Engineering Paper 3 - Chapter 5 and 6

1. A cylinder of mass 5 kg and radius 30 cm, and free to rotate about its axis, receives an angular impulse of 3 kgm^2s^{-1} initially followed by a similar impulse after every 4 sec. what is the angular speed of the cylinder 30 sec after initial imulse ? The cylinder is at rest initially.

(a) 106.7 rad s^{-1}
(b) 206.7 rad s^{-1}
(c) 107.6 rad s^{-1}
(d) 207.6 rad s^{-1}

Answer: a
Solution

Inital angular momentum $= L_i = I\omega_i = I \times 0 = 0$

angular mumentum after initial inpulse $= 3\ kgm^2s^{-1}$

angular mumentum after initial 4sec $= 3 + 3$

$= 6kgm^2s^{-1}$

angular mumentum after initial 8sec $= 6 + 3$

$= 9kgm^2s^{-1}$

angular mumentum after initial 28 sec $= 24\ kgm^2s^{-1}$

angular mumentum after initial 30 sec $= 24\ kgm^2s^{-1}$

$I\omega = 24$ here $I = \frac{MR^2}{2} = \frac{1}{2} \times 5 \times (0.3)^2 = 0.225 kgm^2$

$\therefore \omega = \frac{24}{I} = \frac{24}{0.225} = 106.7\ rad\ s^{-1}$

2. A body of mass m slides down an incline and reaches the bottom with a velocity v. If the same mass were in the form of a ring which rolls down this incline, the velocity of the ring at bottom would have been

(a) v
(b) $\sqrt{2}\,v$
(c) $\frac{1}{\sqrt{2}}v$
(d) $\sqrt{\frac{2}{5}}v$

Answer: c
Solution

When body of mass m slides down an inclined plane then $v = \sqrt{2gh}$

When it is in the form of ring then,

$$v_{Ring} = \sqrt{\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}} = \sqrt{\frac{2gh}{1+1}} = \frac{\sqrt{2gh}}{\sqrt{2}} = \frac{v}{\sqrt{2}}$$

3. Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio

(a) 1 : 2
(b) 2 : 1
(c) 2 : 1
(d) 1 : 2

Answer: d
Solution

$$\frac{L_1}{L_2} = \frac{\sqrt{2IK}}{\sqrt{2(2I)K}} = \frac{1}{\sqrt{2}}$$

4. The moment of inertia of a uniform circular disc of radius R and mass M about an axis touching the disc at its diameter and normal to the disc is

(a) MR^2
(b) $\frac{2}{5}MR^2$
(c) $\frac{3}{2}MR^2$
(d) $\frac{1}{2}MR^2$

Answer: c
Solution

$$I = I_{CM} + MR^2 = \frac{1}{2}MR^2 + MR^2$$

$$= \frac{3}{2}MR^2$$

5. When a disc is rotating with angular velocity ω, a particle situated at a distance of 4 cm just begins to slip. If the angular velocity is doubled, at what distance will the particle start to slip?

(a) 1 cm
(b) 2 cm
(c) 3 cm
(d) 4 cm

Answer: a
Solution

$$\therefore r \propto \frac{1}{w^2}$$

$$\frac{r_1}{r_2} = \frac{w_2^2}{w_1^2}$$

$$\frac{4}{r_2} = \frac{4w^2}{w^2}$$

$$r_2 = 1cm$$

6. A ring of mass 10 kg and diameter 0.4 m is rotated about its axis. If it makes 2100 revolution per minute, then its angular momentum will be

