

# 6

## **ELECTRO MAGNETIC INDUCTION**

### 06.01 Electromagnetic induction

When the magnetic flux linked with a coil or conductor changes, an emf is developed in it. This phenomenon is known as electromagnetic induction. The emf developed is called induced emf. If the circuit is closed, an induced current passes through it.

### 06.02 Experiment to show the production of emf in a coil by electromagnetic induction

Consider an insulated coil. Connect its ends to a galvanometer. Introduce north pole of a bar magnet suddenly into it. The galvanometer shows deflection which indicates the production of current in the coil. When the North Pole is withdrawn, the galvanometer shows deflection in the opposite direction.

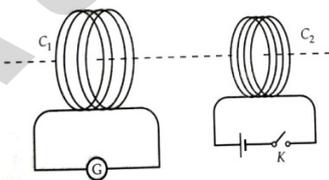
It indicates the production of current in the opposite direction.

In both cases emf is developed across the ends of coil. The experiment can be performed by introducing or withdrawing the south pole. In that case, the deflection of galvanometer will be opposite to that in the first case.

### 06.03 In the figure below $C_1$ and $C_2$ are insulated coils. K is a tapping key. What happens when

- The key is pressed?
- The key K is pressed continuously?
- When the key is released?

Give reasons.



#### **Answer:**

- a) Galvanometer shows a momentary deflection.

When the key is pressed a magnetic field is developed around  $C_2$  (The magnetic flux increases from zero to maximum).

The same flux change is linked with  $C_1$  as it is very close to  $C_2$ . So an emf is developed and an induced current passes through  $C_1$ .

- b) Galvanometer shows no deflection. Here the magnetic flux is not changing. No emf and hence no induced current are developed.

c) Galvanometer shows deflection in the opposite direction.

The change of magnetic flux in this case is in the opposite direction (maximum to zero). Hence emf and induced current are developed in the opposite direction.

#### 06.04 Faraday's law of electromagnetic induction

The magnitude of induced emf in a circuit is equal to the rate of change of magnetic flux linked with it.

$$\text{Induced emf } \epsilon = \frac{-d\phi_B}{dt}$$

-ve sign indicates that induced emf opposes magnetic flux change. If there are N turns

$$\epsilon = - \frac{Nd\phi_B}{dt}$$

#### **Integral form of Faraday's law**

$$\oint E \cdot dl = \text{induced emf} = \frac{-d\phi_B}{dt}$$

$$\text{But } \phi_B = \int B \cdot dA$$

$$\therefore \oint E \cdot dl = \frac{d}{dt} \int B \cdot dA \quad \text{This is called integral form of Faraday's law.}$$

#### 06.05 Lenz's law

The direction of induced emf is given by Lenz's law. The polarity of induced emf is such that it produces current which opposes the magnetic flux change that causes its production.

#### 06.06 Prove that Lenz's law is in accordance with the law of conservation of energy.

Consider an insulated coil, the ends of which are connected to a galvanometer. When North Pole of a bar magnet is introduced into it, the magnetic flux through the coil increases. An induced current is developed in the coil which opposes the increase of magnetic flux. This current makes the near end of the coil (end near the north) north (anti-clockwise current). Thus it is difficult to introduce the North Pole. Some mechanical work has to be done in overcoming repulsion. The energy spent for this appears as electrical energy in the coil.



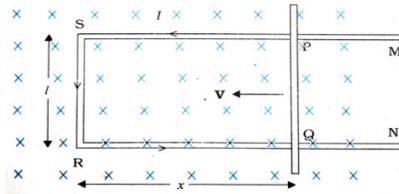
When the North Pole is withdrawn, the magnetic flux through the coil decreases. An induced current is developed in the coil which opposes the decrease of magnetic flux. This current makes the near end of coil south (clock-wise current). Some mechanical work has to be done in overcoming attraction. The energy spent for this appears as electrical energy in the coil. Thus Lenz's law obeys law of conservation of energy.

#### 06.07 Induced emf in a conductor when moved in a magnetic field

Consider a straight rectangular conductor PQRS moving in a uniform magnetic field. Let the direction of field be perpendicular to the plane of paper and into it.

