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MECHANICAL PROPERTIES OF SOLIDS

- **Deforming force**

Deforming force is the force which changes the shape or size of a body.

- **Restoring force**

Restoring force is the internal force developed inside the body which brings the body back to original shape and size when a deforming force acts on it.

- **Elasticity**

It is the property of a body by virtue of which it tends to regain its original shape and size when deforming force is removed.

- **Difference between perfectly elastic bodies and plastic bodies**

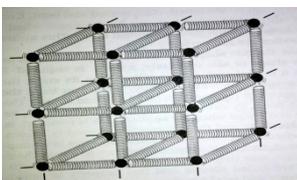
If a body immediately regains its original shape and size when the applied force is removed, it is perfectly elastic.
eg: quartz

If a body does not show any tendency to regain its original shape and size when the applied force is removed, it is a plastic body. eg: putty, mud etc.

- **Why does a solid regain its original shape and size when deforming force is removed?**

Solid molecules are tightly packed. The inter molecular force of attraction is very strong. When deforming forces are acted on it, the inter molecular distance changes. The strong inter molecular force of attraction brings the molecules back to the original positions and thus the body regains its original shape and size.

- **Spring – ball model of a solid**



Balls represent molecules and spring represent inter molecular force.

- **Stress**

It is the restoring force acting per unit area.

$$\text{Stress} = \frac{\text{restoring force}}{\text{area}} . \quad \text{Its unit is } N/m^2$$

- **Strain**

It is the ratio of change in dimension to the original dimension.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Strain has no unit as the numerator and denominator have the same unit.

- **Longitudinal stress or linear stress**

It is the stress when there is change in length.

- **Normal stress or hydraulic stress**

It is the stress when there is change in volume.

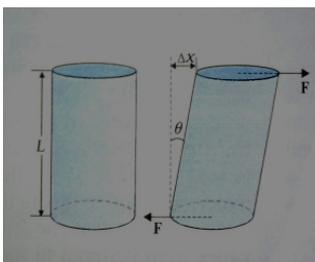
- Shearing stress or tangential stress is the stress when there is change in shape of the body.

- Tensile stress causes increase in length. Compressive stress causes decrease in length.

- Linear strain is the ratio of change in length to the original length.

- Volume strain is the ratio of change in the volume to the original volume.

- Shearing strain indicates the angle through which deformation take place.



Shearing strain = $\theta \approx \tan\theta = \frac{\Delta x}{L}$ where L is the height and Δx is the tangential displacement.

- The strain produced by hydraulic stress is called volume strain.

- **Hooke's law**

For small deformations, the stress is directly proportional to the strain.

Stress \propto strain

Stress = constant (k) X strain or $\frac{\text{stress}}{\text{strain}} = \text{constant}$. This constant is called the modulus of elasticity.

- There are three moduli of elasticity. They are Young's modulus, Bulk modulus and Shear modulus (rigidity modulus)
Unit of modulus of elasticity: N/m^2

- **Young's modulus**

It is the ratio of linear stress to linear strain.

$$\text{Young's modulus} = \frac{\text{linear stress}}{\text{linear strain}}$$

- **Bulk modulus**

It is the ratio of normal stress to volume strain.

$$\text{Bulk modulus} = \frac{\text{normal stress}}{\text{volume strain}}$$

$$\text{Normal stress} = \frac{\text{Normal Force}}{\text{Area}} = \text{Pressure (P)}$$

$$\text{Normal strain} = \frac{\text{change in the volume}}{\text{Original volume}} = \frac{\Delta V}{V}$$

$$\text{Bulk modulus} = P \div \frac{\Delta V}{V} = \frac{PV}{\Delta V}$$

- **Shear modulus**

It is the ratio of shearing stress to shearing strain.

$$\text{Bulk modulus} = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$\text{Shearing stress} = \frac{F}{A}$$

$$\text{Shearing strain} = \frac{\Delta x}{L}$$

$$\text{Bulk modulus} = \frac{F}{A} \div \frac{\Delta x}{L} = \frac{FL}{A\Delta x}$$

- **How will you calculate the Young's modulus of the material of a wire?**

Take two identical wires. Hang them parallel to each other from ceiling. Attach identical weighing pans to both. Attach a vertical scale to the first wire called reference wire. Attach a vernier to the pointer attached to the second wire called experimental wire. Keep identical weights on both pans to keep them upright. Measure the length of the experimental wire (L) Note the reading on the scale at the level of zero of vernier. Add weights to the experimental wire. The wire undergoes elongation. Note the new scale reading.