(a) 44 kg × m^2/s
(b) 88 kg × m^2/s
(c) 4.4 kg × m^2/s
(d) 0.4 kg × m^2/s

Answer: b
Solution

$I = mr^2 = 10 \times (0.2)^2 = 0.4$ kg–m^2

$\omega = 2\pi n = 2\pi \times \frac{2100}{60}$ rad/s

$\therefore L = I\omega = \frac{0.4 \times 2\pi \times 210}{6} = 88$ kg–m^2/s

7. A wheel rotates with a constant angular velocity of 300 rpm. The angle through which the wheel rotates in 1 s is

(a) π rad
(b) 5π rad
(c) 10π rad
(d) 20π rad

Answer: c

Solution

Angular velocity

$$V = \frac{300}{60} = 5rps$$

The angle deseribed by wheel

$$1s = 2\pi \times 5rad$$

$$= 10\pi\, rad$$

8. Two masses of 6 and 2 unit are at positions $(6\hat{\imath} - 7\hat{\jmath})$ and $(2\hat{\imath} + 5\hat{\jmath} - 8\hat{k})$, respectively. The coordinates of the centre of mass are

(a) (2, –5, 3)
(b) (5, –5, -3)
(c) (5, –4, –2)
(d) (5, –4, –4)

Answer: c

Solution

Given, masses $m_1 = 6$units, $m_2 = 2$unit

positions $6\hat{\imath} - 7\hat{\jmath}$ *and* $2\hat{\imath} + 5\hat{\jmath} - 8\hat{k}$

Centre of masss xx_{cm}, y_{cm} *and* z_{cm}

$(6,2) = (x_1, x_2)$

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{6 \times 6 + 2 \times 2}{6 + 2} = 5\hat{\imath}$$

$(-7,5) = (y_1, y_2)$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{6 \times (-7) + 2 \times (5)}{6 + 2} = -4\hat{\jmath}$$

$(0,-8) = (Z_1, Z_2)$

$$Z_{cm} = \frac{m_1z_1 + m_2z_2}{m_1 + m_2} = \frac{6 \times (0) + 2 \times (-8)}{6 + 2} = -2\hat{k}$$

$\therefore$ centre of mass lies on $5\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$

9. A force of $-F\hat{k}$ acts on O, the origin of the co-ordinate system. The torque about the point (1, –1) is

(a) $-F(\hat{\imath} + \hat{\jmath})$

(b) $F(\hat{\imath} + \hat{\jmath})$

(c) $-F(\hat{\imath} - \hat{\jmath})$

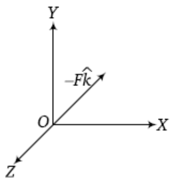
(d) $F(\hat{\imath} - \hat{\jmath})$

Answer: b

Solution

$\vec{F} = -F\hat{k}$

$\vec{r} = (\hat{i} - \hat{j})$



$$\vec{\tau} = \vec{r} \times \vec{F} = (\hat{i} - \hat{j}) \times (-F\hat{k})$$

$$= -F(\hat{i} \times \hat{k}) + F(\hat{j} \times \hat{k})$$

$$\Rightarrow \quad -F(-\hat{j}) + F(\hat{i}) \Rightarrow F\hat{j} + F\hat{i} = F(\hat{i} + \hat{j})$$

10. A thin uniform circular ring is rolling down an inclined plane of inclination 30° without slipping. Its linear acceleration along the inclined plane will be

(a) g/2
(b) g/3
(c) g/4
(d) 2g/3

Answer: c

Solution

$$a = \frac{g \sin\theta}{1 + \frac{K^2}{R^2}} = \frac{g \sin 30°}{1 + 1} = \frac{g}{4}$$

11. An artificial satellite is revolving round the earth in a circular orbit. Its velocity is half the escape velocity. Its height from earth\'s surface is

(a) 6400 km
(b) 12800 km
(c) 3200 km
(d) 1600 km

Answer: a

Solution

$$v = \sqrt{\frac{GM}{R + h}} = \frac{1}{2}\sqrt{\frac{2GM}{R}}$$

$$\Rightarrow 4R = 2\,(R + h)$$

$$\Rightarrow \quad h = R = 6400 \text{ km.}$$

12. Orbital velocity of an artificial satellite does not depend upon

(a) Mass of the earth
(b) Mass of the satellite
(c) Radius of the earth
(d) Acceleration due to gravity

