3

CURRENT ELECTRICITY

3.01 Electric current

Electric current is the charge crossing any section of a conductor in unit time.
If the current is steady, current $I = \frac{q}{t}$
If the current is not steady, $I = \text{Lt}_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$
It is a scalar quantity.
Ampere or coulomb/second is the SI unit of current.

Electromagnetic unit (emu) of current is ab ampere.

$$1\text{A} = \frac{1}{10} \text{ emu}$$

Electrostatic unit (esu) of current is stat ampere.

$$1\text{A} = 3\times10^9 \text{ esu}$$

The current with constant magnitude is called steady current.

> ***Note***
> The direction of current is usually taken as from positive terminal to negative. This current is taken as conventional current. The electrons move from negative terminal to positive. This current is called electronic current.
> Though current has both magnitude and direction, it does not obey the rules of vector addition. Hence it is a scalar quantity.

3.02 Current produced by orbiting electron

For an electron revolving in an orbit of radius r with a speed v , the period of revolution $\tau = \frac{2\pi r}{v}$
Current at any point of the orbit $I = \frac{q}{t} = e \div \frac{2\pi r}{v} = \frac{ev}{2\pi r}$

3.03 Ohm's law

At constant temperature, the current through a conductor is directly proportional to the potential difference between its ends.

If I is the current and V is the P.D, then $I \propto V$ or $V \propto I$

$V = R\ I$ where R is called resistance of the conductor. The unit of resistance is ohm.

3.04 Conductance

The reciprocal of resistance is called conductance.
Ohm^{-1} or siemens is the unit of conductance.

3.05 Factors affecting the resistance of a conductor

a. Length of the conductor (l)

Resistance is directly proportional to the length of the conductor.

$R \propto l$

b. Area of cross-section (A)

Resistance is inversely proportional to the area of cross-section of the conductor.

$R \propto \frac{1}{A}$

c. Temperature

As the temperature of a metal is increased, its resistance also increases.

d. Nature of material

Different materials have different resistances.

Resistance , $R= \frac{\rho l}{A}$ where is called resistivity or specific resistance. Its SI unit is ohm metre.

> ***Note***
>
> The reciprocal of resistivity is called conductivity.
> Mho/metre or Siemens/metre is the unit of conductivity.

3.06 Difference between the following situations

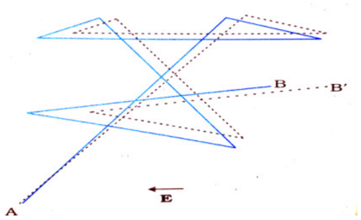
a. Conductor not connected to battery **b. Conductor connected to battery**

a. Conductor not connected to battery

Consider a conductor which is not connected to any battery. The free electrons are in random motion. They collide against each other and also with the positive ions. During the collision between an electron and a positive ion, the initial and final velocities of the electron are the same. But direction of motion is random. Hence the path of electrons becomes zig-zag. As large number of electrons move in random directions, the number of electrons crossing an infinite simully small area from one side will be nearly equal to the number of electrons crossing from the other side in a given interval. The current through the area is therefore zero.

b. Conductor connected to battery

When the conductor is connected to a battery of voltage V, an electric field E = $\frac{V}{l}$ is created across the ends of conductor (Here l is the length of the conductor).The electric field exerts a force -Ee on each electron. Due to this force the electrons modify their random motion. The electrons are drifted to positive terminal side. Each electron gets an acceleration of $\frac{-Ee}{m}$.



> ***Note***
>
> The average thermal speed of electrons in a conductor is 10^5 to 10^6 m/s.
>
> The average drift speed of electrons is very small (nearly 1mm/s)

3.07 Reason why a bulb glows instantly when switch is closed even though the average drift speed of electrons is very small

Electrons are present everywhere in the electric circuit. When a P.D is applied, an electric field is developed at the speed of light. The electrons are drifted under the effect of this field and the bulb glows instantly.

3.08 Relation connecting current and drift velocity

Consider a conductor of length l, area of cross section A containing n electrons per unit volume. When it is connected to a battery, an electric field E = $\frac{V}{l}$ is developed across its ends, where V is the voltage. The electric field exerts a force F= —Ee on each electron. The electrons are drifted to the positive terminal side.

Each electron gets an acceleration of $\frac{-E_e}{m}$

Volume of the conductor = $A\ l$

Number of electrons in the conductor = $A\ l$ n

Total charge= Alne where e is the charge of one electron.

The current through the conductor, I = $\frac{charge}{time}$

$\therefore$ I = $\frac{Alne}{t}$

But $\frac{l}{t} = v_d$, drift velocity v_d

∴ $I = A\, v_d n e$ OR $I = v_d n e A$

It is clear that $I \propto v_d$

3.09 Current density

Current density(J) is the current passing normally through unit area of a conductor.

It is a vector quantity. Its direction is the same as that of current.

A/m^2 is the SI unit of current density.

> ***Note***
> Current $I = \vec{J}.\vec{A}$

3.10 Relation between current density and electric field

The resistance of a conductor, $R = \frac{\rho l}{A}$

Also V = I R

$V = I\frac{\rho l}{A}$

For a conductor connected to battery, $E = \frac{V}{l}$ OR $V = El$

$El = I\frac{\rho l}{A}$

$E = \frac{\rho I}{A}$

$\frac{I}{A}$ = J, current density

∴ $E = J\rho$

But $\rho = \frac{1}{\sigma}$

∴ $E = J \times \frac{1}{\sigma}$ or $J = \sigma E$

or $J \propto E$

3.11 Vector form of Ohm's law

The resistance of a conductor, $R = \frac{\rho l}{A}$

Also V = I R

$V = I\frac{\rho l}{A}$

For a conductor connected to battery, $E = \frac{V}{l}$ OR $V = El$

$El = I\frac{\rho l}{A}$

$E = \frac{\rho I}{A}$

$\frac{I}{A}$ = J, current density

∴ $E = J\rho$

But $\rho = \frac{1}{\sigma}$

$\therefore$ E = J $\times \frac{1}{\sigma}$ or J = σE or J $\propto$ E

Current density vector has the same direction as that of electric field ($\vec{E}$).

$\therefore$ $\vec{J} = \sigma \vec{E}$. This is known as **vector form** of Ohm's law.