Let the arm PQ be movable. Its length is  $l$ .

RQ =  $x$



The magnetic flux enclosed by PQRS is

$$\phi_B = \mathbf{B} \cdot \mathbf{A}$$

$\phi_B = B A$  [ as the angle between  $\mathbf{B}$  and  $\mathbf{A}$  is zero]

But  $A = lx$

$$\phi_B = Blx$$

PQ is moved with a velocity  $\mathbf{v}$  as in figure. When it is moved, the magnetic flux decreases as the area decreases.

Induced emf in it,  $\epsilon = - \frac{d\phi_B}{dt} = - \frac{d}{dt} Blx$  .

$$\epsilon = - Bl \frac{dx}{dt}$$

$\frac{dx}{dt}$  is taken as  $-v$

$$\therefore \epsilon = -Bl \times -v$$

$$\epsilon = Blv$$

If R is the resistance of the conductor,

$$\text{Induced current, } I = \frac{\epsilon}{R} = \frac{Blv}{R}$$

**Note :**

The arm carrying current  $I$  is in the magnetic field  $B$  .

It experiences a force  $F = I(l \times B)$

Here  $\theta = 90^\circ$   $\therefore F = IlB$

$$F = \frac{Blv}{R} \times lB = \frac{B^2 l^2 v}{R}$$

(in the opposite direction of  $v$ )

To move the arm , power required ,  $P = \text{Force} \times \text{velocity}$

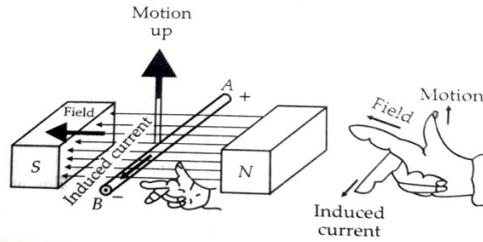
$$P = \frac{B^2 l^2 v}{R} \times v$$

$$P = \frac{B^2 l^2 v^2}{R}$$

The mechanical energy spent appears as electrical energy .

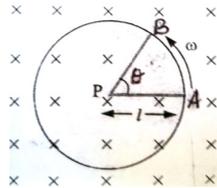
### 06.08 Direction of induced emf in a conductor when moved in a direction perpendicular to a magnetic field

It can be found by using **Fleming's Right Hand Rule**: Stretch the forefinger, middle finger and thumb of right hand in mutually perpendicular directions. If fore finger represents the direction of magnetic field, thumb represents the direction of motion of conductor then the middle finger represents the direction of induced emf or current (also called generator rule).



**06.09 Induced emf developed in a rod when rotated in a magnetic field**

Consider a rod of length  $l$  rotated in a uniform magnetic field  $B$  (into the plane paper and perpendicular) with an angular velocity  $\omega$ .



Angle swept  $\theta = \frac{AB}{l} \therefore AB = l\theta$

Area swept in time  $t = \frac{1}{2}l \times l\theta$

$A = \frac{1}{2}l^2\theta$

Flux linked  $\phi = BA \cos \theta = BA \cos 0 = BA = B \times \frac{1}{2}l^2\theta = \frac{1}{2}Bl^2\theta$

Magnitude of induced emf  $\epsilon = \frac{d\phi}{dt} = \frac{d}{dt} \frac{1}{2}Bl^2\theta = \frac{1}{2}Bl^2 \frac{d\theta}{dt} = \frac{1}{2}Bl^2\omega$

Induced current  $= \frac{\epsilon}{R} = \frac{Bl^2\omega}{2R}$

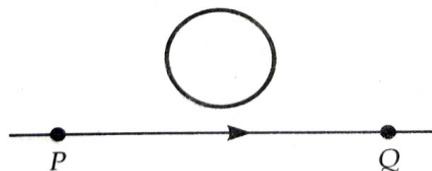
**Note**

Induced emf between the centre and rim of a disc radius  $R$ , rotating with a constant angular speed  $\omega$  in a magnetic field  $B$  along the axis of rotation of the disc  $= \frac{1}{2}Br^2\omega$

**06.10 When a conductor is moved parallel to the magnetic field, no emf is developed in it. Why?**

Here the magnetic flux linked with the conductor does not change. So no emf is developed.