Answer: b

Solution

$$v = \sqrt{\frac{GM}{r}}$$

13. At what distance from the centre of the earth, the value of acceleration due to gravity g will be half that on the surface (R = radius of earth)

(a) 2 R
(b) R
(c) 1.414 R
(d) 0.414 R

Answer: c

Solution

$$g' = g\left(\frac{R}{R + h}\right)^2 \Rightarrow \frac{1}{\sqrt{2}} = \frac{R}{R + h}$$

$$\Rightarrow R + h = \sqrt{2}\,R$$

$$\Rightarrow \quad h = (\sqrt{2} - 1)\,R = 0.414\,R$$

Hence, distance from centre $= R + 0.414\,R = 1.414\,R$

14. Escape velocity on the earth

(a) Is less than that on the moon
(b) Depends upon the mass of the body
(c) Depends upon the direction of projection
(d) Depends upon the height from which it is projected

Answer: d

Solution

$$v_e = \sqrt{\frac{2GM}{(R + h)}}$$

15. ~~In a satellite if the time of revolution is T, then K.E is proportional~~ to

(a) $\frac{1}{T}$

(b) $\frac{1}{T^2}$

(c) $\frac{1}{T^3}$

(d) $T^{-2/3}$

Answer: d

Solution

$v = \sqrt{\frac{GM}{r}} \therefore K.E. \propto v^2 \propto \frac{1}{r}$ and $T^2 \propto r^3$

$\therefore \quad K.E. \propto T^{-2/3}$.

16. A body is acted upon by a force towards a point. The magnitude of the force is inversely proportional to the square of the distance. The path of body will be

(a) Ellipse
(b) Hyperbola
(c) Circle
(d) Parabola

Answer: a

Solution

When a body is acted on by the force towards a point and the magnitude of force is inversely proportional to the square of distance. It means it obeys inverse square law and represents ellipse, for example path of the planet around the sun and the force acts between sun and planet proportional to $\frac{1}{r^2}$.

17. Assertion : During orbital motion of planet around the sun work done by the centripetal force is not zero at all points on the orbit.
Reason : Planet is revolving around the sun in elliptical orbit.

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but the reason is not the correct explanation of the assertion.
(c) If the assertion is true but the reason is false.
(d) If both assertion and reason are false.

Answer: a

Solution

During motion of a planet around sun, the centripetal force is not always perpendicular to the velocity of planet in an elliptical orbit. Thus work done is not zero. Although, incase of circular orbits centripetal force is always perpendicular to velocity.

18. An object weighs 72 N on earth. Its weight at a height of R/2 from earth is

(a) 32 N
(b) 56 N
(c) 72 N
(d) Zero

Answer: a

Solution

Acceleration due to gravity at a height h above the earth's surface is $g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

where g is the acceleration due to gravity on the earth's surface.

At $\quad h = \frac{R}{2}, g_h = \frac{g}{\left(1+\frac{R}{2R}\right)^2} = \frac{4g}{9}$

At $\quad h = R, g_h = \frac{g}{\left(1+\frac{R}{R}\right)^2} = \frac{g}{4}$

Acceleration due to gravity at a depth d below the earth's surface is

$$g_d = g\left(1-\frac{d}{R}\right)$$

At $\quad d = \frac{R}{2}, g_d = \left(1-\frac{2}{2R}\right) = \frac{g}{2}$

At the centre of earth, $d = R$

$$g_d = g\left(1-\frac{R}{R}\right) = 0$$

Thus, the acceleration due to gravity is maximum on the earth's surface.