3.12 Microscopic form of Ohm's law

The resistance of a conductor, R= $\frac{\rho l}{A}$

Also V = I R

$$V = I \frac{\rho l}{A}$$

For a conductor connected to battery, E = $\frac{V}{l}$ OR $V = El$

$$El = I\frac{\rho l}{A}$$

$$E = \frac{\rho I}{A}$$

$\frac{I}{A}$ = J, current density

$\therefore$ E = Jρ

But $\rho = \frac{1}{\sigma}$

$\therefore$ E = J $\times \frac{1}{\sigma}$ or J = σE

J $\propto$ E

Current density is the current passing normally through unit area.

J = σE is known as microscopic form of Ohm's law.

3.13 Relation connecting conductivity and relaxation time

For a conductor connected to a battery, I = v_d enA

But v_d= aτ where a is the acceleration

$\therefore$ I = aτ enA

Magnitude of acceleration a = $\frac{F}{m} = \frac{Ee}{m}$

$$I = \frac{Ee}{m}\tau \text{ enA}$$

$$I = \frac{Ee^2}{m} \tau \text{ nA}$$

$$\frac{I}{A} = \frac{Ee^2 \tau \text{ n}}{m}$$

But $\frac{I}{A}$ = J, current density

$\therefore$ J = $\frac{Ee^2\tau\,\text{n}}{m}$ -------------------(1)

Also J = Eσ -------------------(2)

Comparing (1) and (2) we get

Conductivity, $\sigma = \frac{e^2\tau\,\text{n}}{m}$

Resistivity, $\rho = \frac{1}{\sigma} = \frac{m}{e^2\tau\,\text{n}}$

3.14 Mobility

Mobility is the ratio of magnitude of drift velocity to electric field.

$$\mu = \frac{|v_d|}{E}$$

But v_d = a τ OR $v_d = \frac{Ee}{m}\tau$

$$\therefore \mu = \frac{\frac{Ee\tau}{m}}{E} = \frac{e\tau}{m}$$

SI unit of mobility is m^2/Vs OR mC/Ns.
Mobility is positive.

3.15 Relation between current and mobility

$I = v_d$enA

But $\mu = \frac{v_d}{E}$

$\therefore$ $v_d = \mu E$

$\therefore$ $I = \mu$EenA

3.16 Deduction of Ohm's law

Consider a conductor of length l ,area of cross section A containing n electrons in unit volume. When it is connected to a battery, an electric field E = $\frac{V}{l}$ is developed across its ends, where V is the voltage. The electric field exerts a force F= −Ee on each electron. The electrons are drifted to the positive terminal side. Each electron gets an acceleration of magnitude $\frac{Ee}{m}$.

Drift velocity v_d = aτ where a as the acceleration and τ is the relaxation time(Relaxation time is the average time interval between two successive collisions of electrons.)

Magnitude of acceleration $a = \frac{F}{m} = \frac{Ee}{m}$

$\therefore\ v_d = \frac{Ee}{m}\tau$

But $E = \frac{V}{l}$

$\therefore\ v_d = \frac{Ve\tau}{lm}$

$I = v_d enA$

$I = \frac{Ve\tau}{lm} enA$

$\frac{V}{I} = \frac{lm}{e^2\tau nA}$

At a particular temperature, l,τ,A are constants . Also m, e and n are constants.

i.e. $\frac{V}{I}$ = constant OR $V \propto I$ which is Ohm's law.

3.17 Cause of electrical resistance

When a PD is applied across a conductor, the electrons are accelerated. They frequently collide with the positive ions. Motion of electrons is opposed which is the reason for resistance.

If the area of cross section is more, the number of collisions decreases and resistance decreases. Number of collisions depends on the arrangement of atoms. Different materials have different arrangement of atoms. Hence resistance varies with nature of material.

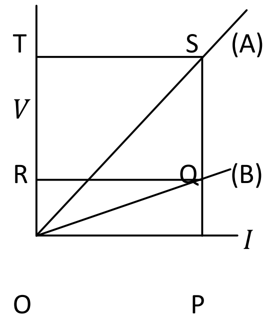
3.18 The graph given shows the variation of I with V for two different materials A and B. Identify the material having more resistance.

For A

Resistance = $\frac{V}{I} = \frac{PS}{OP}$

For B

Resistance = $\frac{V}{I} = \frac{PQ}{OP}$



PS > PQ $\therefore$ Resistance is more for A

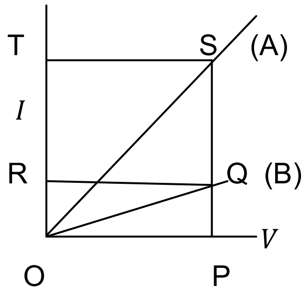
3.18 The graph given shows the variation of I with V for two different materials A and B . Identify the material having more resistance.

For A

Resistance = $\frac{V}{I} = \frac{OP}{PS}$

For B

Resistance = $\frac{V}{I} = \frac{OP}{PQ}$

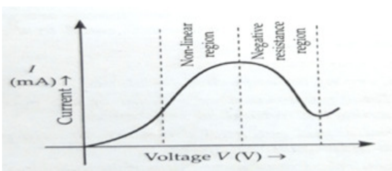


PS > PQ ∴ Resistance is more for B.

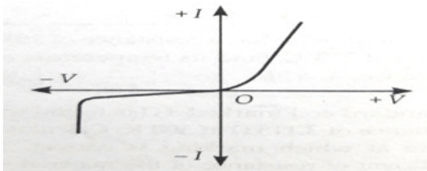
3.19 Limitations of Ohm's law

1. Ohm's law is not obeyed by electrolytes.
2. Ohm's law is not obeyed by semi-conductor devices like diodes, transistors etc.
3. Even for conductors when large currents are passed large amount of heat is produced. In such a case V will not be proportional to I.

3.20 Graph showing the variation of I with V for GaAs and a diode



(a)



(b)

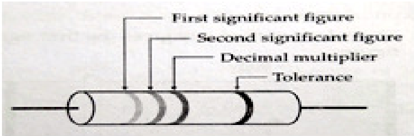
> ***Note***
>
> Alloys like manganin, constantan, nichrome etc are used to make high resistance wires.