**06.11 What is the magnitude of induced current in the loop of radius  $r$  if the straight wire PQ carries (1) steady current  $I$  (2) increased current from P to Q?**



(1) Zero as the current is steady

(2) The magnetic field above the wire is perpendicular to the plane of paper and outwards by right hand thumb rule. The induced current in the loop must oppose it. Hence the current must be clock-wise.

#### **06.12 When a light is switched off, spark is produced in the switch. Why?**

When the circuit is switched off, the current in the circuit decreases suddenly. Large induced emf is developed between the contacts of the switch which tries to maintain current in the circuit. Air in between the contacts loses the insulation property and spark is developed.

#### **06.13 Eddy currents**

When the magnetic flux linked with a bulk piece of conductor changes, a circulating current is produced in it. This current is called eddy current (Foucault current).

Eddy currents are like eddies (like whirling air or water).

Resistance of a metal is low. So eddy currents are large. Eddy currents heat metal. It is disadvantageous for transformers, induction coil etc. Eddy currents can be minimized by making slots in the metal. (When slots are made, the available area decreases). In a transformer, to reduce eddy currents the core is laminated by varnishing it.

#### **06.14 A copper plate is suspended between pole pieces of a magnet. What happens when it is oscillated? What is the change when slots are made in the copper plate. Explain?**

It comes to rest in a short time. It is because eddy currents are developed in it during motion. During motion the magnetic flux linked with the plate changes as it moves in and out of the region between poles. Eddy current obeys Lenz's law. The tendency of current is to decrease the speed of motion. In the next case, when slots are made, the available area decrease. So eddy current decreases. The plate will not come to rest suddenly.

#### **06.15 A ferrous bar falling vertically through the hollow region of a thick cylindrical shell made of copper experiences retarding force. What is special about the bar?**

Bar experiences retarding force means it is a magnet. When it falls down, eddy currents are developed in the shell which opposes motion.

#### **06.16 Magnetic braking system in trains**

In some trains there are electromagnets. When these are activated, strong magnetic field is created over the rails (under the compartments). Eddy current are developed in the rails as the magnetic flux is changing due to the motion of train. Eddy current obeys Lenz's law. The tendency of this eddy current is to stop the train.

### 06.17 Induction furnaces

In an induction furnace a strong magnetic field is created by passing high frequency alternating current through a coil. The metal to be melted is kept at its centre. Eddy currents are developed in it. Eddy currents heat the metal to extremely high temperatures and it melts.

### 06.18 Induction cooker

In an induction cooker a strong magnetic field is created by passing alternating current through a coil. The steel vessel is kept in this field. Eddy currents are developed in the vessel. Eddy currents heat the vessel to high temperatures and food gets cooked.

### 06.19 An aluminium disc is placed over an electro magnet. What happens when the supply is switched on? Why?

The disc will be thrown up. When the supply is switched on, the magnetic flux linked with the disc changes. Eddy currents are developed in it. So the disc is slightly magnetized. If the upper end of the coil is north then the eddy current makes the bottom part of the disc north. Due to the repulsion between the poles, the disc flies up.

### 06.20 Working of speedometer

In a speedometer, a magnet rotates inside an aluminium cylinder, according to the speed of the vehicle. Eddy currents are developed in the aluminium cylinder which rotates it. A pointer attached to it indicates the speed of the vehicle.

### 06.21 Dead beat galvanometer

In a galvanometer a fine insulated coil is wound over an aluminium frame. The coil is between the cylindrical magnetic poles. When current is passed through the coil magnetic flux linked with the aluminium frame changes. Eddy currents are developed in it which opposes the motion of the coil (as it obeys Lenz's law). This type of stopping or damping is called **electromagnetic damping**.

### 06.22 Electric power meter

In a power meter an aluminium disc is kept near a coil. When ac is passed through the coil, the magnetic flux linked with the disc changes. Eddy currents are developed in it and it rotates.

### 06.23 Induction motor

In an induction motor there is stationary coil and a rotor that can be rotated freely. When alternating current is passed through the stator coil, large eddy currents are developed in the rotor and it rotates.

