# ***Electrostatic Potential and Capacitance***

*Examples from NCERT Text Book*

1. *(a) Calculate the potential at a point P due to a charge of $4 \times 10^{-7} C$ located 9 cm away.*

*(b) Hence obtain the done in bringing a charge of $2\times 10^{-9} C$ from infinity to the point P.*

*Does the answer depend on the path along which the charge is brought?*

(a) V = $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

$= 9 \times 10^9 \times \frac{4\times 10^{-7}}{0.09}$

$= 4 \times 10^4$V

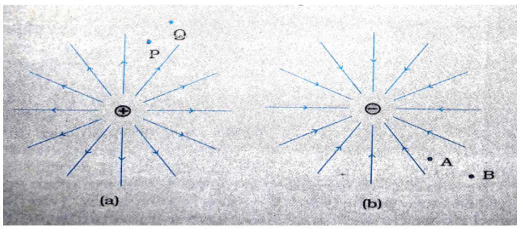
(b) W = qV

$= 2\times 10^{-9} \times 4 \times 10^4$

$= 8 \times 10^{-5} J$

No , work done is independent of path as electrostatic field is conservative.

**2.** *(a) and (b) show the field lines of positive and negative point charge respectively.*




*(a) Give the signs of the potential difference $V_P - V_Q$ and $V_B - V_A$.*

*(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P ; A and B*

*(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.*

*(d) Give the sign of the work done by the external agency in moving a small negative charge from B to A.*

*(d) Does the kinetic energy of small negative charge increase or decrease in going from B to A?*

(a)As V ∝ $\frac{1}{r}$ , $V_P > V_Q$.

Thus $(V_P - V_Q)$ is positive.

$V_B$ is less negative than $V_A$ .

Thus $V_B > V_A$ is positive.

(b) Potential energy,U= $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

Potential energy is negative for a (+)ve and (-)ve

When (-)ve charge is kept at Q or P, PE will be (-)ve.

$U_Q > U_P$

$U_Q - U_P$ is (+)ve.

When (-)ve charge is kept at A or B, PE will be (+)ve.

$U_A > U_B$

$U_A - U_B$ is (+)ve.

(c) $V_P > V_Q$.

In moving a small positive charge from Q to P , work is done against repulsion. Hence the work done by an external agency will be positive.

Work done by an electric field will be negative.

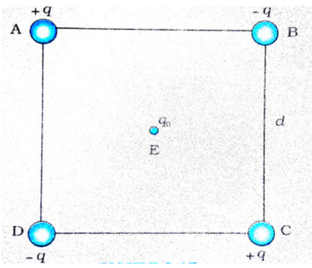
(d) In moving a small negative charge from B to A , work is done against repulsion. Hence the work done by an external agency will be positive.

(e) Velocity and hence the kinetic energy decreases in going from B to A due to force of repulsion on the negative charge.

3. *Four charges are arranged at the corners of a square ABCD of side d, as shown in figure*

*(a) Find the work required to put together this arrangement.*

*(b) A charge $q_0$ is brought to the centre E of the square, the four charges being held fixed at its corners .How much extra work is needed to do this?*




(a) Work needed to bring charge +q to A when other charges are absent is zero.

Work needed to bring –q to B when +q at A

= charge × (potential at B due to charge +q at A)

= -q × $\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{d}\right)$

= $\frac{-q^2}{4\pi\varepsilon_0 d}$

Work needed to bring charge +q to C when +q is at A and–q is at B= (charge at C) ×(potential at C due to charges at A and B)

= +q$\left(\frac{1}{4\pi\varepsilon_0}\frac{+q}{d\sqrt{2}} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{d}\right)$

= $\frac{-q^2}{4\pi\varepsilon_0 d}\left(1 - \frac{1}{\sqrt{2}}\right)$

Work needed to bring –q to D when +q at A,-q at B , and +q at C= (charge at D) × (potential at D due to charges at A, B and C )

= -q $\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{d} + \frac{1}{4\pi\varepsilon_0}\frac{+q}{d\sqrt{2}} + \frac{1}{4\pi\varepsilon_0}\frac{q}{d}\right)$

= $\frac{-q^2}{4\pi\varepsilon_0 d}\left(2 - \frac{1}{\sqrt{2}}\right)$

The total work required

= $\frac{-q^2}{4\pi\varepsilon_0 d}\left\{(0) + (1) + \left(1 - \frac{1}{\sqrt{2}}\right) + \left(2 - \frac{1}{\sqrt{2}}\right)\right\}$

= $\frac{-q^2}{4\pi\varepsilon_0 d}\left(4 - \sqrt{2}\right)$

(b) The extra work necessary to bring a charge $q_0$ to the point $E$ when the four charges are at A,B,C and D = $q_0$ ×(potential at E due to the charges at A,B,C and D )

The electrostatic potential at E =0

∴ Work required =0

4. *(a) Determine the electrostatic potential energy of a system consisting of two charges -2μC and 7 μC (and with no external field ) placed at (-9 cm , 0 ,0 ) and (9 cm, 0,0) respectively.*

*(b) How much work is required to separate the two charges infinitely away from each other?*
*(c) Suppose that the same system of charges is now placed in a external electric field E = A $(1/r^2)$; A = 9 × $10^5 C\,m^{-2}$ what would the electrostatic energy of the configuration be?*

(a) U = $\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

$= 9 \times 10^9 \times \frac{7 \times 10^{-6} \times (-2) \times 10^{-6}}{0.18}$

$= -0.7$ J

(b) W = $U_2 - U_1$

= 0− U

= 0 −(−0.7)

