dx} = -mg = F$ (Gravitational force)

ie, $F = -\frac{dV}{dx}$

or $Fdx = -dV$

Integrating, $\int_{xi}^{xf} F \quad dx = \int_{Vi}^{Vf} -dV$

$\int_{xi}^{xf} F \quad dx = -[V]_{V_i}^{V_f}$

$\int_{xi}^{xf} F \quad dx = -[V_f - V_i]$

$\int_{xi}^{xf} F \quad dx = V_i - V_f$

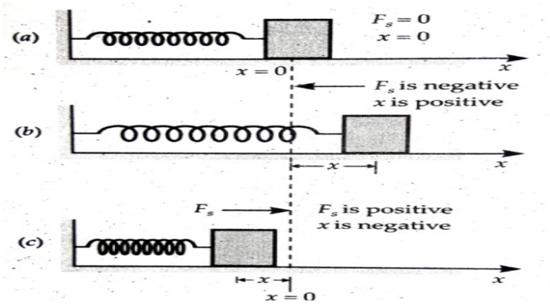
ie, $W = V_i - V_f$

Work done by gravitational force depends on initial potential energy and final potential energy only.

OR It depends on initial position and final position only.

∴ Gravitational force is conservative.

- **Potential energy stored in a stretched spring**



Consider a spring whose one end is attached to a rigid support. The other end is attached to a mass. The spring is resting on a frictionless a horizontal table. The position of the mass is taken as '0'. It is stretched to distance x by applying a force F. Thus the final position is x.

$$F \propto x$$

But the applied force , $F = -Fs$

$\therefore -F_S \propto x$ OR $F_s \propto -x$

$F_s = -k\,x$ where k is called spring constant or force constant. –ve indicates that spring force and displacement are in the opposite direction.

Work done in producing an extension x is $W = \int_{xi}^{xf} F\,dx = \int_{xi}^{xf} -F_S\,dx$

$W = \int_{xi}^{xf} kxdx \quad = k\int_{xi}^{xf} xdx$

$W = k\,[\frac{x^2}{2}]_{xi}^{xf} = \frac{k}{2}[x_f^2 - x_i^2]$

Here, $x_i = 0$ and $x_f = x$ ∴ $W = \frac{k}{2}[x^2 - 0^2] \ \frac{1}{2}kx^2$

This work is stored as potential energy in the spring.

∴ PE of spring $= \frac{1}{2}kx^2$

Note : Work done by spring $= -\frac{1}{2}kx^2$

- **Prove that spring force is conservative.**

Consider a spring whose on end is attached to rigid support. It is resting horizontally on a frictionless table. A mass 'm' is attached to it. When it is pulled to new position, the work done by the spring,

$$=\int_{X_i}^{x_f} F_s dx = \int_{x_i}^{x_f} -kx dx$$

$$=-k \int_{x_i}^{x_f} x dx = -k \left[\frac{x^2}{2}\right]_{x_i}^{x_f} = -\frac{k}{2}[x_f^2 - x_i^2]$$

$$=\frac{k}{2}[x_i^2 - x_f^2]$$

The spring finally arrives at the initial position, $x_i = x_f$

Work done by the spring $= \frac{k}{2} \times 0 = 0$

Work done in this closed path is zero. We know that the work done by a conservative force in a closed path is zero.

∴ The spring force is conservative.

- **Velocity of a spring when it is stretched maximum and released**

Consider a spring of spring constant k. If it is stretched to a maximum distance x_m, the P.E. stored $= \frac{1}{2} k x_m^2$

When it is released, it crosses a point P at a distance 'x' from the mean position.

Total energy at P = KE + PE $= \frac{1}{2} mv^2 + \frac{1}{2} kx^2$

$$\frac{1}{2} k x_m^2 = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$$

$$k x_m^2 = mv^2 + kx^2$$

$k(x_m^2 - x^2) = mv^2$ OR $\frac{k(x_m^2 - x^2)}{m} = v^2$

$$v = \sqrt{\frac{k(x_m^2 - x^2)}{m}}$$

For mean position, x =0

$$\therefore v = \sqrt{\frac{k(x_m^2)}{m}}$$

For the extreme position, $x = x_m$ ∴ v = 0

- **Law of conservation of mechanical energy**

The total mechanical energy of a system is conserved, if the force doing work is conservative.

