# 2

# ELECTROSTATIC POTENTIAL AND CAPACITANCE

## 2.01 Electrostatic potential at a point in an electrostatic field

Electrostatic potential at a point is the work done in moving a unit positive charge from infinity to that point against electrostatic force without accelerating the charge.

Electrostatic potential is a scalar quantity.

Unit: volt or J/C

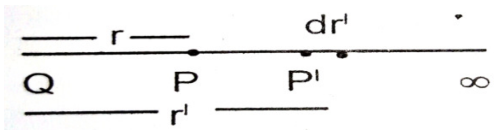
Potential $= \frac{Work}{Charge}$

## 2.02 Potential difference between two points in an electrostatic field

It is the work done in moving a unit positive charge from one point to other against electrostatic force without accelerating the charge.

Unit : volt or J/C.

## 2.03 Electrostatic potential at a point due to a point charge




Consider a charge +Q. Let P be a point at a distance r from it.

Potential at P = Work done in bringing a unit positive charge from infinity to P.

Take a point $P'$ in the path from infinity to P at a distance $r'$ from +Q.

When +1C is at $P'$, the force acting on it,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q\times(+1)}{(r')^2}$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(r')^2}$$

Consider a small path $dr'$ at $P'$.

Work done in moving +1 C along $dr'$ is,

$dW = F_{ext}\, dr'$

But $F_{ext} = -F$

$\therefore \quad dW = -F dr'$

$$dW = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{(r')^2}\, dr'$$

Work done in bringing +1C from infinity to P is given by

$$W = \int_{\propto}^{r} - \frac{1}{4\pi\varepsilon_o}\frac{Q}{(r')^2}\ dr'$$

$$= -\frac{Q}{4\pi\varepsilon_o}\int_{\propto}^{r}\frac{1}{(r')^2}\ dr'$$

$$= -\frac{Q}{4\pi\varepsilon_o}\int_{\propto}^{r}(r')^{-2}\ dr'$$

$$= -\frac{Q}{4\pi\varepsilon_o}\left[\frac{(r')^{-2+1}}{-2+1}\right]_{\propto}^{r}$$

$$= -\frac{Q}{4\pi\varepsilon_o}\left[\frac{(r')^{-1}}{-1}\right]_{\propto}^{r}$$

$$= -\frac{Q}{4\pi\varepsilon_o}\left[\frac{-1}{(r')}\right]_{\propto}^{r}$$

$$= \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{(r')}\right]_{\propto}^{r}$$

$$= \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{r} - \frac{1}{\alpha}\right]$$

But $\frac{1}{\alpha} = 0$

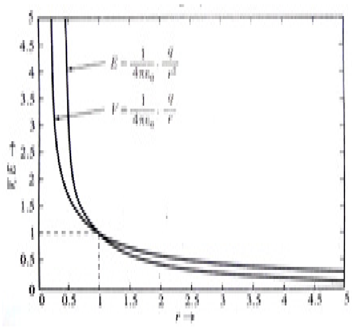
$\therefore$ W $= \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{r}\right]$

This work is the potential at P.

$\therefore$ Potential, V $= \frac{1}{4\pi\varepsilon_o}\ \frac{Q}{r}$

It is clear that V $\alpha\ \frac{1}{r}$

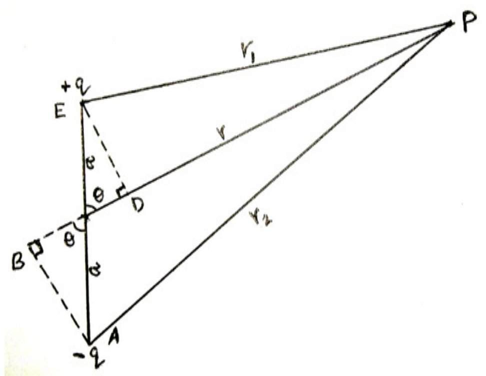
## 2.04 Graph showing the variation of V or E with r for a point charge



***Important !!!***

Electric potential decreases along the electric field line.

## 2.05 Potential at any point due to an electric dipole




Consider an electric dipole with charges +q and -q and length 2a. Let P be a point at a distance r from its centre. The distance from +q to P is $r_1$ and -q to P is $r_2$.

From ΔABC, $\cos\theta = \frac{BC}{a}$ or $BC = a\cos\theta$

From ΔCDE, $\cos\theta = \frac{CD}{a}$ or $CD = a\cos\theta$

$r_1 \approx PD = r - a\cos\theta$

$r_2 \approx PB = r + a\cos\theta$

Potential at P due to +q is $V_1 = \frac{1}{4\pi\varepsilon_o}\left[\frac{+q}{r_1}\right]$

Potential at P due to -q is $V_2 = \frac{1}{4\pi\varepsilon_o}\left[\frac{-q}{r_2}\right]$

Total Potential $V = V_1 + V_2$

$$= \frac{1}{4\pi\varepsilon_o}\frac{+q}{r_1} + \frac{1}{4\pi\varepsilon_o}\frac{-q}{r_2}$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{+q}{r - a\cos\theta} + \frac{1}{4\pi\varepsilon_o}\frac{(-q)}{r + a\cos\theta}$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{(r + a\cos\theta) - (r - a\cos\theta)}{(r - a\cos\theta)(r + a\cos\theta)}\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{(r + a\cos\theta - r + a\cos\theta)}{(r - a\cos\theta)(r + a\cos\theta)}\right]$$

$= \frac{q}{4\pi\varepsilon_o}[\frac{(2aCos\theta)}{(r^2 - a^2\, Cos^2\theta)}]$

But 2a x q = P (dipole moment)

$\therefore$ V $= \frac{1}{4\pi\varepsilon_o}[\frac{(Pcos\theta)}{(r^2 - a^2\, cos^2\theta)}]$

As 'a' is very small, $a^2Cos^2\theta$ is neglected.

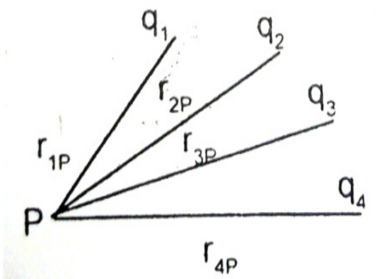
$\therefore$V $= \frac{1}{4\pi\varepsilon_o}[\frac{(Pcos\theta)}{r^2}]$

If the point is on the axial line $\theta = 0^o$

$\therefore$ V $= \frac{1}{4\pi\varepsilon_o}[\frac{P}{r^2}]$

If the point is on the equatorial line $\theta = 90^0$ $\therefore$ V = 0

## 2.06 Potential at a point due to a system of charges




Let the potential at P due to $q_1$ , $q_2$ and $q_3$ be $V_1$ , $V_2$ and $V_3$ respectively.

Potential at P , V $= \frac{1}{4\pi\varepsilon_o}[\frac{q_1}{r_{1p}}] + \frac{1}{4\pi\varepsilon_o}[\frac{q_2}{r_{2p}}] + \frac{1}{4\pi\varepsilon_o}[\frac{q_3}{r_{3p}}] + \frac{1}{4\pi\varepsilon_o}[\frac{q_4}{r_{4p}}]$

## 2.07 Potential due to a charged shell of radius R

(a) At a point outside the shell:

V $= \frac{1}{4\pi\varepsilon_o} \frac{Q}{r}$ where r is the distance from the centre of the shell to the point.