= 0.7 J

(c) The net electrostatic energy

$= q_1 V_1 + q_2 V_2 + \frac{q_1 q_2}{4\pi\varepsilon_0 r}$

= A $\frac{7 \times 10^{-6}}{0.09}$ + A $\frac{(-2) \times 10^{-6}}{0.09} - 0.7$

= 70−20−0.7

= 49.3 J

**5**. *A molecule of a substance has a permanent electric dipole moment of magnitude $10^{-29}$ Cm. A mole of this substance is polarized (at low temperature) by applying a strong electrostatic field of magnitude $10^6 V\,m^{-1}$. The direction of field is suddenly changed by an angle of $60^o$. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarization of the sample.*

Dipole moment of one molecule = $10^{-29}$ Cm

Total dipole moment of all the molecules in one mole , P = $6 \times 10^{23} \times 10^{-29}$ Cm

= $6 \times 10^{-6}$ Cm

Initial potential energy , $U_i = -PE\cos\theta$

$= -6 \times 10^{-6} \times 10^6 \cos 0^0$

$= -6$J

Final potential energy (when $\theta = 60^o$ ),

$U_f$ = - $6 \times 10^{-6} \times 10^6 \cos 60^0$

= -3J

Change in potential energy = $-3 - (-6) = 3$J

There is loss in potential energy .

This energy is released in the form of heat in aligning its dipoles.

6. *A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but a thickness (3/4)d , where d is the separation of the plates . How is the capacitance changed when the slab is inserted between the plates?*

$E_0 = V_0/d$

If the dielectric is now inserted, the electric field in the dielectric will be E = $E_0$/K

The potential difference , V = $E_0(\frac{1}{4}d) + \frac{E_0}{K}(\frac{3}{4}d)$

$= E_0 d\,(\frac{1}{4} + \frac{3}{4K})$

$= V_0 \frac{K+3}{4K}$

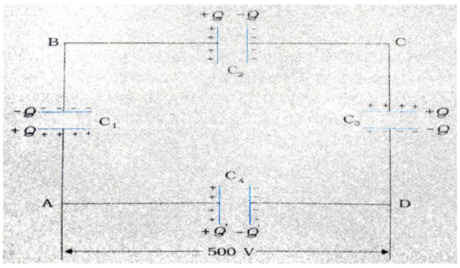
The potential difference decreases by the factor (K +3)/K.

The free charge $Q_0$ on the plate remains unchanged.

$\therefore$ C = $\frac{Q_0}{V} = \frac{4K}{K+3}\frac{Q_0}{V_0} = \frac{4K}{K+3} C_0$

The capacitance thus increases.

7. *A network of four 10 μF capacitors is connected to a 500V supply, as shown in figure. Determine (a) the equivalent capacitance of the network and (b) the charge on each capacitor. (Note: The charge on a capacitor is the charge on the plate with higher potential, equal and opposite to the charge on the plate with lower potential.)*




a) $C_1, C_2$ and $C_3$ are in series .

$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$\therefore C_1 = C_2 = C_3 = 10\mu$F

The effective capacitance C'=(10/3)$\mu$F

C'and$C_4$ connected in parallel.

The equivalent capacitance C =C'+$C_4$

= $(\frac{10}{3})$ + 10 = 13.3$\mu F$

(b)The charge on each of the capacitors,$C_1$,$C_2$ and $C_3$ is the same( say Q).

Let the charge on $C_4$ be Q'.

Potential difference across AB is Q/$C_1$,across BC is Q/$C_2$,across CD is Q/$C_3$,

$\frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = 500$V

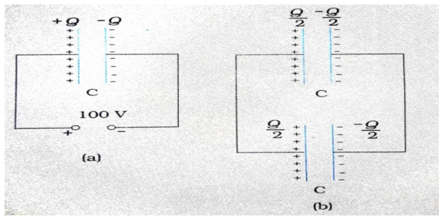
Q'/$C_4$ = 500V

Q = 500$\times \frac{10}{3} \times 10^{-6}$ =1.7$\times 10^{-3}$C

Q'= 500 $\times$10$\times 10^{-6}$ = 5.0$\times 10^{-3}$C

8. *(a) A 900 pF capacitor is charged by 100V battery . How much electrostatic energy is stored by the capacitor?*

*(b) The capacitor is disconnected from the battery and connected to another 900 pF capacitor. What is the electrostatic energy stored by the system?*




(a) The charge on the capacitor ,

Q = CV

= 900 $\times 10^{-12} \times 100$

= 9 $\times 10^{-8}$C

The energy stored = (1/2)$CV^2$

= (1/2) QV

=(1/2)$\times$9$\times 10^{-8}$ $\times$100

= 4.5 $\times 10^{-6}$J

(b)The common potential difference be V'.

The charge on each capacitor is then Q' = CV'

By charge conservation, Q' = Q/2

or V' = V/2

The total energy of the system is

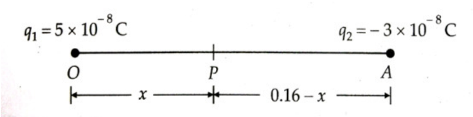
= 2 $\times \frac{1}{2}$Q' V'

= $\frac{1}{4}$ QV = 2.25 $\times 10^{-6}$J

Energy is lost in the form of heat and electromagnetic radiation.

# ***Numerical Problems***

1. *Two charges 5× $10^{-8}$C and -3× $10^{-8}$ C are located 16 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.*




Let the potential be zero at the point P.