19. ~~Two satellites of equal mass are revolving around earth in elliptical~~ orbits of different semi-major axis. If their angular momenta about earth centre are in the ratio 3 : 4 then ratio of their areal velocity is

(a) $\frac{3}{4}$

(b) $\frac{2}{3}$

(c) $\frac{1}{3}$

(d) $\frac{4}{3}$

Answer: a

Solution

Areal velocity, $\frac{\Delta A}{\Delta t} = \frac{|\vec{L}|}{2m} = v_A$

$\vec{L}$ is the angular momentum of satellite, m is the mass of satellite,

$\Rightarrow \quad \frac{v_{A1}}{v_{A2}} = \frac{|\vec{L}_1|}{|\vec{L}_2|} = \left(\frac{3}{4}\right)$

20. Two point masses m and 4m are separated by a distance d on a line. A third point mass m0 is to be placed at a point on the line such that the net gravitational force on it is zero. Image The distance of that point from the m mass is

(a) $\frac{d}{2}$

(b) $\frac{d}{4}$

(c) $\frac{d}{3}$

(d) $\frac{d}{5}$

Answer: c

Solution

Net force = 0

$\Rightarrow F_1 = F_2$

$$\frac{Gmm_0}{r^2} = \frac{4Gmm_0}{(d-r)^2}$$

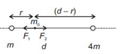
$\Rightarrow (d-r)^2 = (2r)^2$

$\Rightarrow d - r = 2r$

$\Rightarrow d = 3r$

Force of gravitation on m_0 due to $m = \frac{Gmm_0}{r^2} = F_1$

Force of gravitation on m_0 due to $4m = \frac{G4mm_0}{(d-r)^2} = F_2$



Thus, $r = \frac{d}{3}$

21. Air contains 79% N_2 and 21% $O_2$by volume. If the barometric pressure is 750mm Hg, the partial pressure of oxygen is

(a) 157.5 mm of Hg
(b) 175.5 mm of Hg
(c) 3125.0 mm of Hg
(d) None of these

Answer: a

Solution

$\therefore$ Partial pressure of $O_2 = 0.21 \times 750$

$= 157.5$ mm of Hg

22. A weather balloon filled with hydrogen at 1 atm and 27ºC has volume equal to12000 litres. On ascending it reaches a place where temperature is – 23ºCand pressure is 0.5 atm. The volume of the balloon is

(a) 24000 litres
(b) 20000 litres
(c) 10000 litres
(d) 12000 litres

Answer: b

Solution

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \frac{1 \times 12000}{300} = \frac{0.5 \times V_2}{250}, V_2 = 20000L$

23. Two glass bulbs A and B are connected by very small tube having a stop cock.Bulb A has a volume of 100 ml and contained the gas while bulb B was empty and had a volume of 150 ml. On opening the stop-cock, the pressure of the gas in bulb A will fall down to

(a) 0.8
(b) 0.6
(c) 0.4
(d) 20%.

Answer: c
Solution

$$\text{Now}, P_1V_1 = P_2V_2 \text{ i.e. } P \times 100 = P_2 \times 250$$

$$\text{or } P_2 = \frac{100}{250}P = 0.40\,P = 40\% \text{ of } P$$

24. If the density of air at 298 K and 101.325 kPa is 1.161 kg m^{-3}, then assuming air to behave like an ideal gas, the average molar mass of air (g mol^{-1}) is

(a) $\frac{1.161 \times 0.821 \times 298}{101.325}$

(b) $\frac{1.161 \times 8.314 \times 298}{101.325}$

(c) $\frac{1.161 \times 8.314 \times 298}{101.325 \times 10^3}$

(d) $\frac{1.161 \times 0.0821 \times 298}{101.325 \times 10^3}$

Answer: b
Solution

$$m = d\frac{RT}{P} = \frac{1.161 \times 8.314 \times 298}{101.325}$$

25. A gaseous mixture contains 56 g of N_2 , 44 g of CO_2 and 16 g of CH_4. The total pressure of mixture is 720 mm of Hg. The partial pressure of methane is

(a) 75 atm
(b) 160 atm
(c) 180 atm
(d) 215 atm

Answer: c
Solution

$\therefore$ number of moles(n) $= \frac{\text{mass(m)}}{\text{Molar mass(M)}}$

$\therefore$ The number moles of $N_2 = \frac{56}{28} = 2$mole

number of moles of $CO_2 = \frac{44}{44} = 1$mole

Number of moles of $CH_4 = \frac{16}{16} = 1$ mole

$\therefore$ The total number of moles in a gaseous mixture

$= 2 + 1 + 1 = 4$mole.