3.21 Resistivity of some materials

Silver	1.6 x 10^{-8}Ωm
Copper	1.7 x 10^{-8}Ωm
Aluminium	2.7 x 10^{-8}Ωm
Tungsten	5.6 x 10^{-8}Ωm

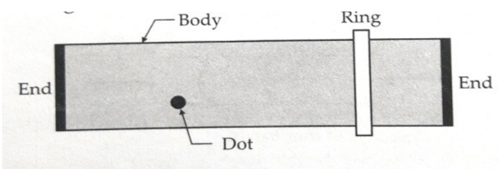
3.22 <u>Colour code of carbon resistors</u>

Carbon resistors are made of semi – conductors. In a carbon resistor, coloured rings will be marked. Each colour has a code number. The first two colours represent the first two digits in the resistance value. The third ring represents the multiple of 10. The fourth ring represents the percentage of tolerance (percentage of accuracy).For gold it is $\pm$5% ,for silver it is $\pm$10% and if no line, it is $\pm$20%



Colour	Number	Multiplier
Black	0	10^0
Brown	1	10^1
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Grey	8	10^8
White	9	10^9
Gold	-1	10^{-1}
Silver	-2	10^{-2}

Another system of colour code



- The colour of body indicates the first significant figure in resistance value.
- The colour of the end indicates the second significant figure in resistance value.
- The colour of dot gives the multiplier of ten.
- The colour of the ring gives tolerance.

3.23 Resistivity of metallic conductor

$\rho_T = \rho_o [1 + \propto (T - T_o)]$

where ρ_T is the resistivity at T^oC, ρ_o is the resistivity at $T_o{}^oC$ and $\propto$ is the temperature coefficient of resistance.

We know that $R_T = R_o [1 + \propto (T - T_o)]$

$$R_T = R_o + R_o \propto (T - T_o)$$

$$\propto = \frac{R_T - R_o}{R_o(T - T_o)}$$

Temperature coefficient of resistance is the ratio of increase in resistance to resistance at a reference temperature T_o for 1^o rise of temperature.

Its unit is $^oC^{-1}$ or K^{-1}

$\propto$ is positive for a metal. $\propto$ for a semi-conductor is negative.

3.24 Reason why resistivity of a metal increases with temperature

We know that resistivity $\rho = \frac{m}{ne^2\tau}$

When temperature is increased the speed of collision of electrons increases. Relaxation time decreases.

$\rho \propto \frac{1}{\tau}$

When relaxation time decreases, resistivity increases.

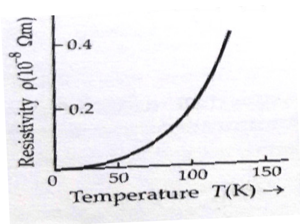
3.25 Reason why resistivity of a semi conductor decreases with increase of temperature

Resistivity $\rho = \frac{m}{ne^2\tau}$

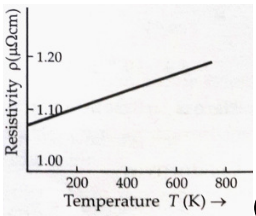
When temperature is increased, relaxation time decreases. At the same time number of electrons increases. This increase is more compared to the decrease of relaxation time. Therefore resistivity decreases.

3.26 Graph showing the variation of resistivity with temperature for (i) Copper (ii) nichrome (iii) semi-conductor

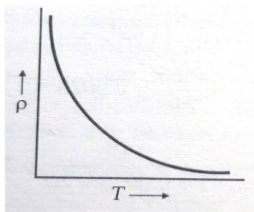
(i)



(ii)



(iii)

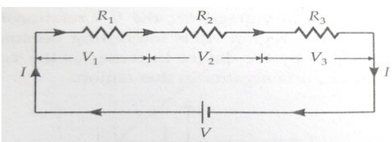


3.27 Properties of alloys which enable from to use for making standard resistance

1. They have high resistivity.
2. They have small value of temperature coefficient of resistance.
3. They are least affected by atmospheric conditions like moisture, air etc.

3.28 Series and parallel combination of resistors

<u>Series</u>



The resistors are connected as in figure. Let I be the main current and V_1 , V_2 and V_3 be the P.D across R_1 , R_2 and R_3 respectively.

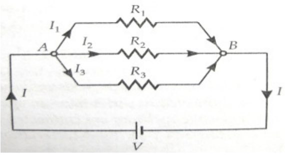
$V = V_1 + V_2 + V_3$

$IR = IR_1 + IR_2 + IR_3$ where R is the effective resistance of the combination.

$R = R_1 + R_2 + R_3$

ie, The total resistance is the sum of resistances. This combination is used to increase resistance.

Parallel



The resistors are connected as in figure .Here the P.D. across all the resistors will be the same (V). The current through R_1 , R_2 and R_3 are I_1, I_2 and I_3 respectively.

$$I = I_1 + I_2 + I_3$$

$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$ where R is the resistance of the combination.

OR $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ OR $R = [\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}]^{-1}$

This combination is used to decrease resistance. The total resistance is always less than the least value of resistance used.

Note

- Wire bound resistors are resistors made of high resistance wires like nichrome, constantan or manganin. Their resistivity is almost independent of temperature.
- The free electrons of a metal are in random motion .They collide against each other. The average time interval between two successive collisions of an electron is called relaxation time.
- A long wire offers more resistance, as the chance of collision of electron with positive ion is more.

3.29 Reason for large resistivity of alloys

In an alloy like nichrome Ni^{2+} and Cr^{3+} ions have different charge and size. They are in random positions. Hence the chance of collision of electron with them is more. Relaxation time decreases and thus the resistivity increases.

3.30 Relation between electric current and mobility for a semi - conductor

$I = AeE\,(n_e\mu_e + n_h\mu_h)$ where n_e is the number of electrons in unit volume n_h is the number of holes in unit volume μ_e is the mobility of electron and μ_h is the mobility of holes.

- **Conductivity of semi- conductor**

  $\sigma = e(n_e\mu_e + n_h\mu_h)$

- For semi – conductors and insulators, the number density of electrons at temperature T is

  n(T) = $n_0 e^{-E_g/K_B T}$ where E_g is the energy gap between valence band and conduction band and K_B is Boltzmann constant(1.38× 10^{-23} J/K).

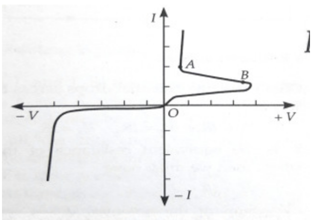
- **For an electrolyte**

  As the temperature increases the speed of ions increases and conductivity increases.

3.31 Ohmic and non-Ohmic conductors

The conductors which obey Ohm's law are called Ohmic conductors. The conductors which do not obey Ohm's law are called non – Ohmic conductors. Eg. Diode, thyristor etc.

Thyristor consists of four alternate layers of p and n type semi – conductors. The $I - V$ graph for a thyristor is shown in the next page.



3.32 Super conductivity

When a metal is cooled, at a perpendicular temperature called transition temperature or critical temperature it completely loses its resistivity. Now they are called super conductors and this phenomenon is called super conductivity.