Taking OP = $x$

$V_1 + V_2 = 0$

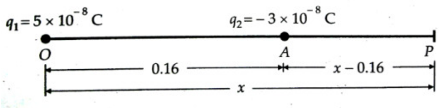
$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{x} + \frac{q_2}{0.16-x}\right] = 0$

$9 \times 10^9 \left[\frac{5 \times 10^{-8}}{x} - \frac{3 \times 10^{-8}}{0.16-x}\right] = 0$

$\frac{5}{x} - \frac{3}{0.16-x} = 0$

or $x$ = 0.10 m = 10cm

If P is on the extended line OA,




$V_1 + V_2 = 0$

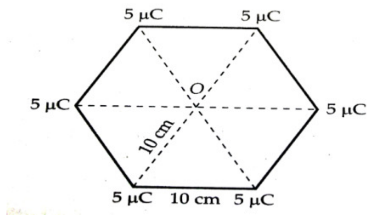
$\therefore \frac{1}{4\pi\varepsilon_0}\left[\frac{5 \times 10^{-8}}{x} - \frac{3 \times 10^{-8}}{x-0.16}\right] = 0$

$\frac{5}{x} - \frac{3}{x-0.16} = 0$

or $x$ = 0.40 m = 40cm

Electric potential is zero at 10 cm from $5 \times 10^{-8}$ C and 40 cm away from $5 \times 10^{-8}$ C **on the side of the negative charge** .

2. *A regular hexagon of side 10 cm has a charge 5μC at each of its vertices. Calculate the potential at the centre of the hexagon.*




Distance of each charge from the centre is

r = 10cm = 0.10m

q = 5 $\mu C$= 5 × $10^{-6} C$

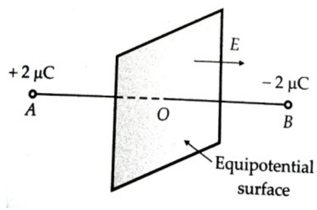
V = 6× $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

= $\frac{6 \times 9 \times 10^9 \times 5 \times 10^{-6}}{0.10}$

= 2.7 × $10^6$ V

3. *Two charges 2μC and -2μC are placed at points A and B 6 cm apart.*
*a) Identify an equi potential surface of the system*
*b) What is the direction of the electric field at every point on this surface?*

(i)The equi-potential surface will be a plane normal to AB and passing through its midpoint O, as shown in figure . The potential at any point on it is zero.




(ii) The direction of electric field is normal to the plane in the direction from positive to negative charge.

4. *A parallel plate capacitor with air between the plate has a capacitance of 8 pF (1pF = $10^{-12}$ F). What will be the capacitance if the distance between the plates is reduced by half, and the space*

*between them is filled with a substance of dielectric constant 6?*

Capacitance of the capacitor with air between its plates, C= $\frac{\varepsilon_0 A}{d}$ = 8pF

When there is dielectric, C' = $\frac{K\varepsilon_0 A}{d}$ = $\frac{6\times\varepsilon_0 A}{d/2}$

= 12 $\frac{\varepsilon_0 A}{d}$

= 12 × 8 = 96 pF

5. *Three capacitors each of capacitance 9 pF are connected in series.*

*a) What is the total capacitance of the combination?*

*b) What is the potential difference across each capacitor if the combination is connected to 120 V supply?*

*a)* $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$= \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$

C = 3 pF

PD across each capacitor is same as capacitance are equal. Let the PD be V'.

*b)* V = $V_1 + V_2 + V_3$

$= V' + V' + V'$

$= 3V'$

or $V' = \frac{V}{3} = \frac{120}{3} = 40$ V

6. *Three capacitors each of capacitance 2 pF ,3 pF and 4pF are connected in parallel*

*a) What is the total capacitance of the combination?*

*b) Determine the charge on each capacitor if the combination is connected to 100 V supply?*

*a)* C = $C_1 + C_2 + C_2$

= 2 +3+4 = 9 pF

*b)* $q_1 = C_1V = 2 \times 10^{-12} \times 100 = 2 \times 10^{-10} C$

$q_2 = C_2V = 3 \times 10^{-12} \times 100 = 3 \times 10^{-10} C$

$q_2 = C_3V = 4 \times 10^{-12} \times 100 = 4 \times 10^{-10} C$

7. *In a parallel plate capacitor with air between the plates, each plate has an area of 6× 10-3 m2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?*

C = $\frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$ = 18pF

V= 100V

q= CV = 18× $10^{-12}$ × 100 = 1.8 × $10^{-9}$C

8. *Explain what would happen if in the capacitor a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates in the above question,*

*a) while the voltage supply remained connected.*

*b) after the supply has disconnected?*

C = 18× $10^{-12}$ F = 18 pF

q = 1.8 × $10^{-9} C$

K = 6

*a)* When the voltage supply remains connected, the potential difference between capacitor plates remains same i.e., 100 V

∴ New capacitance, C' = K C = 6 × 18 = 108 pF

New charge, q' = CV = 108 × $10^{-12}$ × 100

= 1.08 × $10^{-8}$C

*b)* After the supply is disconnected, the charge on the capacitor plates remains same.

i.e, q = 1.8 × $10^{-9}$C

New capacitance, C' = K C = 108 pF

New PD, V' = $\frac{V}{K}$ = $\frac{100}{6}$ = 16.6 V

9. *A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?*

C = 12 pF = $12 \times 10^{-12}$ F

V = 50 V

U = $\frac{1}{2}\ CV^2$

= $\frac{1}{2} \times 12 \times 10^{-12}\ \times (50)^2$

= $1.5 \times 10^{-8} J$

10. *A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?*

$C_1$ = 600 pF, $V_1$ = 200 V, $C_2$ = 600 pF, $V_2$ = 0

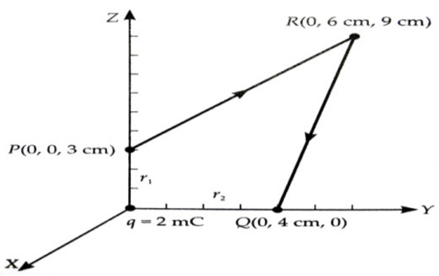
Common potential,

V = $\frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ = $\frac{600 \times 10^{-12} \times 200 + 600 \times 10^{-12} \times 0}{(600+600) \times 10^{-12}}$