### 06.24 Disadvantages of eddy currents

- ❖ Eddy currents develop heat in core of devices like transformers which causes loss of electrical energy.
- ❖ Heat developed in core may cause burning of the insulation of coil.
- ❖ Eddy currents may cause unwanted damping effect.

## 06.25 Inductance

An emf is developed in a coil by the flux change produced by the same coil or flux change produced by another coil in its vicinity.

If  $\phi_B$  is the flux and  $I$  is the induced current,  $\phi_B \propto I$

If there are  $N$  turns,  $N \phi_B \propto I$

$$N \phi_B = \text{constant} \times I$$

The constant in this situation is called inductance. It is a scalar quantity.

Its dimension is  $ML^2T^{-2}A^{-2}$ . Its SI unit is henry (H).

## 06.26 Mutual induction

The phenomenon of production of emf in a coil due the change in the magnetic flux in the neighboring coil caused due to a varying current is called mutual induction.

Consider two co-axial insulated coils of area  $A$  and length  $l$ . The number of turns in the inner coil is  $N_1$  and the number of turns in the outer coil is  $N_2$ . The inner coil is  $C_1$  and outer coil is  $C_2$ . When current  $I_2$  is set up in  $C_2$ , a magnetic flux  $\phi_1$  is set up in  $C_1$ .

Then

$$N_1 \phi_1 \propto I_2 \quad \text{or} \quad N_1 \phi_1 = M_{12} I_2 \dots\dots\dots(1)$$

$M_{12}$  is the mutual inductance of  $C_1$ .

$$N_1 \phi_1 = n_1 l \times B_2 A$$

$$N_1 \phi_1 = n_1 l \times \mu_0 n_2 I_2 A \dots\dots\dots(2) \text{ where } n_1 \text{ and } n_2 \text{ represent the number of turns in unit length of } C_1 \text{ and } C_2.$$

$$\text{From (1) and (2) } M_{12} I_2 = n_1 l \times \mu_0 n_2 I_2 A$$

$$\text{or } M_{12} = \mu_0 n_1 n_2 A l \dots\dots\dots(A)$$

Similarly when current  $I_1$  is set up in  $C_1$  a magnetic flux  $\phi_2$  is set up in  $C_2$ . Then  $N_2 \phi_2 \propto I_1$

$$N_2 \phi_2 = M_{21} I_1 \dots\dots\dots(3)$$

where  $M_{21}$  is the mutual inductance of  $C_2$

$$N_2 \phi_2 = n_2 l \times B_1 A$$

$$N_2 \phi_2 = n_2 l \times \mu_0 n_1 I_1 A \dots\dots\dots(4)$$

$$M_{21} I_1 = n_2 l \times \mu_0 n_1 I_1 A$$

$$\text{or } M_{21} = \mu_0 n_1 n_2 A l \quad (B)$$

$$M_{12} = M_{21} = M \text{ (say) .}$$

where  $M$  is the mutual inductance or coefficient of mutual induction. This is known as the reciprocity theorem of mutual inductance.

### 06.27 Self induction

The phenomenon of production of emf in a coil due to the change in the magnetic flux in the same coil due to a varying current is known as self induction.

Consider a coil of  $N$  turns carrying  $I$ . Magnetic flux linked with it

$$N \phi_B \propto I$$

$$N \phi_B = LI$$

$$N \phi_B = n l B A = n l \times \mu_0 n I A$$

$$\text{or } LI = n l \times \mu_0 n I A \quad \text{OR } L = \mu_0 n n A l$$

$$L = \mu_0 n^2 A l$$

The emf produced as a result of self induction is called back emf. Thus when the magnetic flux linked with a coil changes, an emf is developed in it in the opposite direction of applied emf. This emf is called back emf. It opposes the growth and decay of current in the circuit.

#### Important points

- **Self induction** is known as inertia of electromagnetic induction because it opposes the growth and decay of current.