We know that,

Partial pressure = Total presssure × Mole fraction.

$\therefore$Partial pressure of CH_4 = Total pressure × mole fraction of CH_4

Total pressure × $\frac{\text{number of moles of } CH_4}{\text{Total number of moles of given mixture}}$

$= 720 \times \frac{1}{4}$

$\therefore$Partial pressure of $CH_4 = 180$atm.

26. For an ideal gas, number of mol per litre in terms of its pressure *p*, temperature *T* and gas constant *R* is

(a) pT / R
(b) pRT
(c) p / RT
(d) RT / p

Answer: c
Solution

Form ideal gas equation

$$pV = nRT$$

$$n = \frac{pV}{RT}$$

$$If\ V = 1L \text{ then } n = \frac{p}{RT}$$

27. The pressure and temperature of 4 dm^3 of carbon dioxide gas are doubled, then the volume of carbon dioxide gas would be

(a) 2 dm^3
(b) 3 dm^3
(c) 4 dm^3
(d) 8 dm^3

Answer: c
Solution

Given, $p_1 = p_1$ atm, $p_2 = 2p_1$ atm

$T_1 = T_1$, $T_2 = 2T_1$

$V_1 = 4dm^3$

$V_2 = ?$

From ideal gas equation,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\Rightarrow V_2 = \frac{p_1V_2T_2}{p_2T_1}$$

$$\Rightarrow V_2 = \frac{p_1 \times 4 \times 2T_1}{2p_1 \times T_1}$$

$$\Rightarrow V_2 = 4dm^3$$

28. Root mean square velocity of a gas molecule is proportional to

(a) $m^{1/2}$
(b) m^o
(c) $m^{-1/2}$
(d) m

Answer: c
Solution

$$PV = \frac{1}{2}\,m\,n\,u^2. \text{Hence } u^2 \propto \frac{1}{m} \text{ or } u \propto m^{-1/2}$$

29. If two molecules of *A* and *B* having mass 100 kg and 64 kg and rate of diffusion of *A* is 12×10^{-3} , then what will be the rate of diffusion of *B* ?

(a) 15×10^{-3}
(b) 64×10^{-3}
(c) 5×10^{-3}
(d) 46×10^{-3}

Answer: a
Solution

From Graham's law of diffusion,

$$\frac{r_1}{r_2} \propto \frac{\sqrt{M_2}}{\sqrt{M_1}} \propto \frac{\sqrt{d_2}}{\sqrt{d_1}}$$

$$Or\frac{r_1}{r_2} = \frac{\sqrt{M_2}}{\sqrt{M_1}} = \frac{\sqrt{d_2}}{\sqrt{d_1}} \rightarrow (1)$$

And Molecular weight = 2× V.D

$$V.D = \frac{M}{2}$$

By given data

$$V.D_A = \left(\frac{100}{2}\right)\frac{\text{kg}}{\text{molecule}}$$

$$V.B_B = \left(\frac{64}{2}\right)\text{kgmolecule}^{-1}$$

$$r_A = 12 \times 10^{-3}$$

$$r_B = ?$$

Substitute the values of M_A, M_B and r_A in equation (1),

$$\frac{12 \times 10^{-3}}{r_B} = \sqrt{\frac{64 \times 2}{100 \times 2}}$$

$$\frac{12 \times 10^{-3}}{r_B} = \sqrt{\frac{64}{100}}$$

$$\Rightarrow \frac{12 \times 10^{-3}}{r_B} = \frac{8}{10}$$

$$\Rightarrow r_B = \frac{10 \times 12 \times 10^{-3}}{8}$$

$$\Rightarrow r_B = 15 \times 10^{-3}$$

Therefore the diffusion of B is 15×10^{-3}

30. A gas is heated through 1°C in a closed vessel and so the pressure increases by 0.4%. The initial temperature of the gas was