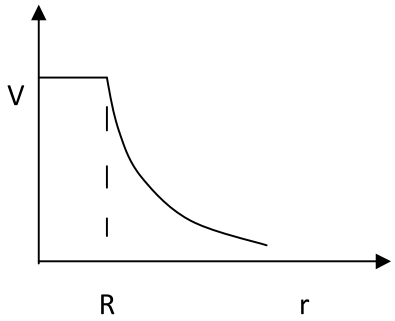
(b) At a point on its surface:

V$= \frac{1}{4\pi\varepsilon_o} \frac{Q}{R}$ where R is the radius of the shell.

(c) At a point in the interior:

V $= \frac{1}{4\pi\varepsilon_o} \frac{Q}{R}$ (as he potential on the surface and interior are equal)

## 2.08 Variation of V with r for a charged shell




## 2.09 Equi-potential surface and its properties

Equi- potential surface is a surface at every point of which the potential is the same.

No work is done in moving a charge from one point to other on an equi-potential surface.

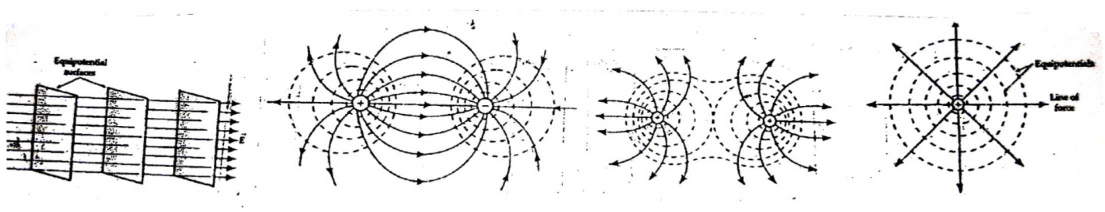
Two equi-potential surfaces never intersect.

In the case of a point charge, equi-potential surfaces are concentric spheres with the charge at centre.

In a uniform electric field equi-potential surfaces are planes. Each plane is perpendicular to electric field line.

Surface of a charged conductor is an example for equi- potential surface.

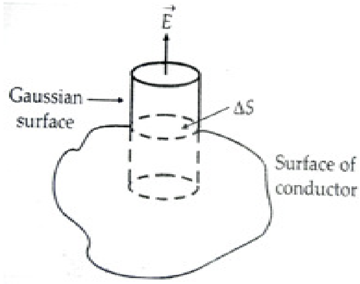
## 2.10 Equi-potential surfaces for a uniform electric field, for a dipole, two identical positive charges and around a positive charge



> ***Note:***
> - In an electric field, proton moves from higher potential region to a lower potential region.
> - One electron volt (eV) is the kinetic energy acquired by an electron when accelerated through a potential difference of one volt.

## 2.11 Electric field on the surface of a charged conductor is $\frac{\sigma}{\varepsilon_o}\hat{\mathbf{n}}$ (Proof)

Consider a charged conductor of irregular shape. Let $\sigma$ be the surface charge density(say positive) at any point on its surface. We consider a short cylindrical (pill box shaped) Gaussian surface at this point. It lies partly inside and partly outside the conductor. Let ΔS be its cross sectional area. (Height of the cylinder is negligible).




Inside the conductor, the electric field is zero. Outside, it is perpendicular to the end face of the pill box. The flux through the lateral face is zero.

The net flux, $\emptyset = \vec{E}.\overrightarrow{\Delta S}$ = EΔS (as the angle between $\vec{E}$ and $\overrightarrow{\Delta S}$ is zero)

$\sigma = \frac{q}{\Delta S}$

OR $q = \sigma\Delta S$

By Gauss's theorem $\emptyset = \frac{1}{\varepsilon_o}$ x q

$= \frac{1}{\varepsilon_o}$ x $\sigma\Delta S$

$E\Delta S = \frac{1}{\varepsilon_o}$ x $\sigma\Delta S$

OR $E = \sigma/\varepsilon_0$

Vectorially, $\vec{E} = \sigma/\varepsilon_0\ \hat{n}$ where $\hat{n}$ is a unit vector outwards from the surface.

## 2.12 Potential energy of a system of two charges

It is the work done in assembling the system from infinite separation to the given points. Consider a charge $q_1$. Let P be a point at a distance r from it. Potential at P,

$V = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r}$

Let a charge $q_2$ be at infinity. To bring it to P, work has to be done. This work is stored as potential energy of the system.

W = Potential energy

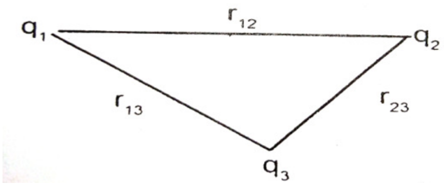
= Potential x charge

$= \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r}$ x $q_2$

$= \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r}$

Potential energy = $\frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r}$

## 2.13 Potential energy of a system of 3 charges as in figure




$$V = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_o} \frac{q_2 q_3}{r_{23}} + \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_3}{r_{13}}$$

## 2.14 Potential energy of a single charge when kept in an external field

Potential energy = Vq where V is the potential at that point due to the external field.

## 2.15 Potential energy of a system of 2 charges when kept in an external field

Potential energy = $V_1q_1 + V_2q_2 + \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r}$ where $V_1$ and $V_2$ are the potentials due to the external field and r is the distance between the charges.

## 2.16 Potential energy of a dipole which is kept in an external field

Consider a dipole with charges +q and -q and length 2a kept in a uniform electric field E. It experiences a torque $\tau = PE\ Sin\theta$

To turn the dipole through a small angle from $\theta_0$ to $\theta_1$ against this torque, external torque $\tau_{ext}$ is applied just to neutralize the torque.

Work done in turning through an angle $d\theta$ is $dw = \tau_{ext} d\theta$

Work done in turning from $\theta_o$ to $\theta_1$ is

$$W = \int_{\theta_o}^{\theta_1} \tau d\theta = \int_{\theta_o}^{\theta_1} PESin\theta d\theta = PE[-Cos\theta]_{\theta_o}^{\theta_1}$$

$$= -PE[Cos\,\theta_1 - Cos\theta_0]$$

$$W = PE\,[Cos\,\theta_o - Cos\theta_1]$$

This work is stored as potential energy of the system .

Potential energy, U = $PE\,[Cos\,\theta_o - Cos\theta_1]$

If $\theta_0 = 90^o$ and $\theta_1 = \theta^o$ then U = $-PECos\theta$

When $\theta = 0^o$, potential energy = $-$PE[minimum] ie, most stable equilibrium.

When $\theta = 180^o$, potential energy= PE[maximum] ie, most unstable equilibrium.

***Care !!!***

- Potential energy of two like charges is positive. While the potential energy of two unlike charges is negative.
- Positive potential energy implies that positive work has to be done against electrostatic repulsive force between two charges to assemble them from infinite separation to the given points.
- Negative potential energy implies that work has to be done by the external agency against electrostatic attractive force to bring the charges to infinite separation.

### 2.17 Electrostatic properties of conductors

(1) The electrostatic field is zero in the interior of a conductor (neutral or charged).

(2) On the surface of a charged conductor electrostatic field is normal to the surface at every point.