- **Proof of the law of conservation of mechanical energy**

Consider a conservative force F acting on a body. According to work- energy theorem,
Work done = Change of kinetic energy

$$F\,\Delta x = \Delta K \text{ - - - - - } (1)$$

Also we have, $F\,\Delta x = -\Delta V$ - - - - - (2)

(1)-(2) gives $F\,\Delta x - F\,\Delta x = \Delta K - (-\Delta V)$

$0 = \Delta K + \Delta V$ OR $0 = \Delta(K+V)$

ie, Change of mechanical energy is zero.

OR Mechanical energy is conserved.

- **Prove the law of conservation of mechanical energy in the case of a freely falling body**.

Consider a body which is dropped from a point A at a height h. Gravitational force (which is conservative force) acts on the body downwards.

P.E of the body =mgh As the velocity is zero , KE = 0 A

Total energy = PE + KE = mgh + 0 = mgh

The body covers a distance 'x' and reaches the point B.

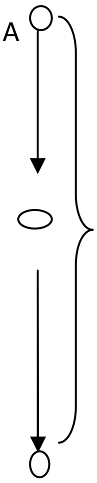
PE of the body = mg(h-x) = mgh – mgx

KE of the body = $\frac{1}{2}mv^2$ But $v^2 - {v_0}^2 = 2aS$

Here v_0 = 0 , a = g and S = x B

$\therefore v^2 - o^2 = 2gx$ or $v^2 = 2gx$ $\therefore$ KE =$\frac{1}{2}m \times 2gx = mgx$ h Total energy =

$PE + KE = mgh - mgx + mgx = mgh$



When the body reaches the point C on ground, h-x

$PE = 0$ $KE = \frac{1}{2}mv^2$ But $v^2 - {v_0}^2 = 2aS$ Here v_0 = 0 , a = g and S = h C

Total energy = $PE + KE = 0 + mgh = mgh$

Total energy at A = Total energy at B = Total energy at C.

ie,Total energy is conserved.

- **Prove that the work done by a conservative force in moving body along a closed path is zero.**

For a conservative force, we know that $\int_{Xi}^{xf} F \, dx = -[V]_{Vi}^{Vf}$

$\int_{xi}^{xf} F \, dx = V_i - V_f$----------(1)

Also, $\int_{xi}^{xf} F \, dx = K_f - K_i$-----------(2)

L.H.S. is same for both.

L.H.S. = $\int_{xi}^{xf} F \, dx = W$

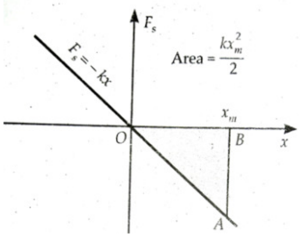
$\therefore$ W = $V_i - V_f = K_f - K_i$

If the body is brought back to the initial position, $V_i = V_f$

$\therefore$ W =0

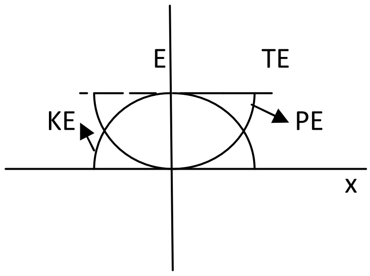
ie, Work done in moving the body along a closed path is zero.

- **Graphical representation of the work done by a stretched spring**



Area of shaded portion = $\frac{1}{2}$ bh =$\frac{1}{2}x\ F_s xx$ = $\frac{1}{2}x(-k\ x)\ xx$ = $-\frac{1}{2}k\ x^2$

- **The graph showing potential energy, kinetic energy and total energy of a spring - mass system set to oscillation**



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- When we rub our palm each other, the work done against friction appears as heat energy.

- When an object is heated, the internal energy of its molecules increases.

- A stable chemical compound has less energy than its constituent elements.

- If the total energy of reactants is more than products, heat is released during the reaction and the reaction is called exothermic.

- If the total energy of reactants is less than products, heat is absorbed during the reaction and the reaction is called endothermic.

- Chemical energy is associated with the forces which give stability to substances. These forces bind atoms into molecule, molecules into isomeric chains etc.