= 100V

Loss of energy = $\frac{C_1C_2\,(V_1 - V_2)^2}{2(C_1 + C_2)}$ = $6 \times 10^{-6} J$

11. *A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge -2$\times\ 10^{-9}C$ from a point P(0,0,3cm) to point Q ( 0,4cm,0), via a point R (0,6cm,9cm).*




The work done in taking a charge from one point to another is independent of the path followed .

W = $q_0\ \left[V_Q - V_P\right]$ = $q_0\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{r_2}\ - \frac{1}{4\pi\varepsilon_0}\ \frac{q}{r_1}\right]$

= $\frac{q_0 q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

q = 8mC = $8 \times 10^{-3}$ C , $q_0$ = -2 $\times 10^{-9}$C

$r_1$= 3cm = $3 \times 10^{-2} m$ , $r_2$ = 4cm = $4 \times 10^{-2} m$

$W = -2\times 10^{-9} \times 8 \times 10^{-3} \times 9 \times 10^{9}\left[\frac{1}{4 \times 10^{-2}} - \frac{1}{3 \times 10^{-2}}\right]$

= 1.2 J

12. *A cube of side b has a charge q at each of its vertices. Determine the potential due to this charge array at the centre of the cube.*

Length of diagonal of the cube

= $\sqrt{b^2 + b^2 + b^2}$ = $\sqrt{3}$ b

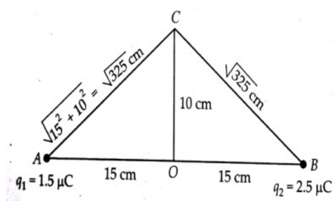
Distance of each charge from the centre of the cube

r = $\frac{\sqrt{3}}{2} b$

V = 8 $\times \frac{1}{4\,\pi\varepsilon_0}\frac{q}{r}$ = 8 $\times \frac{1}{4\,\pi\varepsilon_0}\frac{2q}{\sqrt{3}b}$

= $\frac{4q}{\sqrt{3}\pi\varepsilon_0\, b}$

13. *Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential*
*a) at the mid-point of the line joining the two charges*
*b) at a point 10 cm from this mid-point in plane normal to the line and passing through the mid-point .*



$r_1 = r_2$ = 15cm = 0.15m

∴ Potential at the mid –point O

$q_1$=1.5 $\times$ $10^{-6} C$          $q_2$=2.5 $\times$ $10^{-6}$ C

$V_O = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$

$= 9 \times 10^9 \left[\frac{1.5 \times 10^{-6}}{0.15} + \frac{2.5 \times 10^{-6}}{0.15}\right]$

$=2.4 \times 10^5$ V

$r_1 = r_2 = \sqrt{10^2 + 15^2} = \sqrt{325} \approx 18$ cm

= 0.18 m

$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$

$= 9 \times 10^9 \left[\frac{1.5 \times 10^{-6}}{0.18} + \frac{2.5 \times 10^{-6}}{0.18}\right]$

$= 2 \times 10^5\ V$

14. *A spherical conducting shell of inner radius $r_1$ and outer radius $r_2$ has a charge Q. A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?*

The charge q placed at the centre of the shell induces a charge –q on the inner surface of the shell and charge +q on its outer surface.

∴ Surface charge density on the inner surface of the shell $= \frac{charge}{surface\ area} = -\frac{q}{4\pi r_1^2}$

Surface charge density on the outer surface of the shell $= \frac{Q+q}{4\pi r_2^2}$

15. *a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by $(E_2 - E_1).\hat{n} = \frac{\sigma}{\varepsilon_0}$ , where $\hat{n}$ is a unit vector normal to the surface charge density at that point.(The direction of $\hat{n}$ is from side 1 to side*

*2) Hence show that just outside a conductor, the electric field is $\sigma\ \hat{n}/\varepsilon_0$ .*

*b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another.*

(a) Electric field near a plane sheet of charge ,

$E = \frac{\sigma}{2\varepsilon_0}$

If $\hat{n}$ is a unit vector normal to the sheet from side 1 to side 2,

Electric field on side 2

$\vec{E_2} = \frac{\sigma}{2\varepsilon_0}\hat{n}$ (normal to the side 2 and outwards)

Electric field on side 1 is $\vec{E_1} = \frac{\sigma}{2\varepsilon_0}\hat{n}$ (normal to the side 1 and outwards)

$\therefore \left(\vec{E_2} - \vec{E_1}\right) . \hat{n} = \frac{\sigma}{2\varepsilon_0} - \left(-\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$

$\vec{E_1}$ and $\vec{E_2}$ act in opposite directions.

Hence there is discontinuity at the sheet of charge.

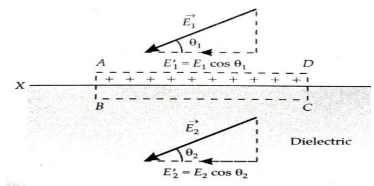
The electric field inside a conductor is zero, therefore

$\vec{E_1} = 0$

Outside the conductor, the electric field is

$\vec{E} = \vec{E_2} = \frac{\sigma}{\varepsilon_0}\hat{n}$

(b)Let XY be the charged surface of a dielectric and $\vec{E_1}$ and $\vec{E_2}$ be the electric field on the two sides .