(3) The interior of a conductor has no excess charge in static situation.

(4) Electrostatic potential is constant throughout the volume of a charged conductor and has the same value as on surface.

(5) Electric field on the surface of a charged conductor = $\sigma/\varepsilon_0\ \hat{n}$ where $\hat{n}$ is a unit vector perpendicular to the surface in the outward direction (if the conductor is positively charged).

(6) Electrostatic shielding: Inside the cavity of a conductor, the electric field is zero.
This vanishing of electric field inside a cavity of a conductor is called electrostatic shielding.

### 2.18 Applications / effects of electrostatic shielding

- In a co axial cable, the outer earthed conductor provides electrostatic shielding to the signals carried by the inner cable.
- **To protect certain instruments from external electric fields, they are kept in metal boxes**
  The external field can induce charge only on the surface of the box. Inside it the electrostatic field is zero. This is a practical application of electrostatic shielding. Such boxes are called **Faraday's cages.**
- **It is safe to sit inside a car during lightning**
  Car has the shape of a shell. The charges due to lightning spread uniformly on the surface of the car. Inside the car the electrostatic field is zero. Hence it is safe to sit inside a car. This is a practical application of electrostatic shielding.

### 2.19 Reason why electrostatic field is normal to the surface of a charged conductor

If electric field is not normal, then it will have a non-zero component tangential to the surface of the conductor. It causes the flow of charges producing surface currents. But no such currents exist under static conditions. Hence electric field is perpendicular to the surface.

### 2.20 The net charge in the interior of a charged conductor is zero. Excess charge resides on the surface. Explain.

Consider a conductor carrying excess charge q with no currents flowing in it.

Choose a Gaussian surface very close to the boundary of the conductor.

Inside the conductor, E = 0

$\therefore \; \emptyset = \int E.ds \; = \frac{q}{\epsilon_o} \; = 0$

OR $\frac{q}{\epsilon_o} = 0$ OR q =0

ie, No charges inside the conductor.

∴ Charges reside on the surface.

### 2.21 For a charged conductor, potential at every point inside is same and has the same value as on surface. Explain.

$E = -\frac{dV}{dr}$ . Inside the conductor E = 0

$\therefore \; -\frac{dV}{dr} = 0$ OR V is a constant.

### 2.22 Reason why the electric field is perpendicular to equi-potential surface at every point.

If the electric field is not normal, then it will have a non zero component tangential to the surface. Therefore to move the test charge against this component, work has to be done. But for an equi-potential surface, P.D between any two points is zero. So no work is required in moving a charge from one point to another. Therefore electric field must be normal to the equi-potential surface.

### 2.23 Equi-potential surfaces are closer in the region of strong electric field

We know, $E = -\frac{dV}{dr}$

When dV is constant $dr \propto \frac{1}{E}$

ie, If E is strong the distance between equi-potential surfaces is small.

**2.24 No two equi-potential surfaces intersect each other. Give reason.**

If two equi-potential surfaces intersect, at the point of the intersection there are two values for electrostatic potential, which is impossible. Hence equi-potential surfaces never intersect each other.

**2.25 Importance of equi-potential surfaces**

Equi-potential surfaces give a visual picture (both magnitude and direction) of electric field in region of space. In a region of strong electric field, they are closer. Also electric field is normal to equi-potential surface, at every point.

**2.26 Prove that the electrostatic field inside the cavity of a charged conductor is zero.**

Consider a charged conductor having a cavity. The charges reside only on the surface of the conductor. The net charge inside = 0

Take a Gaussian surface very close to the boundary and inside the conductor.

By Gauss' theorem , $\int E.ds = \frac{1}{\epsilon_o} \times q$ Here q = 0 ∴ E = 0

**2.27 No work is done in moving a charge from one point to another on an equi- potential surface. Why?**

For an equi-potential surface, at every point the potential is same.

∴ P.D = 0 But, work = P.D $\times$ charge As P.D is zero, work is zero.

**2.28 No work is done in moving a charge from one point to another, inside a charged conductor. Why?**

For a charged conductor , the potential at every point inside is same.
∴ P D between two points is zero.

Work = P.D $\times$ charge

As P.D is zero, work is zero.

***Note:***

Work done in moving a charge from one point to another in an electric field does not depend on the path along which it is taken as the electrostatic field is conservative.

**2.29 Prove that the eectric field is equal to negative potential gradient.**

Consider an electric field due to a charge at origin. Take two points A and B of potential $V + dV$ and $V$ respectively. The distance between the points is $dr$.

The work done to move a charge $q_o$ against the field ,

$F.dr = -Eq_o.dr$

W = PD × charge

W= $dV \times q_o$

i.e. $-E\ q_o.dr = q_o.dV$

OR E = $\frac{-dV}{dr}$

i .e Intensity of electric field is negative potential gradient.

***Care !!!***

- Potential is negative line integral of electric field.
- Potential gradient is a vector quantity . It is in a direction opposite to electric field.

**2.30 Method of finding the potential at a point if E is known**

E = $\frac{-dV}{dr}$ OR dV = −Edr

$\int_{V_1}^{V_2} dV = \int_{r_1}^{r_2} -E.dr$

$[V]_{V_1}^{V_2} = -\int_{r_1}^{r_2} E.dr$

$V_2 - V_1 = -\int_{r_1}^{r_2} E.dr$

If $r_1$ is at infinity then $V_1$ =0

On taking $V_2 = V$ and $r_2 = r$

Potential $= -\int_{\alpha}^{r} E.dr$

**2.31 Capacitors**

Capacitor is an arrangement of two conductors(plates) having equal and opposite charges separated by an insulating medium. Capacitors are used to store electric charges(electric energy).The plate on which the charge is stored is called posiive plate or collecting plate.The other plate which is earthed is called negative plate or condensing plate.The charge on a capacitor indicates the positive charge on the plate and not the total charge on the two plates, which is zero.

If V is the potential difference between the plates and Q is the charge, then capacitance,
C $= \frac{Q}{V}$

Unit of capacitance: farad or C/V

**2.32 Principle of a capacitor**

Consider a conductor. It is given charge Q. Let V be the P.D between its ends.

Its capacitance C = $\frac{Q}{V}$ -------(1)

When an earthed conductor is placed near the first, it gets opposite charge by induction.

Potential of the first conductor decreases to $V'$ and capacitance becomes $C'$

Charge remains same as Q.

New Capacitance C' = $\frac{Q}{V'}$--------(2)

Comparing (1) & (2)

$V' < V$

Therefore $C' > C$

Thus the presence of an earthed conductor increases the capacitance. This is the principle of a capacitor.

**2.33 Capacitance of a parallel plate capacitor**

Parallel plate capacitor consists of two parallel metal plates of plate area A and plate separation d.

Its capacitance C = $\frac{Q}{V}$ -------(1)

But , surface charge density, $\sigma = \frac{Q}{A}$

$\therefore\ Q = \sigma A$ where σ is the surface charge density.

V =E d $= \frac{\sigma}{\varepsilon_0}$ d

Putting in (1)

C = $\frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d} = \frac{\varepsilon_0 A}{d}$

**2.34 Effect of capacitance of a parallel plate capacitor on introducing a dielectric**

Consider a parallel plate capacitor of plate area A and plate separation d.