- Energy associated with electric current is electrical energy.

- **Equivalence of mass and energy**

  According to Einstein, mass and energy are equivalent. Matter of mass m is equivalent to energy E = mc^2 where c is the velocity of light.
- Calculate the energy equivalent to one kilogram of mass.

  Here m = 1 kg c = 3×10^8 m/s E = mc^2 = $1\times(3 \times 10^8)^2$ = $9 \times 10^{16}\ J$

- **Methods to get energy from nucleus**

  Energy can be obtained from nucleus by nuclear fission and nuclear fusion. The splitting of a nucleus with the release of energy is called nuclear fission.
  The combining of two or more lighter nuclei to form a heavier nucleus with the release of energy is called nuclear fusion.

- Uncontrolled chain reaction(nuclear fission) is the principle of atom bomb.
- Controlled chain reaction(nuclear fission) is utilized in a nuclear reactor.

- The total mass of products in a fission/fusion is less than the total mass of reactants. This decrease in mass appears as energy according to E = mc^2. This is the reason why we get large amount of energy in fission/fusion.
- Nuclear fusion is the principle of hydrogen bomb.

- **Mass defect and binding energy**

  Mass defect is the deference between the rest mass of nucleus and the sum of masses of protons and neutrons. The energy equivalent to mass defect is called binding energy. It holds the nucleons together inside the nucleus.

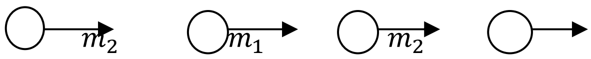
- **Difference between elastic and inelastic collisions**

  Those collisions in which both momentum and kinetic energy are conserved are called elastic collisions. Those collisions in which momentum is conserved and kinetic energy is not conserved are called inelastic collisions.

- **Elastic collision in one dimension**

u_1 u_2 $v_1 v_2$

m_1 m_2 m_1 m_2



Before collision **After collision**

Consider two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 in a straight line($u_1 > u_2$). After collision they move along the same line with velocities v_1 and v_2 .

Total momentum before collision = Total momentum after collision

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ------------------------ (1)

Total kinetic energy before collision = Total kinetic energy after collision

$\frac{1}{2}m_1 {u_1}^2 + \frac{1}{2}m_2 {u_2}^2 = \frac{1}{2}m_1 {v_1}^2 + \frac{1}{2}m_2 {v_2}^2$ -------------(2)

$m_1 {u_1}^2 + m_2 {u_2}^2 = m_1 {v_1}^2 + m_2 {v_2}^2$

$m_1({u_1}^2 - {v_1}^2) = m_2({v_2}^2 - {u_2}^2)$

$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2)$ ---------------(3)

(3) ÷ (1) $u_1 + v_1 = v_2 + u_2$

$(u_1 - u_2) = v_2 - v_1$-----------------(4)

$u_1 - u_2 = -(v_1 - v_2)$

i.e. , The relative velocity before collision is numerically equal to the relative velocity after collision.

From (4) $v_1 = v_2 + u_2 - u_1$ ---------------(5)

Putting this in(1) $m_1[u_1 - (v_2 + u_2 - u_1)] = m_2(v_2 - u_2)$

$m_1 u_1 - m_1 v_2 - m_1 u_2 + m_1 u_1 = m_2 v_2 - m_2 u_2$

$2 m_1 u_1 - m_1 u_2 + m_2 u_2 = m_2 v_2 + m_1 v_2$

$2 m_1 u_1 + (-m_1 + m_2) u_2 = (m_2 + m_1) v_2$

$v_2 = \frac{2m_1u_1+(-m_1+m_2)u_2}{(m_1+m_2)}$ OR $v_2 = \frac{(m_2-m_1)u_2}{(m_1+m_2)} + \frac{2m_1u_1}{(m_1+m_2)}$ --------------(5)

From (4) $v_2 = v_1 - u_2 + u_1$ ------------(6)

Putting this in (1) $m_1(u_1 - v_1) = m_2(v_1 - u_2 + u_1 - u_2)$

$$m_1u_1 - m_1v_1 = m_2v_1 - m_2u_2 + m_2u_1 - m_2u_2$$

$$m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2 = m_2v_1 + m_1v_1$$

$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$

$v_1 = \frac{2m_2u_2+(m_1-m_2)u_1}{(m_1+m_2)}$ $v_1 = \frac{(m_1-m_2)u_1}{(m_1+m_2)} + \frac{2m_2u_2}{(m_1+m_2)}$ ---------(7)

Special cases(i) If $m_1 = m_2 = $ m $v_2 = u_1$ and $v_1 = u_2$

(ii) If $m_1 = m_2 = $ m and $u_2 = 0$ then, $v_2 = u_1$ and $v_1 = 0$

ie, The first body comes to rest and the second body moves with the speed of first.