Super conductivity was invented by Kammerlingh Onnes.

Applications

- Super conductors are used in the transmission of electric power without loss.
- They are applicable in super computer research.
- They are useful in the construction of very sensitive galvanometer.
- They are useful in the construction of electro magnets without any energy loss.

3.33 Joule's law of electrical heating

The heat developed in a current carrying conductor, $H = I^2Rt$ where I is the current, R is the resistance and t is the time for which the current is passed.

The heat developed in a current carrying conductor is directly proportional to the square of current, resistance of the conductor and time for which current is passed. This law is known as Joule's law of electrical heating.

3.34 Fuse wire

Fuse wire is made of an alloy of tin and lead. It has high resistivity and low melting point. When excess current comes in the circuit , more heat is developed in the wire. It melts (due to its low melting point) and breaks the circuit.

> ***Note***
>
> - Commercial or trade unit or BOT(Board of trade) unit of electrical energy is **kwh (**called as **unit).**
>
>   1 kwh = 3.6×10^6 J

3.35 Expression for electric power

$P = IV$

$P = I^2R$

$P = \frac{V^2}{R}$

3.36 Power consumed by parallel combination of bulbs

Consider three bulbs of powers P_1, P_2 and P_3 which work on the same voltage V.

Resistances are, $R_1 = \frac{V^2}{P_1}$ $R_2 = \frac{V^2}{P_2}$ $R_3 = \frac{V^2}{P_3}$

When they are in parallel $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Multiplying by V^2

$\therefore \frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3}$

i.e. P = $P_1 + P_2 + P_3$

Effective power is the sum of powers.

3.37 Efficiency of emf source

$\eta = \frac{Output\ power}{input\ power} = \frac{IV}{IE} = \frac{V}{E} = \frac{IR}{I(R+r)} = \frac{R}{R+r}$

> ***Points to remember***
>
> - Load in an electric circuit indicates the current drawn by the circuit from supply line. If the current in the circuit exceeds a safe value, we say that the circuit is over loaded.
> - When resistors are connected in parallel, the PD will be the same for all resistors.
>
>   P = $\frac{V^2}{R}$ or P $\propto \frac{1}{R}$ and $I = \frac{V}{R}$ $I \propto \frac{1}{R}$

- The temperature up to which the wire gets heated when current is passed is directly proportional to the square of current and inversely proportional to the cube of radius.

When resistors are connected in series, the current will be the same for all resistors.
∴ P = I^2R or P ∝ R V = $I\,R$ or V ∝ R

3.38 Electromotive force (emf)

Emf is the maximum P.D between the terminals of a cell when it is in open circuit. For a closed circuit, it is the work done by the source in moving a unit charge once in the circuit.

Its unit is volt.
Emf has non-electrostatic origin.

3.39 Some important terms

a)Internal resistance - It is the resistance offered by a cell when it is in closed circuit.

b)Lost volt - It is the voltage dropped across a cell due to its internal resistance (Ir).

c)Terminal potential difference - It is the potential difference between the terminals of a cell when it is in a closed circuit (IR).

Emf = Terminal PD + Lost volt OR $E = IR + Ir$

3.40 Difference between emf and P.D

emf	P.D
1. It is the work done by source in moving a unit charge once in the circuit	1. It is the work done in moving a unit charge from one point to another
2. Emf exists even if the circuit is not closed	2. P.D exists only when the circuit is closed
3. It has non electrostatic origin	3. It is due to the electrostatic field set up by the charges at the terminals of source
4. It is the cause of P.D	4. P.D is an effect due to emf
5. It is more than the P.D across any circuit element	5. It is always less than the emf

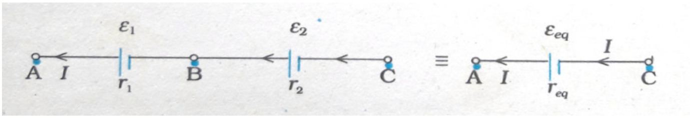
3.41 <u>Factors affecting internal resistance of a cell</u>

Temperature, nature of electrodes used in the cell, nature of electrolyte used in the cell, distance between the electrode plates, area of plates of electrode.

> ***Note***
> - When a cell of emf ε and internal resistance r is connected to a resistor R, the current in the circuit, $I = \frac{\varepsilon}{R+r}$

3.42 Series and parallel combination of cells

Series



Consider two cells of emf's ε_1 and ε_2 and internal resistances r_1 and r_2.

P.D between positive and negative terminals of first cell = $\varepsilon_1 - Ir_1$

P.D between positive and negative terminals of second cell = $\varepsilon_2 - Ir_2$

P.D between A and C $= \varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2$

$= \varepsilon_1 + \varepsilon_2 - I(r_1 + r_2)$

If the cells are replaced by a single cell of emf ε_{eq} and internal resistance r_{eq}.

P.D between A and C = $\varepsilon_{eq} - Ir_{eq}$

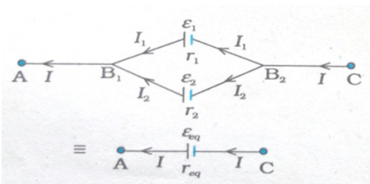
ie., $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ and $r_{eq} = r_1 + r_2$

Total emf is the sum of emf's. Total internal resistance is the sum of internal resistances.

If the negative terminals were connected together

$\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2$

Parallel



Here $I = I_1 + I_2$

P.D between P and Q due to first cell (V) = $\varepsilon_1 - I_1 r_1$ ----------------(1)

P.D between P and Q due to second cell (V) = $\varepsilon_2 - I_2 r_2$--------------(2)

From (1) $I_1 = \frac{\varepsilon_1 - V}{r_1}$ and from (2) $I_2 = \frac{\varepsilon_2 - V}{r_2}$ I_1