The difference between the reading gives the extension (ΔL).

$$\text{Linear strain} = \frac{F}{A} = \frac{Mg}{\pi r^2} \quad \text{where } M \text{ is the mass added and } r \text{ is the radius of the wire.}$$

$$\text{Linear strain} = \frac{\text{increase in length}}{\text{Original length}} = \frac{\Delta L}{L}$$

$$\text{Young's modulus} = \frac{\text{linear stress}}{\text{linear strain}} = \frac{Mg}{\pi r^2} \div \frac{\Delta L}{L} = \frac{MgL}{\pi r^2 \Delta L}$$

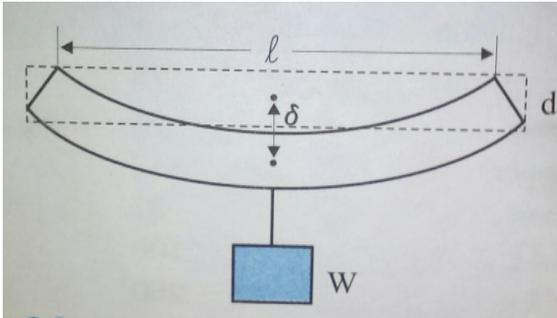
- Reciprocal of bulk modulus is called compressibility.

- **The idea of elasticity helps in determining the area of ropes used in crains. Explain.**

The extension of the rope should not exceed the elastic limit of the material of the rope. The yield strength of the material of the rope (steel) is $300 \times 10^6 \text{ N/m}^2$.

Area of cross – section of the rope $\geq \frac{W}{S_y}$ where W is weight attached and S_y is the yield strength. (Here $W = mg$).

- **Ideas to be kept during the construction of a beam for building**

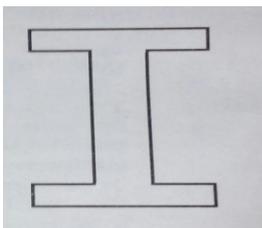


The sag (δ) produced in the beam when weight is placed over it, $\delta = \frac{Wl^3}{4bd^3Y}$ where W is the weight, l is the length of the beam, b is the breadth of the beam, d is the depth of the beam and Y is the young's modulus of the material of the beam. To reduce sag, length is to be reduced, breadth and depth are to be increased and the young's modulus of the material must be large. It will be effective to increase the depth compared to breadth as the sag is inversely proportional to the cube of depth.

- **Cross – sectional shape is ideal for load bearing bars. Why?**

This shape provides maximum load bearing surface. As the depth of it is more, the sag produced in it will be less. As it requires less material, cost can be reduced. Weight of beam is minimum as the material required is minimum.

At the same time it provides maximum strength.



- **The maximum height of a mountain on the earth's surface is 10Km.Why?**

The base of a mountain is under non – uniform compression. Hence the base experiences large shearing stress. The stress must be less than elastic limit of the rocks at the base of the mountain. The elastic limit of rock is $300 \times 10^6 \text{ N/m}^2$

ie, $h \rho g = 300 \times 10^6 \text{ N/m}^2$

Or

$$h = \frac{300 \times 10^6}{g\rho} = \frac{300 \times 10^6}{10 \times 3 \times 10^3} = 10\text{km. ie, maximum height is 10km.}$$

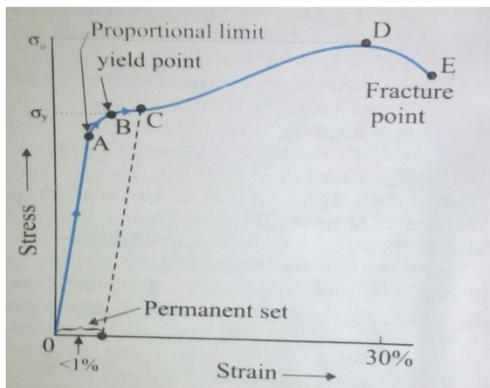
- **Explain the stress-strain graph for a metal**

Consider a metallic wire whose one end is attached to some rigid body. A force is applied at the other end. The stress and the corresponding strain values are noted and a graph is plotted. The stress – strain graph for the metal is shown .

In the first section, the stress and strain are proportion up to a limit called proportional limit (point A in figure). If force is removed, the body returns to its original position. The body behaves as elastic.

In the section from A to B the stress and the strain are not in direct proportion. But if force is removed, the body returns to its original position. The point B is called elastic limit or yield point.

The corresponding stress is called yield strength.



From B onwards if stress is increased, the strain increases much up to the point D. When stress is removed from B to D (at C) the body will not regain its original shape and size. It attains permanent set. The deformation is called plastic deformation. The point D is called ultimate yield point.

Beyond it additional strain is produced even for decreased stress and the body breaks at the point E called fracture point (breaking point).

If D and E are close, the material is brittle.

If they are far, it is ductile.

Substances which can be extended to produce large values of strain are called elastomers. eg: rubber, elastic tissue of aorta etc.

- **The stress- strain graph for the elastic tissue of aorta**

Here the elastic region is large. But it does not obey Hooke's law.