Take a rectangular loop ABCD of length $l$ and negligible breadth.

Line integral along the closed path ABCD

$\int \vec{E} \, . \, \vec{dl} = \vec{E_1} \, . \, \vec{l} - E_2 \, .l = 0$

$E_1 \; l \cos\theta_1 - E_2 \; l \cos\theta_2 = 0$

$(E_1 \cos\theta_1 - E_2 \cos\theta_2) \, l = 0$

Taking $E_1 \cos\theta_1 = E'_1$ and $E_2 \cos\theta_2 = E'_2$

$\therefore (E'_1 - E'_2) \, l = 0$

$\because l \neq 0 \qquad E'_1 - E'_2$ or $E'_1 = E'_2$

Hence the tangential components of the electric field is continuous across the surface.

16. *In a hydrogen atom ,the electron and proton are bound at a distance of about 0.53* Å

*a) Estimate the potential energy of the system in eV taking the zero of the potential energy at infinite separation of the electron from proton.*
*b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in the above?*
*c) What are the answers to a) above if the zero of potential energy is taken at 1.06 Å separation?*

*a)* $q_1 = -1.6 \times 10^{-19}$C

$q_2 = +1.6 \times 10^{-19}$C

r = 0.53 Å = $0.53 \times 10^{-10}$m

$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$= 9 \times 10^9 \times \frac{(-1.6 \times 10^{-19}) \times 1.6 \times 10^{-19}}{0.53 \times 10^{-10}}$

$= -\frac{9 \times 1.6 \times 1.6 \times 10^{-19}}{0.53} J$

$= -\frac{9 \times 1.6 \times 1.6 \times 10^{-19}}{0.53 \times 1.6 \times 10^{-19}}$ eV

$\approx$ - 27.2 eV

*b)*K.E. of the electron in the orbit

$= \frac{1}{2}$ P.E. $= \frac{1}{2} \times 27.2$ eV = 13.6eV

$\therefore$ Total energy of the electron = P.E +K.E

= (-27.2 + 13.6)eV = - 13.6 eV.

Minimum work required to free the electron

= 0 – (-13.6) = 13.6 eV.

*c)* When the zero of potential energy is not taken at infinity, the potential energy of the system

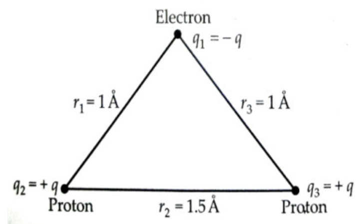
$U = \frac{q_1 \, q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$= 9 \times 10^9 \times (-1.6 \times 10^{-19}) \times 1.6 \times 10^{-19} \times \left[\frac{1}{0.53 \times 10^{-10}} - \frac{1}{1.06 \times 10^{-10}}\right]$

$= \frac{9 \times 10^9 \times -1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19} \times 0.53 \times 10^{-10}}\left[1 - \frac{1}{2}\right]$ eV

= -13.6eV

17.*If one of the two electrons of a $H_2$ molecule is removed ,we get a hydrogen molecular ion $H_2^+$ .In the ground state of a $H_2^+$ the two protons are separated by roughly 1.5* Å *and the electron is roughly 1* Å *from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.*




Charge on an electron, $q_1$ = -e = $-1.6 \times 10^{-19}$ C

Charge on proton, $q_2 = q_3$ = + e

= $+1.6 \times 10^{-19}$ C

If the zero of potential energy is taken at infinity , the potential energy of the system is

$U = U_{12} + U_{23} + U_{13}$

$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 \, q_2}{r_1} + \frac{q_2 \, q_3}{r_2} + \frac{q_1 \, q_3}{r_3}\right]$

$= \frac{1}{4\pi\varepsilon_0}\left[\frac{(-e)e}{1 \times 10^{-10}} + \frac{e.e}{1.5 \times 10^{-10}} + \frac{(-e)e}{1 \times 10^{-10}}\right]$

$= \frac{e^2}{4\pi\varepsilon_0 \times 10^{-10}}\left[-1+\frac{1}{1.5}-1\right]$

$= \frac{(1.6\times10^{-19})^2\times9\times10^9}{10^{-10}}\times\left(\frac{-4}{3}\right)$ J

$= \frac{-(1.6\times10^{-19})^2\times9\times10^9\times4}{1.6\times10^{-19}\times3\times10^{-10}}$ eV

$= -19.2$ eV

18. *Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.*

The charge will flow between the two spheres till their potential become equal(say V).

The charges on the two spheres $\frac{Q_1}{Q_2} = \frac{C_1 V}{C_2 V} = \frac{C_1}{C_2}$

C ∝ radius

$\therefore \frac{C_1}{C_2} = \frac{a}{b}$

$\frac{Q_1}{Q_2} = \frac{a}{b}$

The ratio of the electric fields ,

$\frac{E_1}{E_2} = \frac{\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1}{a^2}}{\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_2}{b^2}} = \frac{Q_1 b^2}{Q_2 a^2} = \frac{a}{b}\,\frac{b^2}{a^2} = \frac{b}{a}$ ---------(1)

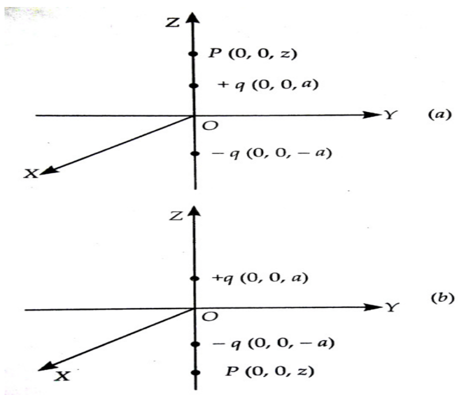
$\frac{\sigma_1}{\sigma_2} = \frac{\frac{Q_1}{4\pi a^2}}{\frac{Q_2}{4\pi b^2}} = \frac{Q_1 b^2}{Q_2 a^2} = \frac{a}{b}\,\frac{b^2}{a^2} = \frac{b}{a}$ ---------(2)

From (1) and (2)

$\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{b}{a}$

ie, Surface charge densities are inversely proportional to the radii of the spheres. The flat portion is taken as a sphere of large radius and a pointed portion is taken as a sphere of small radius. Hence the surface charge density is more on the sharp and pointed ends.