$I = I_1 + I_2 = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2} = \frac{\varepsilon_1}{r_1} - \frac{V}{r_1} + \frac{\varepsilon_2}{r_2} - \frac{V}{r_2}$

$$= \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - \frac{V}{r_2} - \frac{V}{r_1} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - \left[\frac{V}{r_2} + \frac{V}{r_1} \right]$$

$$I = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - V\left[\frac{1}{r_2} + \frac{1}{r_1} \right]$$

$$= \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - V\left[\frac{r_1 + r_2}{r_1 r_2} \right]$$

$$V\left[\frac{r_1 + r_2}{r_1 r_2} \right] = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - I$$

$$V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} \div \left[\frac{r_1 + r_2}{r_1 r_2} \right] - I \div \left[\frac{r_1 + r_2}{r_1 r_2} \right]$$

$$= \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} \times \left[\frac{r_1 r_2}{r_1 + r_2} \right] - I \times \left[\frac{r_1 r_2}{r_1 + r_2} \right]$$

$$V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I\left[\frac{r_1 r_2}{r_1 + r_2} \right]$$

$$V = \varepsilon_{eq} - I r_{eq}$$

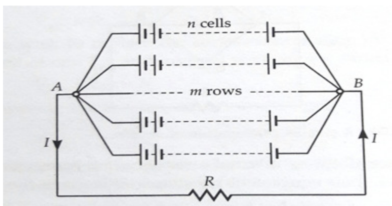
$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

If negative terminal of second is connected to positive of first, (1) and (2) are still valid. (But take $\varepsilon_2 = -\varepsilon_2$)

3.43 Mixed grouping of cells

Consider some identical cells of emf E and internal resistance r connected as in figure. Let there be n cells in each row. Let there be m rows.



Total number of cells = mn

Emf of one row = nE

Emf of m rows = nE

Internal resistance of one row = nr

Internal resistance of m rows r = $\left[\frac{1}{nr} + \frac{1}{nr} + \cdots m\ numbers\right]$

$$= \left(\frac{m}{nr}\right)^{-1}$$

$$= \frac{nr}{m}$$

Total resistance = $\frac{nr}{m} + R$

Current in a circuit = $\frac{total\ emf}{total\ resistance} = \frac{nE}{\frac{nr}{m}+R} = \frac{nE}{\frac{nr+mR}{m}}$

$$I = \frac{mnE}{nr+mR}$$

Note

Internal resistance of a cell is directly proportional to the concentration of electrolyte.
Internal resistance is directly proportional to the distance between electrode plates.
Internal resistance is inversely proportional to the plate area of electrodes.

Internal resistance of a fresh cell is low. But the internal resistance increases with use.

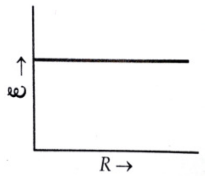
3.44 Primary cells and secondary cells

Primary cells are cells which work as a result of irreversible chemical reactions taking place inside them. They cannot be charged. eg: dry cell

Secondary cells are cells which work as a result of reversible chemical reactions taking place inside them. They can be charged. eg: lead acid cell.

Emf versus R for a cell

Emf is equal to terminal PD when no current is drawn from cell. Hence emf is independent of R.

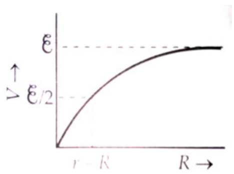


Terminal PD versus R for a cell

$V = IR$

V= $\frac{\varepsilon R}{R+r} = \frac{\varepsilon}{1+\frac{r}{R}}$

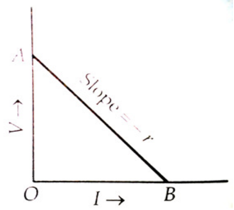
As R increases, V also increases.



Terminal PD versus Current for a cell

$V = \varepsilon - IR$

$V = -IR + \varepsilon$ which is of the form $y = ax + b$



3.45 Seebeck Effet

When two metallic strips, made of different metals are joined at the ends to form a loop and if the junctions are kept at different temperatures, a current is developed in the loop. This effect is called the Seebeck effect and the emf developed is called the Seebeck emf or thermo-emf.

Thermo emf depends on the metals and the temperatures of the hot and cold junctions. Such a combination of two metals is called a thermocouple.

3.46 Thermo electric series

For given temperatures of hot and cold junctions,the direction of the current in a thermocouple depends on the metals forming the thermo couple. Thermo electric series is given below.

Antimony,nichrome,iron,zinc,copper,gold,silver,lead,aluminium,mercury,platinum-rhodium,platinum,nickel,costantan,bismuth.

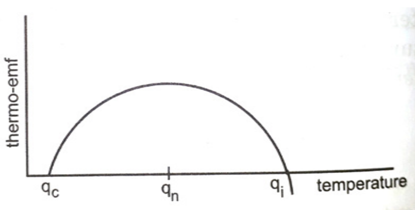
At the cold junction, the direction of current is from the metal coming earlier in the series to the metal coming latter in the series.

3.47 Neutral temperature and Temperature of inversion

The temperature of the hot junction at which the thermo-emf is maximum is called the netural temperature and the temperature at which the thermo-emf changes its sign(current reverses) is called the inversion temperature of inversion.

If θ_c, θ_n and θ_i denote the temperature of the cold junction, the neutral temperature and the inversion temperature respectively, then $\theta_n - \theta_c = \theta_i - \theta_n$

The variation in thermo-emf with the temperature of the hot junction is shown in figure.



If the cold junction is at $0°C$ and the hot junction at θ (in Celsius) , the thermo-emf depends on the temperature as

$$\in = \propto \theta + \frac{1}{2}\beta\theta^2$$

where $\propto$ and β are constants for the pair of metals taken .

$$\frac{d\in}{d\theta} = \propto + \beta\theta$$

The quantity $\frac{d\in}{d\theta}$ is called thermoelectric power at temperature θ.

The emf is maximum when $\frac{d\in}{d\theta} = 0$ or $\theta = \frac{\propto}{\beta}$

This is the neutral temperature .

The emf becomes zero at $\theta = -2 \propto/\beta$.This is the inversion temperature.

3.48 Peltier Effect

When the two junctions of a thermocouple are kept at the same temperature and an electric current is passed through the circuit by using an external battery, then one junction becomes heated and the other gets cooled. This effect and is called Peltier effect.

3.49 Thomson Effect

The liberation or absorption of heat energy at different sections of a wire having non uniform temperature when an electric current is passed through it is known as Thomson effect. This heat is over and above the Joule heat I^2Rt and is called Thomson heat .

If a charge ΔQ is passed through a small section of the wire having a temperature difference ΔT between the ends , the Thomson heat is

$$\Delta H = \sigma(\Delta Q)(\Delta T)$$

where σ is a constant for a given metal at a given temperature.

The quantity $\sigma\Delta T = \frac{\Delta H}{\Delta Q} = \frac{Thomson\ heat}{charge\ transferred}$ is called the Thomson emf.

3.50 Faraday's Laws of Electrolysis

First law

The mass of a substance liberated at an electrode is proportional to the charge passing through the electrolyte.

Second law

The mass of a substance liberated at an electrode by a given amount of charge is proportional to the chemical equivalent of the substance.