Capacitance C = $\frac{\varepsilon_0 A}{d}$

On introducing a dielectric of thickness 't' between plates,

Electric field in the air gap between plates $E_0 = \frac{\sigma}{\varepsilon_0}$

Electric field inside dielectric = $\frac{E_0}{K}$ where K is a dielectric constant.

Potential between the plates, $V' = E_o(d - t) + \frac{E_0 t}{K}$

$V' = E_0\,[d - t + \frac{t}{K}]$

$= \frac{\sigma}{\varepsilon_0}\,[d - t + \frac{t}{K}]$

$= \frac{\sigma\left[d - t + \frac{t}{K}\right]}{\varepsilon_0}$

But charge,Q = σA where σ is the surface charge density.

New Capacitance $C' = \frac{Q}{V'} = \sigma A \div \frac{\sigma}{\varepsilon_0}\,[d - t + \frac{t}{K}]$

$= \sigma A \times \frac{\varepsilon_0}{\sigma[d - t + \frac{t}{K}]}$

$$= \frac{\varepsilon_0 A}{[d - t + \frac{t}{K}]}$$

If the entire space is filled with dielectric , t = d

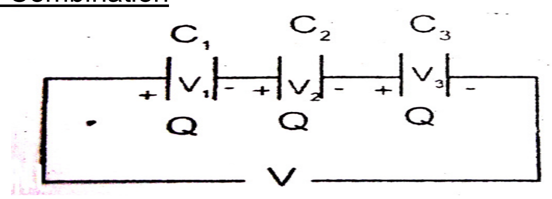
$$C^{I} = \frac{\varepsilon_0 A}{d - d + \frac{d}{K}} = \frac{\varepsilon_0 A K}{d} = \frac{K\varepsilon_0 A}{d}$$

ie, $C^{I}$ = KC

*i.e. Capacitance increases K times.*

## 2.35 Combination of capacitors

Series Combination




The capacitors are arranged as in figure. All of them get the same charge Q. The P.D across each is different. The P.D between plates are $V_1$ , $V_2$ and $V_3$ respectively.

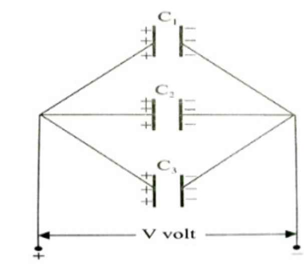
$V = V_1 + V_2 + V_3$

If C is the capacitance of the combination and Q is the charge.

$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$ OR $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ or $C = \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]^{-1}$

This combination is used to decrease capacitance. The total capacitance is always less than the least capacitance value used.

Parallel Combination




The capacitors are arranged as in figure. Let V be the common potential. The charges in the capacitors are $Q_1$, $Q_2$,$Q_3$. If Q is the total charge then $Q = Q_1 + Q_2 + Q_3$

$CV = C_1V + C_2V + C_3V$ where C is the total capacitance

or $C = C_1 + C_2 + C_3$. This combination is used to increase capacitance.

## 2.36 Energy and energy density of a capacitor

Consider a capacitor. Let V be the potential of it at a particular stage of charging. Let the charge be given in installments. To give charge $dq$, work done $dW = Vdq$
Work done in charging from 0 to Q is

$$W = \int_0^Q Vdq = \int_0^Q \frac{q}{C} dq = \frac{1}{C}\int_0^Q qdq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2C}[q^2]_0^Q$$

$W = \frac{1}{2C}[Q^2 - 0^2] = \frac{1}{2C}Q^2$

But $Q = CV$

$\therefore W = \frac{1}{2}CV^2$

This work is stored as potential energy in the electric field between the plates of capacitor.

Potential energy $= \frac{1}{2}CV^2$

Energy Density = Energy/Volume = $\frac{\frac{1}{2}CV^2}{Ad}$

where A is the plate area and d is the distance between plates.

But C = $\varepsilon_0$A/d

$\therefore$ Energy density = $\frac{1}{2}\frac{\varepsilon_0 AV^2}{d} \div Ad = \frac{1}{2}\frac{\varepsilon_0 AV^2}{d} \times \frac{1}{Ad} = \frac{1}{2}\varepsilon_0 \frac{V^2}{d^2}$

But $\frac{V}{d} = E$

$\therefore$ Energy density = $\frac{1}{2}\varepsilon_0 E^2$

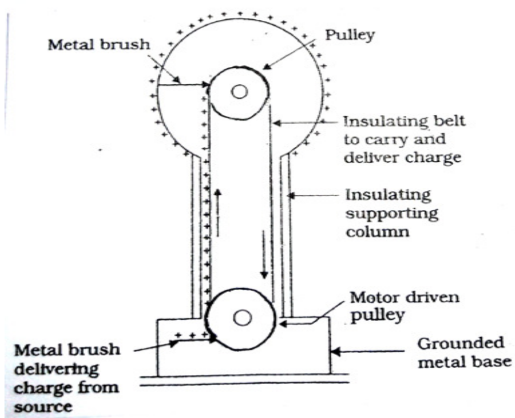
## 2.37 Capacitance of isolated spherical capacitor

An isolated spherical capacitor consists of a metallic sphere . It is given charge Q.

The potential on the surface, $V = \frac{1}{4\pi\epsilon_o}\frac{Q}{R}$

Capacitance, $C = \frac{Q}{V} = Q \times \frac{4\pi\epsilon_o R}{Q} = 4\pi\epsilon_o R$ (applicable for hollow or solid sphere)

## 2.38 Working of a Van de Graaff generator



It is a high voltage generator. This high voltage is used to accelerate charged particles. Consider a shell of radius R. If it is charged Q, the charges spread on the surface. If we keep a charged sphere of radius r and charge q at its centre, the potential of smaller sphere,

$V(r) = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{r} + \frac{Q}{R}\right]$

Potential of bigger $V(R) = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{R} + \frac{Q}{R}\right]$

We assume that q is positive.

The P.D between the spheres = V(r) - V(R)

$= \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{r} + \frac{Q}{R} - \frac{q}{R} - \frac{Q}{R}\right]$

$= \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r} - \frac{1}{R}\right]$

This is a positive answer. If we connect the spheres using a wire, the charges flow from smaller to bigger even though Q is large. Thus the charge on the shell (or sphere) increases to high value. This is the principle used in Van De Graaff Generator.

It consists of a metallic shell supported by insulating stands. At the centre there is a pulley. There is a pulley at the lower part also. An endless belt made of an insulator passes over the pulleys. A metal brush at the bottom delivers charge to the belt. The belt carries the charge to the upper part. Another metal brush collects these charges. As that brush is connected to the shell, the charges move to the surface of shell. After sometime the charge on the shell reaches extremely high values.

Van de Graaff generator is enclosed in an earthed steel tank filled with air under high pressure to prevent the leakage of charge due to ionization. As the pressure is high , the ions formed neutralize soon .

***Note:***

❖ **Conducting belt cannot be used in Van de Graaff generator**
The belt in Van de Graaff generator is made of an insulator. Hence the charge sprayed on to it remain at the place where they are given.As the belt moves, it can carry these charges up. If the belt is conducting, the charges delivered to it get spread in the belt. They cannot be collected as such at the metal brush at top.

***Note:***

❖ **The radius of the sphere of Van de Graaff generator must be sufficiently large**
Potential, $V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R}$ . As the radius increases, it can collect more charge without being raised to large potential.