19. *Two charges -q and +q are located at points (0,0,z) and (0,0,a) respectively . a) What is the electrostatic potential at the points (0,0,z) and (x,y,0)? b) Obtain the dependence of potential on the distance r of a point from the origin when r/a>>1. c) How much work is done in moving a small test charge from the point (5,0,0) to (-7,0,0) along the x-axis ? Does the answer change if the path of the test charge between the same points is not along the x-axis ?*




*a)* When the point P lies closer to the charge +q as in figure (a)

Potential at point P,

$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_1} - \frac{q}{r_2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{z-a} - \frac{1}{z-(-a)}\right]$

$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{z-a} - \frac{1}{z+a}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{z+a-(z-a)}{z^2-a^2}\right]$

$= \frac{q}{4\pi\varepsilon_0}\left[\frac{2a}{z^2-a^2}\right]$

$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{P}{z^2-a^2}$ $[\because P = q \times 2a]$

Point (x,y,0) lies in XY –plane which is perpendicular bisector of Z – axis .

Distances from charges –q and +q to it are equal Hence potential at point (x,y,0) will be zero.

*b)* If the distance of point P from the origin O is r , then $V = \pm\frac{1}{4\pi\varepsilon_0}\,\frac{P}{r^2-a^2}$

If r >> a , we neglect $a^2$

$V = \pm\frac{1}{4\pi\varepsilon_0}\,\frac{P}{r^2}$

∴ For r >>a ,

V $\alpha$ $1/r^2$

*c)* (5,0,0 ) and (-7,0,0) are the points on the X-axis.

i.e, On the perpendicular bisector of the dipole.

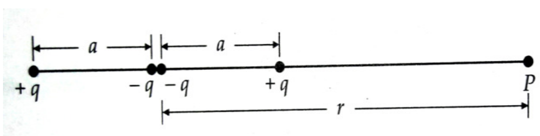
∴ Potential is zero.

Work done in moving the test charge $q_0$ from the point (5,0,0) to (-7,0,0) is

W = q $(V_1 - V_2)$ = q (0-0) = 0

The work done by the electrostatic field between two points is independent of the path.(as it is conservative). The answer will not change.

20.*Figure shows a charge array known as an electric quadrupole .For a point on the axis of the quadrupole. Obtain the dependence of potential on r for r/a>> 1 and contrast your results with that due to an electric dipole and an electric monopole*




Potential at point P is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r-a} - \frac{q}{r} - \frac{q}{r} + \frac{q}{r+a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r-a} - \frac{2q}{r} + \frac{q}{r+a}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r(r+a)-2\,(r-a)(r+a)+r(r-a)}{r(r-a)(r+a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2+ar-2r^2+2a^2+r^2-ar}{r(r^2-a^2)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{2\,a^2}{r(r^2-a^2)} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r(r^2-a^2)}$$

where Q = 2q $a^2$ is the quadrupole moment.

As r >> a

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}$$

ie, V $\alpha$ $1/r^3$

Dipole potential, V $\alpha$ $1/r^2$

Monopole potential , V $\alpha$ 1/r

21.*An electric technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V . Suggest a possible arrangement that requires the minimum number of capacitors.*

Let there be n capacitors of 1 $\mu F$ each in series and m such series combinations to be connected in parallel.

P.D. across each capacitor of a series combination = $\frac{1000}{n}$ = 400

or n = $\frac{1000}{400}$ = 2.5

Number of capacitors cannot be a fraction,

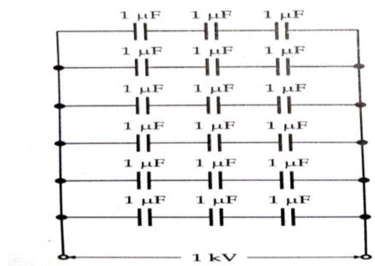
∴ n =3

Total capacitance= $\frac{1}{n}$ .m = 2

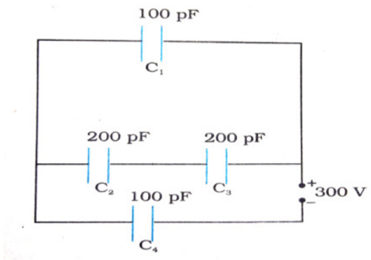
or m = 2n = 6

Total number of capacitors required = 3 ×6 =18

So the capacitors should be connected as in figure to get the required value.




22.*Obtain the equivalent capacitance of the network in figure. For 300 V supply , determine the charge and voltage across each capacitor.*




$C_2$ and $C_3$ are in series.