If an electric current I is passed through an electrolyte for a time t, the amount of charge passed is $Q = It$.

According to the first law , $m \propto Q$ ------------(1)

or $m \propto It$

or $m = Zit$

where Z is a constant called the electrochemical equivalent(ECE) of the substance liberated. The SI unit of ECE is kg C^{-1}.

The chemical equivalent of a substance is the ratio of relative atomic mass to valency.

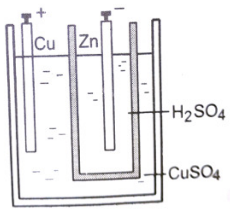
Relative atomic mass of a substance is the ratio of the mass of its atom to $1/_{12}$ of the mass of a C^{12} atom.

eg: The relative atomic mass and valency of oxygen are 16 and 2 respectively. The chemical equivalent is 16/2=8.

If E denotes the chemical equivalent of a substance being liberated at an electrode,
$m \propto E$------------(2)
From (1) and (2)
$m \propto EQ$

Or $m = \frac{1}{K}EQ$
where K is a constant. $K = 9.6485\times 10^{7} C\ kg^{-1}$.

3.51 Daniell Cell

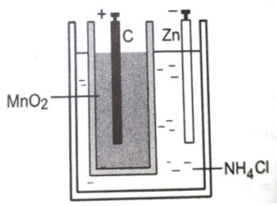


A Daniell cell consists of a zinc electrode in a dilute H_2SO_4 solution and a copper electrode in $CUSO_4$ solution. The two solutions are separated by a porous cup which allows any gas to pass through, but prevents the liquids to mix.

The zinc electrode works as the negative electrode and the copper electrode works as the positive electrode.
Daniel cell is used when a continuous current is required. The emf of a Daniell cell is nearly 1.09V and its internal resistance is about 1Ω.

3.52 Leclanche Cell

A Leclanche cell has a carbon and a zinc electrode in a solution of NH_4CI. The carbon electrode is kept in a porous cup containing MnO_2. The zinc electrode is the negative terminal and carbon is the positive terminal.
NH_4CI is the electrolyte. The cell is used when intermittent currents are needed. Its emf is about 1.4V. Its internal resistance is high.



3.53 Dry Cell

Dry cell consists of a zinc container filled with a paste of NH_4CI and MnO_2 . The zinc container acts as the negative electrode. Carbon rod acts positive electrode. NH_4CI is the electrolyte . The internal resistance of a dry cell is very small, about 0.1Ω.

3.54 Lead Accumulator

A lead accumulator consists of electrodes made of PbO_2 and of Pb immersed in dilute sulphuric acid (H_2SO_4). The specific gravity of the solution should be between 1.20 and 1.28. PbO_2 acts as the positive electrode and Pb as the negative electrode. Its emf is about 2.05V.

3.55 Kirchhoff's rules

1. Junction Rule/Point rule: The algebraic sum of currents meeting any junction of an electric circuit is zero.

OR

The sum of currents entering any junction of an electric circuit is equal to the sum of currents leaving the junction.

2. Loop Rule/Mesh rule:

The algebraic sum of changes in potential around any closed loop consisting of resistors and cells is zero.

OR

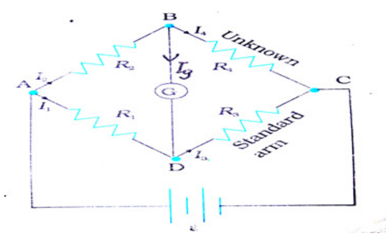
The algebraic sum of product of current and resistance around any closed loop consisting of resistors and cells is equal to the algebraic sum of emf's.

Points to remember

- Kirchhoff's current law(first law) is actually law of conservation of charge. Kirchhoff's second law is actually law of conservation of energy.

3.56 Principle of a Wheatstone's bridge

The bridge consists of four resistors R_1, R_2, R_3 and R_4. The values of these are suitably adjusted such that galvanometer current is zero. The bridge is said to be balanced. A Wheatstone's bridge is shown below.



Applying Kirchhoff's loop rule to the mesh ABDA

$I_2R_2 + I_gG - I_1R_1 = 0$ ----------------(1) where G is the resistance of galvano meter.

Applying Kirchhoff's loop rule to the mesh BCDB

$I_4R_4 - I_3R_3 + I_gG = 0$ ---------------------(2)

In the balanced condition, $I_g = 0$

Equations (1) and (2) become

$I_1R_1 = I_2R_2$ -------------------(3)

$I_3R_3 = I_4R_4$ -------------------(4)

(3) ÷ (4) gives $\frac{I_1R_1}{I_3R_3} = \frac{I_2R_2}{I_4R_4}$

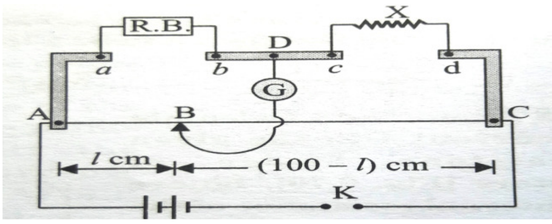
For a balanced bridge, $I_1 = I_3$ and $I_2 = I_4$

Therefore (3) can be written as

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \quad \text{OR} \quad \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

This is the condition for a balanced Wheatstone's bridge.

3.57 The method of calculating unknown resistance using a metre bridge



Metre bridge (also known as Slide Wire Bridge) consists of a uniform resistance wire of 1m long. It is kept stretched between two points A and C. There is an 'L' shaped metal strip at end A and a similar one is there at C. There is a third rectangular metal strip in between them. There are two gaps between the strips. The unknown resistance is connected in the right gap and resistance box is connected in the left gap. The middle strip is connected to a jockey through a galvanometer. Jockey can be slided along the wire.

The positive of accumulator is connected to the end A and negative is connected to end C through a key.

The key is closed. A resistance (say 1 ohm) is taken from resistance box. The jockey is pressed at end A and C. If galvanometer deflections are opposite, the connections are correct.

Metre bridge works on the basis of Wheatstone's principle.

With a suitable resistance in resistance box, the jockey is moved along the wire from end A. At a particular point, galvanometer shows zero. The bridge is said to be balanced. The length is taken as l. The length from the end C is $(100 - l)$.If r is resistance of 1cm wire of metre bridge, the resistance of l cm is $l\,r$ and $(100 - l)$cm is $(100- l)$r.