- **Mutual inductance of two concentric coils of different radii**

$$M = \frac{\mu_0}{4\pi} 2\pi \frac{r_1^2}{r_2^2} \quad (\text{where } r_1 \ll r_2)$$

- **Mutual inductance of two co-axial coils of different radii**

$$M = \frac{\mu_0}{4\pi} \frac{2\pi^2 R^2 r^2}{(R^2 + x^2)^{3/2}} \quad R > r \quad \text{where } x \text{ - is the distance between the centres of two coils}$$

### 06.28 Factors affecting mutual inductance

- ❖ Number of turns of coils
- ❖ Common area of cross-section of coils
- ❖ Relative separation and orientation of two coils
- ❖ Permeability of core material.

**Coefficient of coupling between two coils of self inductance  $L_1$  and  $L_2$**   $k = \frac{M}{\sqrt{L_1 L_2}}$

For tight coupling  $k = 1$ . For loose coupling  $k = 0$ . Generally  $k$  is less than one.

### 06.29 Factors affecting self inductance

- ❖ Number of turns of coil
- ❖ Permeability of core material
- ❖ Area of cross section of the coil.

- Self inductance increases if air core is replaced by soft iron core.
- **Self inductance of series and parallel combination of coils**  
 Series  $L = L_1 + L_2$     Parallel  $L = \frac{L_1 L_2}{L_1 + L_2}$  (Here the assumption is that the magnetic flux due to one coil is not linked with other)

### 06.30 Induced emf is non – conservative

$$\varepsilon = \oint E \cdot dl \quad \text{-----(1)}$$

But Lenz's law  $\varepsilon = -\frac{d\phi_B}{dt}$  ----- (2)

From (1) and (2)  $\oint E \cdot dl = -\frac{d\phi_B}{dt} \neq 0$

$\therefore$  Non conservative.

#### Points to remember

- Induced emf is never greater than emf applied .
- 1 henry =  $10^9$  ab henry or  $10^9$  emu of inductance.
- 1 henry = 1 Wb/ampere
- Induced emf is also called **virtual emf**.
- For a bird sitting on a high voltage line, the wings are repelled due to induction which makes it flies away.
- 1 Wb =  $10^8$  maxwell
- Dimensional formula of magnetic flux  $ML^2T^{-2}A^{-1}$
- Induced emf is called back emf as it opposes the cause for its production.

### 06.31 Induced emf has no direction of its own. Why?

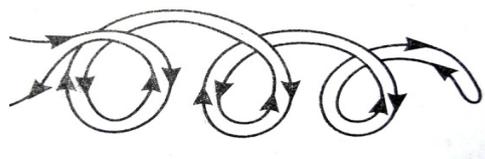
Induced emf depends on increase or decrease of magnetic flux linked with the coil.

#### Note

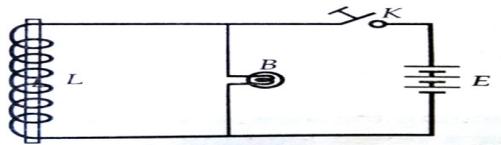
Pure inductor must have negligible resistance.

### 06.32 Non inductive winding

This type of winding is used to reduce self induction. The coil is wound as shown. The magnetic field in the neighboring turns cancel each other as the currents are in the opposite direction. But the resistance is doubled as the length is doubled.

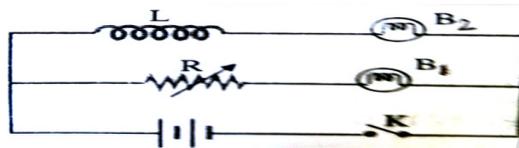


**06.33** In the circuit given, when the key is closed, the lamp glows dimly. When the key is released, the lamp glows brilliantly for a moment. Why?



When the key is closed, the inductor opposes the growth of current and the bulb glows dimly. When the key is released, an induced emf is developed in the coil which opposes the decay of current. Large current flows through the bulb for a moment and it glows brightly.

**06.34** Explain what happens when K is closed in the circuit given?



$B_1$  glows suddenly.

$B_2$  glows slowly. When current passes through L, an emf is developed which opposes the growth of current in it. But no such emf is developed in the resistor R.

**06.35** Energy stored in an inductor

When the magnetic flux linked with a coil changes, a back emf is developed in it. In establishing current, work has to be done against back emf.