Total capacitance= $\frac{C_2C_3}{C_2+C_3}$ = $\frac{200\times200}{200+200}$ = 100 pF

Series combination of $C_2$ and $C_3$ is in parallel with $C_1$. Equivalent capacitance = 100pF +100pF

= 200 pF

The combination of $C_1$, $C_2$ and $C_3$ is in series with $C_4$ Equivalent capacitance of the network

= $\frac{200\times100}{200+100}$ pF = $\frac{200}{3}pF$

Total charge on the network is

q = CV = $\frac{200}{3}\times 10^{-12}\times 300$ = 2× $10^{-8}$ C

= charge on $C_4$

= sum of the charges on the combination of $C_1$, $C_2$ and $C_3$

∴ $q_4$ = q = 2 $\times 10^{-8}$ C

$V_4 = \frac{q_4}{C_4}$ = $\frac{2\times10^{-8}}{100\times10^{-12}}$ V = 200V

P.D between the ends of $C_1$

= V - $V_4$

= (300-200)V

= 100V

∴ $V_1$= 100 V

$q_1 = C_1 V_1 = 100 \times 10^{-12}\times 100 = 10^{-8}$ C

The P.D across the series combination of $C_2$ and $C_3$ = 100V

$C_2 = C_3$

∴ $V_2 = V_3$ = $\frac{100}{2}$ = 50 V

$q_2 = q_3 = 200\times 10^{-12}\times 50 = 10^{-8}$ C

23. *The plates of a parallel plate capacitor have area of 90 cm² each and are separated by 2.5 mm . The capacitor is charged by connecting it to a 400 V supply. How much electrostatic energy is stored by the capacitor? Also calculate the energy density.*

A = 90 $cm^2$ = 90 × $10^{-4}$ $m^2$

d = 2.5 mm = 2.5 × $10^{-3}$ $m$

$\varepsilon_0$ = 8.85 × $10^{-12}$ $C^2$ $N^{-1}$ $m^{-2}$

V = 400V

Capacitance of parallel plate capacitor is

C = $\frac{\varepsilon_0 A}{d}$ = $\frac{8.85\times10^{-12}\times90\times10^{-4}}{2.5\times10^{-3}}$

= 31.86 × $10^{-12}$ F

= 31.86 pF

Energy stored,

U = $\frac{1}{2}CV^2 = \frac{1}{2}\times 31.86\times 10^{-12}\times(400)^2$J

= 2.55 × $10^{-6}$ J

(ii) Energy per unit volume or energy density ,

U = $\frac{U}{Ad}$

= $\frac{2.55\times10^{-6}}{90\times10^{-4}\times2.5\times10^{-3}}$ = 0.113 $J\,m^{-3}$

24.*Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $\frac{1}{2}$QE, where Q is the charge on the capacitor and E is the magnitude of electric field between the plates. Explain the origin of the factor $\frac{1}{2}$*

Let A be the plate area and $\sigma$ be the surface charge density of the capacitor.

Q = $\sigma$A E = $\frac{\sigma}{\varepsilon_0}$

Work done to increase the separation of the capacitor plates by small distance $\Delta x$ against the force F is F. $\Delta x$

If U is the energy density, the increase in potential energy of the capacitor

= U × increase in volume = U .A. $\Delta x$

∴ F. $\Delta x$ = U .A. $\Delta x$

$F = UA = \frac{1}{2}\varepsilon_0 E^2$ A $= \frac{1}{2}(\varepsilon_0 E)$ AE

$= \frac{1}{2}\sigma$A .E $= \frac{1}{2} QE$

Inside the capacitor the field is E.
The electric field outside is zero .
The average value E/2 exerts force.

25. *A spherical capacitor has an inner sphere of radius 12cm and outer sphere of radius 13cm. The outer sphere is earthed and inner is given a charge of 2.5 $\mu C$. The space between the spheres is filled with a liquid of dielectric constant 32.*
*a)Determine the capacitance*
*b) What is the potential of the inner sphere?*
*c) Compare the capacitance of this capacitor with that of an isolated spherical capacitor of radius 12cm. Explain.*

*$C = \frac{4\pi\varepsilon_0 Rr}{R-r}$ , where R and r are the radii of outer and inner spheres, respectively.*

r = 12cm = $12 \times 10^{-2}$ m
R = 13cm = $13 \times 10^{-2}$ cm
q = 2.5 $\mu C$ = $2.5 \times 10^{-6}$ C , K = 32

(a) Capacitance of the spherical capacitor with dielectric is

$C = K .\ 4\pi\varepsilon_0 \frac{Rr}{R-r}$

$= \frac{32}{9\times10^9} \frac{13\times10^{-2}\times12\times10^{-2}}{(13-12)\times10^{-2}} = 5.5\times 10^{-9}$ F

(b) Potential of the inner sphere is

$V = \frac{q}{C} = \frac{2.5\times10^{-6}}{5.5\times10^{-9}} = 4.5\times 10^2 V$

(c) Capacitance of the isolated sphere of radius 12 cm is

$C = 4\pi\varepsilon_0 R = \frac{12\times10^{-2}}{9\times10^9} = 1.3\times10^{-11} F$

The capacitance of an isolated conductor is always small.

26. *Answer carefully:*
*a) Two large conducting spheres carrying charges $Q_1$ and $Q_2$ are brought close to each other .Is the magnitude of electrostatic force between them exactly given by $Q_1 Q_2/4\pi\varepsilon_0 r^2$ , where r is the distance between their centres?*
*b) If coulomb's law involved 1/ $r^3$ dependence, would Gauss's law be still true?*
*c) A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?*
*d) What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?*
*e) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?*

*f) What meaning would you give to the capacitance of a single conductor?*
*g) Guess a possible reason why water has a much greater dielectric constant (= 80) than say , mica (=6)*

(a) No .When the two spheres are brought closer their charge distributions do not remain uniform .So they will not act as point charges.