## 2.39 Loss of energy when two parallelly connected capacitors share their charges

Consider two charged capacitors of capacitances $C_1$ and $C_2$. Let their potential differences be $V_1$ and $V_2$.

If they are connected together, the charges flow from high potential capacitor to the other till they attain a common potential V.

$$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Total energy before sharing $U_i = \frac{1}{2} C_1V_1^2 + \frac{1}{2} C_2V_2^2$

Total energy after sharing, $U_f = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(C_1 + C_2)\left[\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right]^2 = \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$

Loss of energy on sharing $= U_i - U_f = \frac{1}{2} C_1V_1^2 + \frac{1}{2} C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$

$$= \frac{1}{2(C_1 + C_2)}[C_1V_1^2(C_1 + C_2) + C_2V_2^2(C_1 + C_2) - (C_1V_1 + C_2V_2)^2]$$

$$= \frac{1}{2(C_1 + C_2)}[C_1^2V_1^2 + C_1C_2V_1^2 + C_1C_2V_2^2 + C_2^2V_2^2 - (C_1^2V_1^2 + C_2^2V_2^2 + 2C_1C_2V_1V_2)]$$

$$= \frac{1}{2(C_1 + C_2)}[C_1^2V_1^2 + C_1C_2V_1^2 + C_1C_2V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2]$$

$$= \frac{1}{2(C_1 + C_2)}[C_1C_2V_1^2 + C_1C_2V_2^2 - 2C_1C_2V_1V_2)]$$

$$= \frac{C_1C_2}{2(C_1 + C_2)}[V_1^2 + V_2^2 - 2V_1V_2)]$$

$$= \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

This loss of energy appears as heat energy in the connection wires connecting the capacitors. During the process of sharing there is no loss of charges.

## 2.40 Uses of capacitors

1. Capacitors are used in electrical and electronic devices.
2. Capacitors are used in the tank circuit of oscillators.
3. Capacitors are used in tuner circuits.
4. Capacitors are used in the filter circuits of rectifiers.

**2.41** One farad = 1 coulomb/1 volt

But 1volt = 1J/C

∴ One farad = 1 coulomb$^2$/1joule.

As coulomb is very large and joule is small ,one farad is very large.

***Note:***
If the dielectric constant is unity, the permittivity is $8.854 \times 10^{-12} C^2/Nm^2$.

***Note:***
- When a capacitor is charged and discharged repeatedly, its dielectric gets heated .The energy consumed during polarization is not recovered during depolarization. The difference of energy is wasted as heat.

## 2.42 Energy stored in a series combination of capacitors

Consider a series combination of capacitors $C_1, C_2, C_3$…connected to a potential difference of V.

The charge is the same in all capacitors(Q).

Total energy = $\frac{1}{2}CV^2$

Putting V = Q/C

Energy = $\frac{1}{2}C\left(\frac{Q}{C}\right)^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{2}\frac{1}{C}$

Here $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$

∴ Energy = $\frac{Q^2}{2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\right] = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3} + \ldots = U_1 + U_2 + U_3 + \ldots$

*ie*, The total energy is the sum of energy stored in each capacitor.

## 2.43 Energy stored in a parallel combination of capacitors

Consider a parallel combination of capacitors of capacitances $C_1, C_2, C_3$… connected to a potential difference of V.

Total energy = $\frac{1}{2}CV^2$

Here the PD remains the same(V).

$C = C_1 + C_2 + C_3 + \ldots$

Total energy $= \frac{1}{2}CV^2 = \frac{1}{2}(C_1 + C_2 + C_3 + \ldots)V^2$

$= \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2 + \ldots$

$= U_1 + U_2 + U_3 + \ldots$

*ie*, The total energy is the sum of energy stored in each capacitor.

## 2.44 Redistribution of charges if two conductors are touched mutually and separated

Consider two charged conductors of capacitances $C_1$ and $C_2$, charges $Q_1$ and $Q_2$ and PD's $V_1$ and $V_2$.

If they are connected by means of a wire, the charges flow from the high potential conductor to the low potential conductor till they attain a common potential,

$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

If the conductors are far separated and if the connection wire has negligible capacitance,

The new charge on the first, $Q_1' = C_1V$

The new charge on the second, $Q_2' = C_2V$

The ratio of charges = $Q_1'/Q_2' = C_1V/C_2V = C_1 : C_2$

The ratio of charges = The ratio of capacitances

## 2.45 Parallel plate capacitor with metal slab between the plates

The capacitance of a parallel plate capacitor , $C' = \frac{\varepsilon_0 A}{d - t + t/K}$

For a metal, $K = \infty$

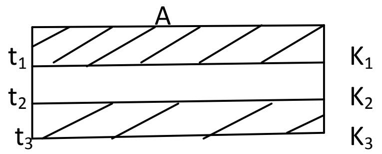
∴ $C' = \frac{\varepsilon_0 A}{d - t} = \frac{\varepsilon_0 A}{d}\frac{d}{d - t} = \frac{d}{d - t}C$

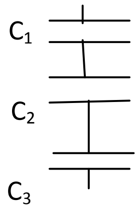
## 2.46 Corona discharge

When a conductor having a pointed end is charged, the charge density is maximum near the pointed end .If the magnitude of charge is too large , discharge takes place through the air in front of the pointed end. The air gets ionized. The air in contact with the pointed end gets the same charge by contact. It gets repelled from the tip. Thus a charged wind is created .This process is called corona discharge.

## 2.47 Parallel plate capacitor with compound dielectric

**Case 1**:




Let there be n dielectrics of thickness $t_1$,$t_2$,$t_3$ etc. and dielectric constants $K_1$,$K_2$,$K_3$ etc. This is equivalent to the combination of n capacitors in series. The area is the same for all.

Here $C_1 = \frac{K_1\varepsilon_o A}{t_1}$ , $C_2 = \frac{K_2\varepsilon_o A}{t_2}$ and $C_3 = \frac{K_3\varepsilon_o A}{t_3}$

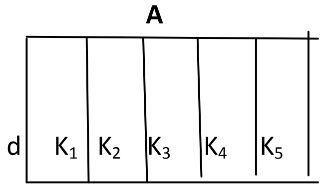
Total capacitance, C' = $\left[\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right]^{-1}$

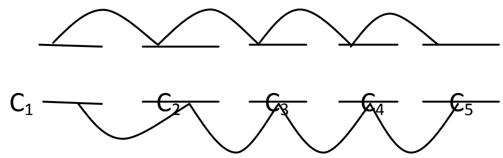
$$C' = \left[\frac{t_1}{K_1\varepsilon_o A}+\frac{t_2}{K_2\varepsilon_o A}+\frac{t_3}{K_3\varepsilon_o A}\right]^{-1}$$

$$C' = \left[\frac{1}{\varepsilon_o A}\right]^{-1}\left[\frac{t_1}{K_1}+\frac{t_2}{K_2}+\frac{t_3}{K_3}\right]^{-1}$$

$$\therefore C' = \frac{\varepsilon_o A}{\left(\frac{t1}{K1}+\frac{t2}{K2}+\frac{t3}{K3}+\cdots\right)}$$

**Case 2** :




Let there be n dielectrics of dielectric constants $K_1$,$K_2$,$K_3$ etc. The distance between their plates is d. This is equivalent to the combination of n capacitors in parallel. Let the areas be $A_1$,$A_2$,$A_3$ etc.