Rate of work is  $\frac{dw}{dt} = |\varepsilon| I$  where  $|\varepsilon|$  is the magnitude of induced emf and  $I$  is the current.

$$\varepsilon = \frac{-d(N\phi_B)}{dt}$$

$$\varepsilon = \frac{-d(LI)}{dt}$$

$$\varepsilon = \frac{-LdI}{dt}$$

$$\therefore \frac{dw}{dt} = \frac{LdI}{dt} I$$

$$dW = LdI \times I = LI dI$$

Total work done increasing the current from 0 to  $I$  is

$$\int dW = \int_0^I LI dI$$

$$W = L \left[ \frac{I^2}{2} \right]_0^I$$

$$W = \frac{1}{2} LI^2$$

This work is stored as magnetic potential energy

$$\therefore \text{Energy stored} = \frac{1}{2} LI^2$$

**06.36 A bar magnet is allowed to fall down along the axis of a solenoid. Compute the acceleration of the magnet with respect to acceleration due to gravity.**

When the bar magnet freely falls towards the solenoid, magnetic flux linked with the coil increases. An emf is induced in it. This emf obeys Lenz's law. Hence there is opposition to the movement of magnet.

Net force on the magnet =  $mg - F$  where  $F$  is the opposing force and  $mg$  is its weight.

Acceleration of the magnet =  $\frac{mg-F}{m} = g - \frac{F}{m}$  which is less than  $g$ .

When the magnet is well inside the solenoid there is no change for the magnetic flux linked with the coil. No emf is developed. So it falls under gravity.

Acceleration of magnet =  $g$

When it comes out of the solenoid the magnetic flux linked with the solenoid decreases. An emf is developed in it.

The motion of magnet is opposed.

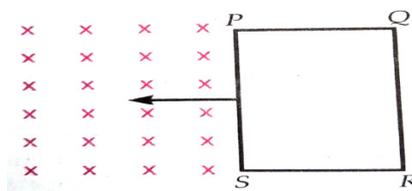
$\therefore$  Acceleration of magnet =  $g - \frac{F}{m}$

**06.37 A rectangular loop of wire is pulled to right from a long straight wire through which a steady current  $I$  is passed upwards. Does the induced current in the loop flow in clockwise or anti clockwise direction?**



The magnetic field at the right side of the wire is into the plane of paper. When the loop is moved, the magnetic flux linked with it decreases. The direction of induced current in the loop must be in such a way that it must oppose the decrease of magnetic flux. So it must be clock-wise (by right hand thumb rule).

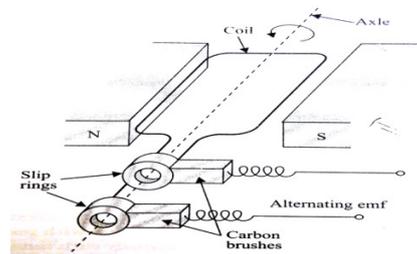
**06.38 A closed loop PQRS is moved into a uniform magnetic field perpendicular to the plane of paper into it. State the direction of induced current in the loop.**



When the loop is moved, the magnetic flux linked with it increases. The direction of induced current in the loop is such that it opposes the increase of magnetic flux (by Lenz's law). So it will be SRQPS (by right hand thumb rule).

### 06.39 Working of an ac generator

It consists of an insulated coil which is free to rotate on a rotor shaft. The coil is kept between two magnetic poles. When the coil is rotated, the magnetic flux linked with it changes. An emf is developed in it. The ends of the coil are connected to external circuit by means of slip ring and carbon brushes. Thus we get current in the external circuit.



The magnetic flux linked with the coil  $\phi_B = B.A = BAN \cos \theta$  where  $\theta$  is the angle between  $\vec{B}$  and area vector  $\vec{A}$ .