According to Wheatstone's principle,

$$\frac{lr}{(100-l)r} = \frac{R}{X} \quad \text{OR} \quad \frac{l}{(100-l)} = \frac{R}{X}$$

$$X = \frac{(100-l)R}{l}$$

The experiment is repeated for different values of R. The experiment is also repeated by interchanging X and R in gaps. When resistance box is in left gap, the length is to be taken from the end A. When resistance box is in right gap, the length is taken from the end C.

3.58 Principle of potentiometer

Consider a wire of L connected to a battery of emf E.

The voltage E is dropped in the length L. If I is the current through wire then the potential drop in 1 cm is Ir where r is the resistance of 1 cm wire.

Consider a secondary circuit consisting of a cell of emf E_1, a jockey and galvanometer. When the jockey is moved along the wire , at a point J, galvanometer shows zero. Let the length AJ be l.

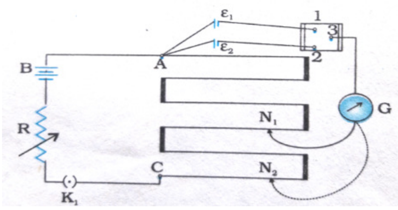
Potential drop in l cm = $I\ r\ l$

Here $E_1 = I\ r\ l$ OR $E_1 \propto l$

ie. Emf cell in secondary is proportional to balancing length. This is the principle of a potentiometer.

3.59 Use of a potentiometer to compare the emf of two primary cells

Potentiometer consists of a uniform resistance wire (normally 10m long) arranged in a zig- zag way on a wooden board. There are the two ends A and C. Potentiometer can be used to find the emf of a cell. It can also be used to find the internal resistance of a cell. The positive of an accumulator is connected to end A. The negative is connected to end C through a rheostat and key. This is primary circuit.



The positive of the two given cells are connected to end A. The negatives are connected to the two ends of a two way key. The middle terminal is connected to a jockey through a galvanometer. This is secondary circuit.

The primary is closed. The first cell of emf E_1 is included in the circuit and balancing length l_1 is found.

By the principle of potentiometer, emf of cell in secondary ∝ balancing length.

$E_1 \propto l_1$

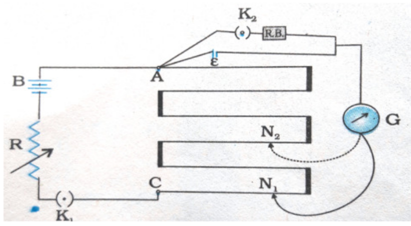
Now the second cell of emf E_2 is included and balancing length l_2 is found.

$E_2 \propto l_2$

OR $\frac{E_1}{E_2} = \frac{l_1}{l_2}$ Thus emf's are compared.

3.60 Use of a potentiometer to find the internal resistance of a cell

The positive of accumulator is connected to end A. Negative is connected to end C through a rheostat and key. This is primary.



The positive of the cell whose internal resistance is to be found is connected to end A. Negative is connected to a jockey through a galvanometer. A resistance box and key are connected across the cell. With the key open, balancing length is found for a resistance from resistance box. If emf of cell in secondary is E and balancing length is l_1

$E \propto l_1$ --------------------(1)

Now with the key in secondary closed, balancing length is found.

Here $\frac{ER}{R+r} \propto l_2$ ------------------ (2)

where l_2 is the new balancing length.

(1) ÷ (2) $E \div \frac{ER}{R+r} = \frac{l_1}{l_2}$

OR $E \times \frac{R+r}{ER} = \frac{l_1}{l_2}$

$\frac{R+r}{R} = \frac{l_1}{l_2}$ OR $l_1R = l_2R + l_2r$

$(l_1 - l_2)R = l_2r$

OR $r = \frac{(l_1 - l_2)R}{l_2}$

Thus internal resistance is calculated.

3.61 Reason why potentiometer is preferred over voltmeter

Voltmeter draws some current for its working. Voltmeter measures the terminal P.D. In the balanced condition, potentiometer does not draw any current for its working. So emf obtained will be accurate.

3.62 Methods of increasing the sensitivity of potentiometer

1. By increasing the length of wire .

2. By decreasing the current through the wire. When the current is reduced the potential gradient decreases and sensitivity increases.

3.63 Reason for using rheostat in the primary of potentiometer

By adjusting the rheostat, resistance of the circuit can be increased. Current through potentiometer wire decreases and potential gradient decreases. Thus sensitivity increases.

3.64 Sometimes balancing point may not be obtained in the potentiometer wire. Give reason.

Emf of cell in primary will be less than the emf of cell in secondary.

3.65 Sometimes galvanometer shows deflection to the same side when the jockey is pressed at A and C. What may be the reason?

1.Emf of cell in primary may be less than emf of cell in secondary.
2. Negative terminal of cell in secondary may be connected to end A.

If deflection at C is more :

In this case negative terminal of cell in secondary may be connected to end A.

If the deflection at A is more :

In this case the emf of cell in primary may be less than emf of cell in secondary.

3.66 Potential gradient

The potential drop per unit length of potentiometer wire is called potential gradient. Its unit is volt/m.

Note

Null method is used in potentiometer. At null point, it does not draw any current from cell. Thus there is no potential drop due to the internal resistance of the cell. Thus the measured P.D in the circuit will be equal to the actual emf of the cell.

3.67 Sensitivity of potentiometer and methods to increase sensitivity

Potentiometer is sensitive if it can measure very small P .D and it shows significant change in balancing length for small change of PD being measured. Smaller the potential gradient, greater will be the sensitivity of the potentiometer.

Methods of increasing sensitivity

- By increasing the length of potentiometer wire.
- By decreasing the current through potentiometer wire.

Note

Potentiometer can be considered as an ideal voltmeter with infinite resistance.
Any physical quantity that can produce a P.D can be measured using potentiometer. Hence it can measure stress, temperature, radiation, p^H etc.

3.68 Advantages of measuring resistance by Wheatstone's method

- It uses null method. Hence internal resistance of cell or resistance of galvanometer do not affect null point.
- As the method does not involve the measurement of current or P.D, the resistance of ammeters and voltmeters do not affect measurements.

Note

- Wheatstone's bridge is **most sensitive** when the **resistance in the four arms are of the same order**.
- Meter Bridge, Carry – Foster's bridge (CF Bridge) and post office box are the applications of wheat stone's bridge.
- For a balanced Wheat stone's bridge, the inter change in the position of galvanometer and battery has no effect in the balancing conditions.
- Wheatstone's bridge method is not suitable for measuring very low and very high resistances.

3.69 If a wire is stretched to double its original length without any loss of mass, how will resistivity be affected?

The resistivity depends on nature of material. Hence increase of length will not change resistivity.