$C' = C_1 + C_2 + C_3 + \ldots\ldots = \frac{K_1\varepsilon_o A_1}{d} + \frac{K_2\varepsilon_o A_2}{d} + \frac{K_3\varepsilon_o A_3}{d} + \ldots..$

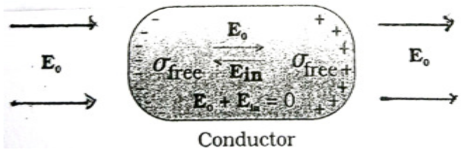
If the area of all the capacitors are equal ,then the area of each = A/n (since the total area is A)

$\therefore C' = \frac{K_1\,\varepsilon_o A_1}{nd} + \frac{K_2\varepsilon_o A_2}{nd} + \frac{K_3\varepsilon_o A_3}{nd} + \ldots..$

> ***Note:***
> - As the charge given to a capacitor is increased, the PD between the plates increases in the same proportion. Hence C remains the same.

## 2.48 Effect when a conductor is placed in an external electric field

When a conductor is placed in an external electric field the free charge carriers move. The electrons move in the opposite direction of external field. An internal field ($E_{in}$) is developed inside the conductor which opposes the external field ($E_o$). The magnitude of internal field will be equal to that of external field. The two fields cancel each other and the net electrostatic field in the conductor is zero.




If $E_0$ is the external field and $E_{in}$ is the internal field then total field $E_o + E_{in} = 0$

## 2.49 Difference between polar and non-polar molecules

In a non-polar molecule, the centres of positive and negative charges coincide. The net dipole moment is zero.

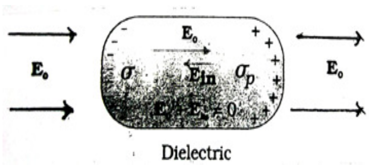
eg: Oxygen, Hydrogen, $CO_2$, $N_2$, $CH_4$ etc.

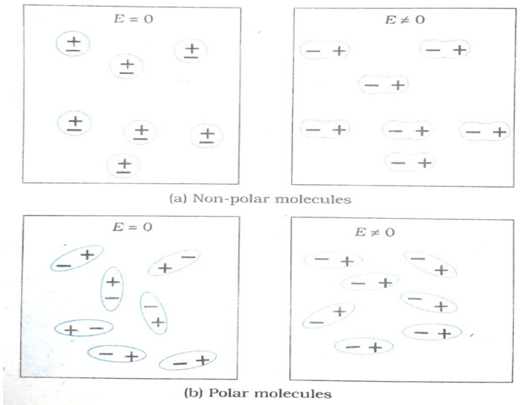
In a polar molecule, the centres of positive and negative charges do not coincide. The molecule will have a permanent dipole moment.

eg: $H_2O$, HCl, $NH_3$, CO, $CH_3OH$, NaCl etc.

## 2.50 Dielectric polarization

When a non-polar molecule is kept in an external electric field, the positive and negative charges are displaced in the opposite directions. The displacement stops when external force on charges is balanced by restoring force due to internal field in the molecule. Thus a dipole moment is developed in the same direction of external field. This is called dielectric polarization.




Consider a polar molecule. In the absence of external electric field the dipoles are oriented in a random way. Total dipole moment is zero. When external field is applied the dipoles get aligned with the field. Thus the molecule develops a net dipole moment in the same direction of external field. The dielectric get polarized. This is called dielectric polarization in the case of a polar molecule.

In either case a dipole moment is developed. Dipole moment per unit volume is called polarization density (P).It is numerically equal to surface charge density.

$P = \varepsilon_o \chi_e E$ where $\chi_e$ is the electric susceptibility.

> ***Note:***
> Dielectric polarization indicates the magnitude of polarization density. The direction of polarization is the same as that of the electric field.

### 2.51 Electric susceptibility(χ)

Electric susceptibility is the ratio of polarization to $\varepsilon_o$ times the net electric field.

$\chi = \frac{P}{\varepsilon_o E}$

> ***Note:***
> $\mathbf{K} = \frac{\overrightarrow{E_o}}{\vec{E}}$ where $E_o$ is the outside field and $E$ is the net external field.

### 2.52 Relation between K and χ

The net electric field in a dielectric kept in external field $\vec{E} = \overrightarrow{E_o} - \overrightarrow{E_p}$

$\overrightarrow{E_p} = \frac{\sigma_p}{\varepsilon_o} = \frac{P}{\varepsilon_o}$

$\vec{E} = \overrightarrow{E_o} - \frac{P}{\varepsilon_o}$

$\vec{E} = \overrightarrow{E_o} - \frac{\varepsilon_o \chi \vec{E}}{\varepsilon_o}$

Dividing by $\vec{E}$

$1 = \frac{\overrightarrow{E_o}}{\vec{E}} - \chi$

$1 = K - \chi$

Or $K = 1 + \chi$

### 2.53 Dielectric breakdown

When a dielectric is kept in a very strong electric field ,the outer shell electrons of dielectric atoms are detached and the dielectric behaves like a conductor. Its insulation property is lost. This process is called dielectric breakdown.

Dielectric strength indicates the maximum electric field that can be applied to the dielectric without causing dielectric breakdown.

### 2.54 Free charges and bound charges

For a metal, the outer shell electrons are almost free. Such free electrons are called free charges. The remaining positive ion consisting of nucleus and the inner shell electrons form bound charges.

## 2.55 Potential due to a linear charge distribution

Consider a linear charge distribution .Take a point P at a distance r from a charge $dq$.

Potential due to the linear charge distribution,

V = $\int \frac{1}{4\pi\epsilon_o}\frac{dq}{r} = \frac{1}{4\pi\epsilon_o}\int \frac{dq}{r}$

dq = $\lambda$dl where $\lambda$ is the linear charge density

V = $\frac{1}{4\pi\epsilon_o}\int \frac{\lambda dl}{r}$

## 2.56 Potential due to a surface charge distribution

Consider a surface charge distribution .Take a point P at a distance r from a charge dq.

Potential due to the surface charge distribution,

V = $\int \frac{1}{4\pi\epsilon_o}\frac{dq}{r} = \frac{1}{4\pi\epsilon_o}\int \frac{dq}{r}$

dq = $\sigma$dS where $\sigma$ is the surface charge density

V = $\frac{1}{4\pi\epsilon_o}\int \frac{\sigma dS}{r}$

## 2.57 Potential due to a volume charge distribution

Consider a volume charge distribution .Take a point P at a distance r from a charge dq.

Potential due to the volume charge distribution,

V = $\int \frac{1}{4\pi\epsilon_o}\frac{dq}{r} = \frac{1}{4\pi\epsilon_o}\int \frac{dq}{r}$

dq = $\rho$dV where $\rho$ is the volume charge density

V = $\frac{1}{4\pi\epsilon_o}\int \frac{\rho dV}{r}$

## 2.58 A parallel plate capacitor is charged and at a particular stage the voltage source is removed. What is the new charge, electric field, potential difference, capacitance and energy stored when a dielectric slab is completely introduced between the plates at this stage?