If there are N turns  $\phi_B = BAN \cos \theta$

Induced emf in the coil  $\epsilon = \frac{-d\phi_B}{dt}$

$$\epsilon = \frac{-d}{dt} (BAN \cos \theta) = -BAN \frac{d}{dt} \cos \theta$$

If  $\omega$  is the angular speed of the coil,  $\omega = \frac{\theta}{t}$  or  $\theta = \omega t$

$$\therefore \epsilon = -BAN \frac{d \cos \omega t}{dt}$$

$$\epsilon = -BAN \times -\sin \omega t \times \omega$$

$$\epsilon = BAN \omega \sin \omega t$$

Maximum emf  $\epsilon_0 = BAN \omega \sin 90^\circ$

$$\epsilon_0 = BAN \omega$$

$$\epsilon = \epsilon_0 \sin \omega t$$

When the plane of coil is perpendicular to magnetic field  $\theta = \omega t = 0^\circ$

$$\therefore \text{Induced emf } \epsilon = \epsilon_0 \sin 0^\circ$$

$$\epsilon = 0$$

When the coil turns through  $90^\circ$  from the initial position,  $\theta = \omega t = 90^\circ$

$$\text{Induced emf } \epsilon = \epsilon_0 \sin 90^\circ$$

$$\epsilon = \epsilon_0 \text{ (maximum emf)}$$

When the coil turns through  $180^\circ$  from the initial position,  $\theta = \omega t = 180^\circ$

$$\text{Induced emf } \epsilon = \epsilon_0 \sin 180^\circ$$

$$\epsilon = 0$$

When the coil turns through  $270^\circ$  from the initial position,  $\theta = \omega t = 270^\circ$

$$\text{Induced emf } \epsilon = \epsilon_0 \sin 270^\circ$$

$$\epsilon = \epsilon_0 \times -1$$

$$\varepsilon = -\varepsilon_0 \quad (\text{Maximum emf in the opposite direction})$$

When the coil turns through  $360^\circ$  from the initial position,  $\theta = \omega t = 360^\circ$

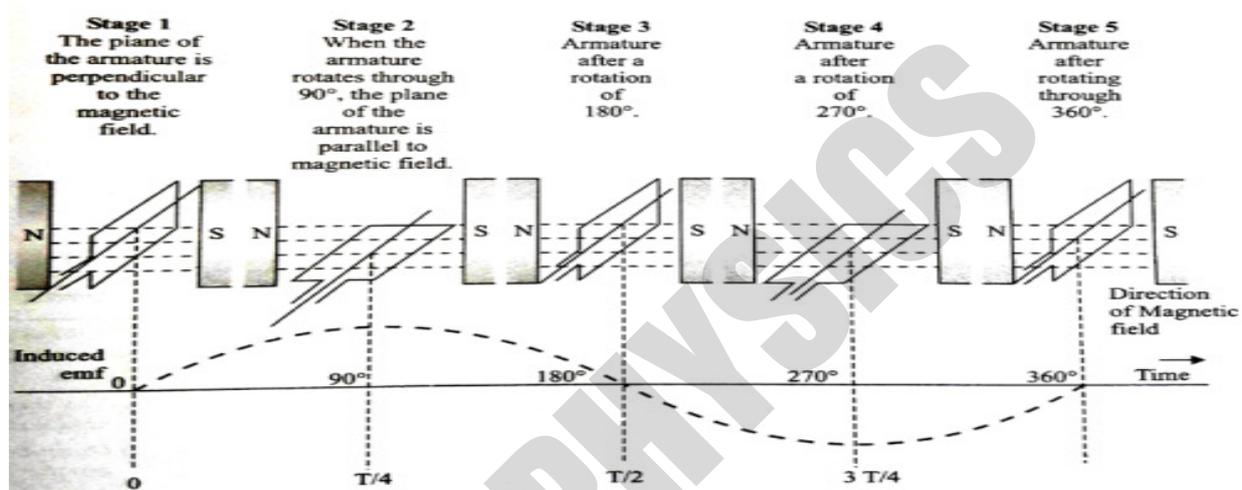
$$\text{Induced emf } \varepsilon = \varepsilon_0 \sin 360^\circ$$

$$\varepsilon = \varepsilon_0 \times 0$$

$$\varepsilon = 0$$

It can be shown graphically as in figure .

As the direction of emf is continuously changing, we get ac in the external circuit.



#### Note

The frequency of ac generated in India is **50Hz**.

#### 06.40 Electro magnetic shielding

When a magnetic field is directed towards a conducting sheet, eddy currents are produced in the sheet. The change in the magnetic field is partially detected at a point on the other side of the sheet. This is called electro magnetic shielding. If the conductivity of the sheet is more, the shielding is more effective.