(b) No. If Coulomb's law is violated Gauss's law also is violated

(c) The small test charge will move along the line of force only if it is a straight line. The line of force gives the direction of acceleration (not that of velocity).

(d) Zero in both cases
(e) No , potential is constant everywhere.
(f) A single conductor is a capacitor with one plate at infinity. It also has capacitance.

(g) Water molecule has bent shape and it contains highly polar O –H bonds. It possesses a permanent dipole moment about 0.6 $\times 10^{-29}$ Cm. Hence water has a large dielectric constant.

27. *A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm . The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 $\mu C$ . Determine the capacitance of the system and the potential of the inner cylinder .*

**L =** 15 cm = 0.15 m ,

q = 3.5 $\mu C$ = 3.5 $\times 10^{-6}$ C , r = 1.4 cm = 0.014 m

R = 1.5 cm = 0.015m

Capacitance of a cylindrical capacitor is given by

$C = \frac{2\pi\varepsilon_0 L}{2.303\log\frac{R}{r}} = \frac{L}{2\frac{1}{4\pi\varepsilon_0}2.303\log\frac{R}{r}}$

$= \frac{0.15}{2\times 9\times 10^9\times 2.303\log\frac{0.015}{0.014}} = \frac{0.15\times 10^{-9}}{18\times 2.303\times 0.03}$

= 1.2 $\times 10^{-10}$ F

Potential ,

$V = \frac{q}{C} = \frac{3.5\times 10^{-6}}{1.2\times 10^{-10}}$ V = 2.9 $\times 10^4$V .

28. *A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about $10^7 Vm^{-1}$. For safety, we should like the field never to exceed, say 10 % of the dielectric strength .What minimum area of the plates is required to have a capacitance of 50 pF?*

Maximum permissible voltage = 1 KV = $10^3 V$

Maximum permissible electric field

= 10 % of $10^7$

= $10^6\ Vm^{-1}$

∴ Minimum separation d required $d = \frac{V}{E} = \frac{10^3}{10^6}$

$= 10^{-3}\ m$

Capacitance of a parallel plate capacitor ,

$C = \frac{K\,\varepsilon_0\,A}{d}$

$\therefore A = \frac{Cd}{K\,\varepsilon_0} = \frac{50\times 10^{-12}\times 10^{-3}}{3\times 8.85\times 10^{-12}}$

$= 18.8\times 10^{-4} m^2$

29. *Describe schematically the equi-potential surface corresponding to*

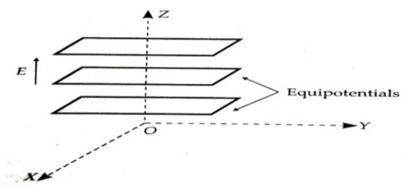
*a) a constant electric field in the z-direction ,*

*b) a field that uniformly increases in magnitude but remains in a constant (say , z) direction*

*c) a single positive charge at the origin*

*d) a uniform grid consisting of long equally spaced parallel charged wires in a plane .*

*a)*For a constant electric field in Z- direction, equi-potential surfaces will be planes parallel to XY- planes, as in figure




*b)* Equi-potential surfaces will be planes parallel to XY –plane .

As field increases, planes will be closer.

*c)*Concentric spheres with the charge at the centre.

*d)* Near the grid equi-potential surface will have varying shapes.

At large distances, the equi-potential surface will be planes parallel to the grid.

30. *In a Van de Graff type generator a spherical metal shell is to be a 1.5$\times 10^6$ V electrode. The dielectric strength of the gas surrounding the electrode is 5$\times 10^7$*

*V m^-1^. What is the minimum radius of the spherical shell required?*

Maximum permissible electric field is

E = 10% of dielectric strength

= 10 % of 5 × $10^7$

= 5 × $10^6 \text{Vm}^{-1}$

For a spherical shell,

V = $\frac{1}{4\,\pi\varepsilon_0}\ \frac{q}{r}$

E = $\frac{1}{4\,\pi\varepsilon_0}\ \frac{q}{r^2} = \frac{V}{r}$

∴ Minimum radius required,

r = $\frac{V}{E} = \frac{1.5\times10^6\text{V}}{5\times10^6\text{V}}$

= $3\times10^{-1}$m = 30cm

*31. Answer the following*
*a) The top of the atmosphere is at about 400 kV with respect to the surface of the earth corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 V m^-1^. Why then do we not get an electric shock as we step out of our house into the open?*

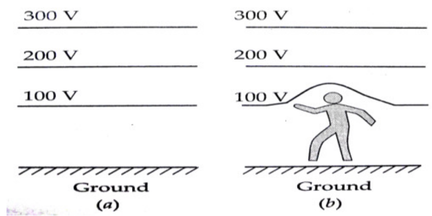
*b) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1 m^2^. Will he get an electric shock if he touches the metal sheet next morning?*
*c)The discharging current in the atmosphere due to the small conductivity of air is known to be 1800A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words what keep the atmosphere charged?*

*d) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?*

*a)*The equi-potential surface are parallel to the surface of the earth as in figure. Our body is a good conductor.

So as we step out , the equi-potential surfaces of open air get modified, keeping our head and the ground at the same potential .

Hence we do not any electric shock.




*b)* Yes. The aluminum sheet and the ground form a capacitor with insulating slab as dielectric.

The discharging current in the atmosphere will charge the capacitor steadily and rise its voltage.

Next morning, if the man touches the metal sheet , he may receive shock depending upon the capacitance of the capacitor formed .

*c)* The atmosphere is charged continuously by thunder storms and lightning but maintains an equilibrium with the discharge of the atmosphere.

*d)*The electrical energy is lost as (i) light energy (during lightning) (ii) heat and sound energy (during thunder).