Let the initial charge be $Q_o$, PD be $V_o$, energy be $U_o$, electric field be $E_o$ and capacitance be $C_o$.

$Q_o = C_o V_o$ $\quad E_o = \frac{V_o}{d}$ $\quad V_o = E_o d$ $\quad U_o = \frac{1}{2} C_o V_o^2$

The charge remains $Q_o$ as the source is removed.

New electric field, $E = \frac{E_o}{K}$ (decreases)

New PD, $V = Ed = \frac{E_o}{K} d = \frac{V_o}{K}$ (decreases)

New capacitance, C = $\frac{Q}{V}$ = $\frac{Q_o}{\frac{V_o}{K}}$ = KC$_o$ (increases K times)

New energy = $\frac{1}{2}CV^2$ = $\frac{1}{2}KC_o\left(\frac{V_o}{K}\right)^2$ =$\frac{1}{K}$ $\frac{1}{2}C_oV_o{}^2$ = $\frac{1}{K}U_o$ (decreases)

**2.59 A parallel plate capacitor is charged by connecting to a source. What is the new charge, electric field, potential difference, capacitance and energy stored when a dielectric slab is completely introduced between the plates at this stage?**

Let the initial charge be $Q_o$,PD be $V_o$, energy be $U_o$ ,electric field be $E_o$ and capacitance be $C_o$.

$Q_o = C_oV_o$ $E_o = \frac{V_o}{d}$ $V_o = E_o$d $U_o = \frac{1}{2}C_oV_o{}^2$

PD remains at $V_o$ as the source is connected.

New capacitance, $C = KC_o$ (increases K times)

New electric field, $E = \frac{V_o}{d} = E_o$ (remains the same).

New energy = $\frac{1}{2}CV^2 = \frac{1}{2}KC_oV_o{}^2 = K$U$_o$ (increases K times).

New charge ,$Q = CV = KC_oV_o = KQ_o$ (increases).

**2.60 Two copper spheres of same radii one hollow and the other solid are charged to the same potential. Which one gets more charge? Why?**

We know that capacitance of isolated spherical capacitor C = $4\pi\epsilon_o$R
C ∝ R.
The spheres have same radius .So capacitance is the same.
They are charged to the same potential.

∴ The charge will be the same.

**2.61 Can we give any charge to a capacitor?**

No. As the charge is increased, V also increases. At a particular stage the electric field between the plates reach the breakdown value of air. Air gets ionized and the charges leak through air.

**2.62 What happens when the plates of a capacitor are suddenly connected by means of a wire?**

Capacitor gets discharged suddenly. Energy stored in the electric field becomes heat energy.

**2.63 Is there any material which when placed between plates of a capacitor reduces the capacitance?**

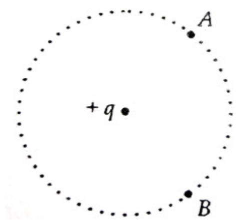
No . For any material , K > 1

∴ Capacitance always increases when a material is kept between plates.

***Note:***

For a parallel plate capacitor of finite plate area, the field lines bend at the edges. This effect is called fringing of field.

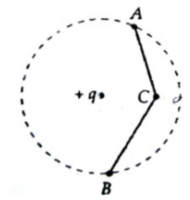
**2.66 In figure, what is the work done in moving a charge from point A to point B?**




Points A and B are at the same potential.(as both points are equi distant from q).The potential difference between A and B is zero.

Work = PD x Charge = 0 x charge = 0

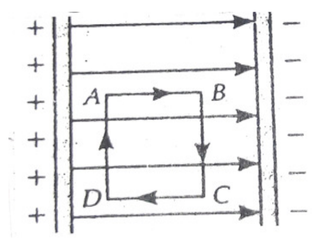
**2.67 If a point charge is taken first from A to C and then from C to B of a circle with another point charge at the centre, then along which path more work is done?**




Points A and B are at the same potential.(as both points are equi distant from q).

Hence, $V_c$ - $V_A$ = $V_c$ - $V_B$ .Hence the work is the same for both cases.

**2.68 In figure what is the work done in moving a charge along the closed path ABCDA?**




Electrostatic field is a conservative field. Hence the work done in moving a charge along the closed path ABCDA is zero.

**2.69 Why should electrostatic field be zero inside a conductor?**

If there is any electric field inside the conductor, it causes motion of electrons without the expense of any energy. This violates the law of conservation of energy.

**2.70 Potential on the surface of the earth is taken as zero. Justify.**

If we consider the earth as a conducting sphere, its capacitance, $C = 4\pi\epsilon_0 R$ .

The radius of the earth is very large. So capacitance is too large. If a finite charge q is given to its surface, the increase of potential on its surface is small as V = $\frac{q}{C}$ .

As C is very large, $\frac{q}{C}$ tends to zero.

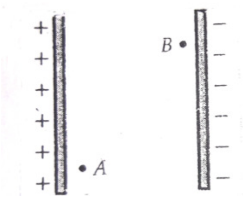
**2.71 Two conductors are carrying equal positive charges. Can there be any PD between them?**

Potential depends not only on charges , but also on geometrical shape and size. So there can be a PD.

**2.72 In a given region the electric potential is constant. What can you think about the electric field?**

E = $\frac{-dV}{dr}$ . If V is a constant, then E is zero.

**2.73 Two protons are kept - one at point A and other at point B as in figure. Will they experience equal or unequal force?**




The electric field is constant between the plates. Force, F = Eq. As E is the same, the force is also the same.

**2.74 In the figure what is $V_A$-$V_B$ if q is positive or negative?**


If q is positive, the potential at A is more than that at B. Therefore $V_A$-$V_B$ is positive.

If q is negative, the negative potential at A is more than that at B. Therefore $V_A$-$V_B$ is negative.

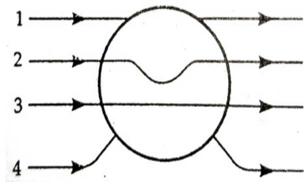
**2.75 Even if the electric field is zero, the potential may be a non – zero constant as E = $\frac{-dV}{dr}$. Give example.**

For example, the electric field inside a charged conducting sphere is zero. But the potential inside has the same value as on the surface. Also the electric field at the mid-point of the line joining two similar charges is zero. But potential at that point is not zero.

**2.76 Potential at a point can be zero even when the electric field is non-zero. Give example.**

For example, the potential at a point on the perpendicular bisector of an electric dipole is zero. But the electric field at that point is non-zero.

**2.77 A metallic sphere is kept in a uniform electric field. Which electric field line shown is correct?**




3 is correct. Electric field lines start and end perpendicular to the surface of a conductor.

**2.78 Can a metal sphere of radius 1cm hold a charge of one coulomb?**

No. Electric field on the surface of the sphere = $\frac{1}{4\pi\epsilon_o}\frac{Q}{R^2}$

Here R= 1cm and Q = 1C

The value of E obtained is 9x $10^{13}$V/m.

This is much greater than the dielectric strength of air(3x $10^{6}$V/m). It causes ionization of surrounding air.

**2.79 What is the effect on capacitance of parallel plate capacitor if a thin metal sheet is placed in the middle of the space between plates?**

When metal plate is introduced, it is equivalent to the series combination of two capacitors of equal capacitance C' with the total capacitance C.