- In a perfectly inelastic collision, the two bodies stick together after collision and start moving together.
- **Coefficient of restitution**

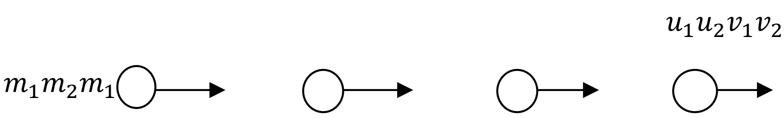
It is the ratio of the velocity of separation to the velocity of approach.

Coefficient of restitution $= e = \frac{v_2 - v_1}{u_1 - u_2}$

e = 1,for elastic collision

e = 0,for perfectly inelastic collision and e< 1 for inelastic collision.

- **Inelastic collision in one dimension**



Before collision After collision

Consider two bodies of masses m_1 *and* m_2 moving with velocities u_1 *and* u_2 in a straight line($u_1 > u_2$).

After collision they move along the same line with velocities v_1 *and* v_2 .

Since the collision is inelastic,

Total momentum before collision = Total momentum after collision

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

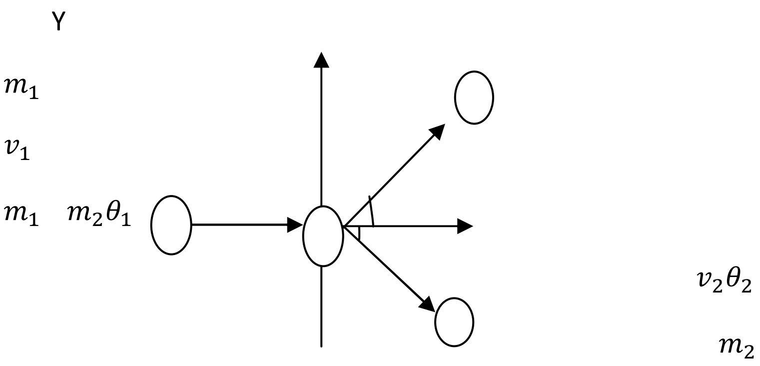
$m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ------------------------ (1)

Here $\frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 < \frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2$

$m_2({v_2}^2 - {u_2}^2) < m_1({u_1}^2 - {v_1}^2)$ or $v_2 - v_1 < u_1 - u_2$

- **Elastic collision in two dimension**

Consider two bodies of masses m_1 *and* m_2 moving with velocities u_1 *and* u_2 in a straight line ($u_1 > u_2$). After collision , m_1 moves with a velocity v_1 making an angle θ_1 with the initial direction and m_2 moves with a velocity v_2 making an angle θ_2 with a initial direction.



Along X-direction:

Total momentum before collision = Total momentum after collision

$m_1u_1 + m_2u_2 = m_1v_1cos\theta_1 + m_2v_2 \cos\theta_2$ where $v_1cos\theta_1$ and $v_2 \cos\theta_2$ are the components of v_1 and v_2 along the X direction.

Along Y – direction:

0 = $m_1v_1sin\theta_1 + -m_2v_2 \sin\theta_2$ where $v_1sin\theta_1$ and $v_2 \sin\theta_2$ are the components of v_1 and v_2 along the Y direction.

Total kinetic energy before collision = Total kinetic energy after collision

Along X-direction:

$\frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2 = \frac{1}{2}m_1(v_1cos\theta_1)^2 + \frac{1}{2}m_2(v_2 \cos\theta_2)^2$

Along Y – direction:

$0 = \frac{1}{2}m_1(v_1sin\theta_1)^2 + \frac{1}{2}m_2(-v_2sin\theta_2)^2$