3.70 Two wires of equal length, one of copper & the other of manganin have same resistance. Which is thicker?

R = $\frac{\rho l}{A}$ OR A = $\frac{\rho l}{R}$ Here R and l are the same

$\rho_{copper} < \rho_{manganin}$ $\therefore$ $A_{copper} < A_{manganin}$

3.71 Two wires of equal area of cross – section one of copper and the other of manganin have the same resistance. Which is longer?

R = $\frac{\rho l}{A}$ OR $l = \frac{RA}{\rho}$ Here R and A are the same $\therefore l \propto \frac{1}{\rho}$

$\rho_{copper} < \rho_{manganin}$ $\therefore$ $l_{copper} > l_{manganin}$

3.72 Two wires A and B of the same material are given. Their lengths are l and 2 l and radii are r and r/2 respectively. What is the ratio of their resistances?

$R_1 = \frac{\rho l_1}{A_1} = \frac{\rho l_1}{\pi {r_1}^2} = \frac{\rho l}{\pi r^2}$ $R_2 = \frac{\rho l_2}{\pi {r_2}^2} = \frac{\rho \times 2l}{\pi (\frac{r}{2})^2} = \frac{8\,\rho l}{\pi r^2}$ = 8 R_1 $R_1 : R_2$ = 1:8

3.73 The current through a conductor is 2mA at 50 V and 3 mA at 60 V. Is it Ohmic or non – Ohmic?

Resistance in the first case $R_1 = \frac{V_1}{I_1} = \frac{50}{2\times10^{-3}}$ = 25000 Ω

Resistance in the second case $R_2 = \frac{V_2}{I_2} = \frac{60}{3\times10^{-3}}$ = 20000 Ω

As the resistance changes with current it is non – Ohmic.

3.74 Two wires A and B of the same metal have the same area of cross – section and lengths in the ratio 2:1. Find the ratio of current through them when the same P.D is applied across the length of each?

$I = \frac{V}{R}$ R = $\frac{\rho l}{A}$ ∴ $I = \frac{V}{\frac{\rho l}{A}} = \frac{VA}{\rho l}$ ρ, A and V are the same

∴ $I \propto \frac{1}{l}$ OR $I_A \propto \frac{1}{2l}$ and $I_B \propto \frac{1}{l}$ $\frac{I_A}{I_B} = \frac{1}{2l} \div \frac{1}{l}$ = 1:2

3.75 Two wires A and B of the same metal and of the same length have their area of cross – sections in the ratio 2:1. If the same P.D is applied across each in turn what will be the ratio of currents in them?

$I = \frac{V}{R}$ R = $\frac{\rho l}{A}$ ∴ $I = \frac{VA}{\rho l}$ ρ, l and V are the same ∴ $I \propto$ A

$I_A \propto$ 2A and $I_B \propto$ A $\frac{I_A}{I_B} = \frac{2A}{A}$ = 2:1

Note:
The drift velocity of electrons in a metallic conductor decreases with increase in temperature. With the increase in temperature frequency of collisions of electron increases.

3.76 A P.D of V is applied across a conductor of length *l*. How is the drift velocity affected when V is doubled and length is halved?

Drift velocity $v_d = \frac{E_e\tau}{m}$ But E = $\frac{V}{l}$ ∴ $v_d = \frac{V_e\tau}{lm}$

Here V → 2V $l \to \frac{l}{2}$

∴ $v_d = \frac{2V_e\tau}{\frac{l}{2}m} = \frac{4V_e\tau}{lm} = 4v_d$ i.e. Drift velocity increases 4 times.

3.77 A uniform resistance wire of 20 Ω is cut in two equal parts. These are now connected in parallel. What is the new resistance?

Resistance of each part = 10 Ω New resistance = $\frac{10\times10}{10+10}$ = 5 Ω

Note

- **Potentiometer wire** is made of **nichrome or manganin.**
- Current should not be passed through potentiometer wire for a long time. If passed, the wire gets heated and the resistance changes. Potential drop per unit length also changes.
- Wheatstone's bridge is called so because this method was first suggested by CF Wheatstone.
- Wheatstone's bridge is sensitive if it produces more deflection in galvanometer for a small change of resistance in resistance arm.

3.78 Why should we get the null point in the middle of the meter bridge wire?

The meter bridge is most sensitive when the four resistances in it are equal. This is possible only if the balance point is near the middle of the wire.

3.79 Three resistors 2 Ω , 3 Ω and 4 Ω are connected to the same battery in turn, in which case the power dissipated will be maximum?

P = $\frac{V^2}{R}$ Here V is the same. ∴ P ∝ $\frac{1}{R}$ ∴ Power dissipation is maximum for 2 Ω.

3.80 Two wires A and B of same material having the same length having their area of cross – section in the ratio 1:4. Find ratio of heat produced if the same voltage is applied across each?

$R_A = \frac{\rho l}{A}$ $R_B = \frac{\rho l'}{A'}$ $R_A = \frac{\rho l}{A}$ $R_B = \frac{\rho l}{4\,A}$ $= \frac{1}{4} R_A$ $\frac{R_A}{R_B} = \frac{1}{4}$

H = I^2Rt H = Pt H = $\frac{V^2}{R}$ t

$\frac{H_A}{H_B} = \frac{\frac{V^2}{R_A}t}{\frac{V^2}{R_B}t} = \frac{R_B}{R_A}$ = 1:4

3.81 A toaster produces more heat than bulb when connected parallel to 220 V mains. Which has more resistance?

H = $\frac{V^2}{R}t$ H ∝ $\frac{1}{R}$ ∴ Bulb has high resistance.

3.82 There bulbs 40 W, 60 W and 100 W are connected to 220 V mains. Which will glow brightly when connected in series?

In series, the same current passes through all. Resistance is more for 40 W bulb(as R ∝ $\frac{1}{P}$). So more heat is developed in it and it glows brightly.

3.83 Explain why an electric bulb dims when an electric heater is switched on?

Heater with more power has small resistance. It draws more current. Hence the bulb glows dim. During working it gets heated and the resistance increases and it takes small current. Current through the bulb increases and its dimness decreases.

3.84 Two 120 V bulbs, one of 25 W and other of 200 W were connected in series across 240 V line. One burnt out instantaneously. Which one was bunt and why?

R = $\frac{V^2}{P}$ ∴ 25 W bulb has high resistance.

In series, the same current passes through both. ∴ More heat is developed in 25 W bulb and it burns out instantaneously.