Total capacitance of the set up = $\frac{(C'xC')}{(C'+C')} = \frac{C'}{2} = C$

OR C' = 2C

ie When 2C and 2C are connected in series, the total capacitance is $\frac{(2Cx2C)}{(2C+2C)}$ = C

ie, The capacitance remains the same.

**2.80 By what factor does the capacitance of a metal sphere increase if its volume is tripled?**

C = $4\pi\epsilon_o R$ ie, C α R

Volume = $\frac{4}{3}\pi R^3$

$V \alpha R^3$   If $V_1$ and $V_2$ are the initial and final volumes,

Here $\frac{V_2}{V_1} = 3$ Or  $V_2 = 3\, V_1$ or $\frac{4}{3}\, \pi R_2{}^3 = 3 \times \frac{4}{3}\, \pi R_1{}^3$  or  $\frac{R_2{}^3}{R_1{}^3} = 3$

OR $\frac{R_2}{R_1} = \sqrt[3]{3} = 1.44$

$\frac{C_2}{C_1} = \frac{R_2}{R_1} = 1.44$

OR New capacitance = 1.44 times the initial capacitance.

**2.81 What happens to the stored energy in a capacitor if the plates are drawn apart if the battery is disconnected?**

When d increases, the capacitance decreases as $C \alpha \frac{1}{d}$.

As the battery is disconnected the charge remains the same.

Energy = $\frac{Q^2}{2C}$.

As C decreases, the energy increases.

**2.82 What happens to the stored energy in a capacitor if the plates are drawn apart if the battery is not disconnected?**

As d increases, the capacitance decreases as $C \alpha \frac{1}{d}$.

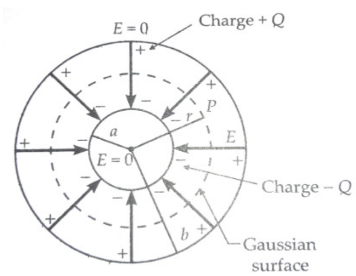
As the battery is connected the PD remains the same.

Energy = $\frac{1}{2} CV^2$ .

As C decreases, the energy decreases.

**2.83 Spherical capacitor**

A spherical capacitor consists of two concentric spherical shells of inner and outer radii a and b. The shells carry charges $-Q$ and $+Q$ respectively.




The electric field inside a hollow conductor is zero.

$\therefore$ $\vec{E} = 0$ for r < a

The field is zero outside the outer shell i.e., $\vec{E} = 0$ for r > b.

A radial field $\vec{E}$ exists in the region between the two shells due to the charge on the inner shell only.

To determine the electric field at any point $P$ at distance $r$ from the centre, consider a concentric sphere of radius r as the Gaussian surface.

Using Gauss's theorem,
$\phi_E$ = E. $4\pi r^2$ = $\frac{Q}{\varepsilon_o}$

or E = $\frac{Q}{4\pi\, \varepsilon_o r^2}$

The potential difference (Caused by the inner sphere alone ) between the two shells
V = $-\int_a^b \vec{E} \, . \, \vec{dr}$

= $\int_a^b Edr$

= $\int_a^b \frac{Q}{4\pi\, \varepsilon_o r^2} \, dr$

[Since $\vec{E}$ points radially inward and $\vec{dr}$ points outward , $\vec{E} \, . \, \vec{dr}$ = $Edr \, 180^0 \;=\; -Edr$]

V = $\frac{Q}{4\pi\, \varepsilon_o} \int_a^b r^{-2} \, dr$

= $\frac{Q}{4\pi\, \varepsilon_o} \left[-\frac{1}{r}\right]_a^b$

= $\frac{Q}{4\pi\, \varepsilon_o} \left[\frac{1}{a} - \frac{1}{b}\right]$

The capacitance of the spherical capacitor is
C = $\frac{Q}{V}$

= $\frac{Q}{\frac{Q}{4\pi\, \varepsilon_o}\left[\frac{1}{a} - \frac{1}{b}\right]}$

or C = $\frac{4\pi\, \varepsilon_o \, a\, b}{b - a}$

**2.84 Cylindrical capacitor**

A cylindrical capacitor consists of two coaxial conducting cylinders of inner and outer radii a and b. The cylinders have uniform linear charge densities of $\pm \lambda \, Cm^{-1}$ . The length L of the capacitor is so large (L >> a or b ).

The electric field in the region between the two cylinders is only due to the inner cylinder . To calculate the electric field E at any point P in between the two cylinders at a distance r from the central axis, we consider a coaxial Gaussian cylinder of radius r.

Electric flux at end face = 0

Electric flux at curved face = $\oint \vec{E} \, . \, \vec{dS}$

$= \oint EdS \cos 0$

$= \oint EdS$

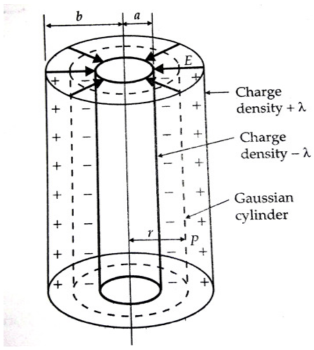
$= E \oint dS$

$= E. 2\pi r L$

Total flux $\phi_E = E. 2\pi r L$

By Gauss's theorem,

$\phi_E = \frac{q}{\varepsilon_o}$




$E. 2\pi r L = \frac{\lambda L}{\varepsilon_o}$

$E = \frac{\lambda}{2\pi \varepsilon_o r}$

∴ Potential difference between the two cylinders is

$V = -\int_a^b \vec{E} . \overrightarrow{dr}$

$= \int_a^b \vec{E} \ \overrightarrow{dr}$   [∵ $\vec{E}$ and $\overrightarrow{dr}$ are in opposite directions]

$= \int_a^b \frac{\lambda}{2\pi \varepsilon_o r} dr$

$= \frac{\lambda}{2\pi \varepsilon_o} \int_a^b \frac{1}{r} dr$

$= \frac{\lambda}{2\pi \varepsilon_o} [\ln r]_a^b$

$= \frac{\lambda}{2\pi \varepsilon_o} [\ln b - \ln a]$

Or   $V = \frac{\lambda}{2\pi \varepsilon_o} \ln \frac{b}{a}$

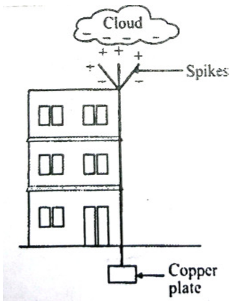
Total charge on each cylinder is $Q = \lambda L$

∴ Capacitance of cylindrical capacitor,

$C = \frac{Q}{V} = \frac{\lambda L}{\frac{\lambda}{2\pi \varepsilon_o} \ln \frac{b}{a}}$

or   $C = \frac{2\pi \varepsilon_o L}{\ln \frac{b}{a}}$

### 2.85 Working of lightning conductor




Lightning conductor is used to protect buildings from the danger of lightning. If consists of a metallic rod having pointed ends. The device will be earthed.

If a charged cloud (say negative) passes near the device, the pointed ends get positive by induction and the other end gets negative. The negative charge disappears immediately as that end is earthed. The charge density at the pointed ends will be more. The air in contact with the positive tip gets positive charge by contact. It gets repelled from the tip. Thus a positively charged cloud is created.

Now discharge takes place between this positively charged cloud and the negatively charged cloud. The building is safe.