(a) -23° C
(b) +23° C
(c) 250°C
(d) 523°C

Answer: a
Solution

Let, $T_1 = T \Rightarrow T_2 = (T+1)$

and $p_1 = p$ and $p_2 = p + \frac{0.4}{100} = \frac{100.4p}{100}$

From general ideal gas equation

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\Rightarrow \frac{pV}{T} = \frac{100.4p}{100} \times \frac{V}{T+1}$$

$$\Rightarrow \frac{1}{T} = \frac{100.4}{100} \times \frac{1}{T+1}$$

$$\Rightarrow 100T + 100 = 100.4T$$

$$\Rightarrow 100.4T - 100T = 100$$

$$\Rightarrow 0.4T = 100$$

$$\Rightarrow T = \frac{100}{0.4} = 250K$$

$$\Rightarrow (250 - 273)°C$$

$$\Rightarrow -23°C$$

31. For a reversible isothermal process in equilibrium, the entropy change is given by the expression

(a) $\Delta S = \frac{T}{q_{rev}}$

(b) $\Delta S = \frac{q_{rev}}{T}$

(c) $\Delta S = \frac{\Delta V}{T}$

(d) $\Delta S = \frac{\Delta E}{T}$

Answer: b
Solution

$$\Delta S = \frac{q_{rev}}{T}$$

32. For the reaction, $2H_2(g) + O_2(g) \rightarrow 2H_2O(g)$; $\Delta H = -573.2$ kJ
The heat of decomposition of water per mole is

(a) 286.6 kJ
(b) 573.2 kJ
(c) - 28.66 kJ
(d) zero

Answer: a
Solution

Consider the given reaction,

$2H_2(g) + O_2(g) \rightarrow 2H_2O(g)$; $\Delta H = -573.2$ KJ

Then , the decomposition of water is given as

$H_2O(g) \rightarrow H_2(g) + \frac{1}{2}O_2$; $\Delta H_1 = ?$

Comparing above two equations

$\Delta H_1 = \frac{573.2}{2} = 286.6$ KJ/mol.

∴ The heat of decomposition of water per mole is 286.6 KJ.

33. The bond dissociation energy of gaseous H_2, Cl_2 and HCl are 104, 58 and 103 kcal mol^{-1} respectively. The enthalpy of formation for HCl gas will be

(a) – 44.0 kcal
(b) – 22.0 kcal
(c) 22.0 kcal
(d) 44.0 kcal

Answer: b
Solution

$H_2(g) \rightarrow HCl(g)$; $\Delta H = 104\ Kcal \rightarrow (1)$

$Cl_2(g) \rightarrow 2Cl(g)$; $\Delta H = 58\ kcal \rightarrow (2)$

$HCl(g) \rightarrow H(g) + Cl(g)$; $\Delta H = 103\ kcal \rightarrow (3)$

Heat of formation for HCl

$H_2(g) + Cl_2(g) \rightarrow HCl(g)$; $\Delta H = ?$

Divide equation (1)and (2)by 2, and the add

$H_2(g) + cl_2 \rightarrow H(g) + Cl(g)$; $\Delta H = 81\ Kcal \rightarrow (4)$

subtracting equation (3)from equation (4)

$HCl(g) \rightarrow H(g) + Cl(g)$; $\Delta H = 103\ Kcal \rightarrow (3)$

$H_2(g) + Cl_2(g)\ HCl(g)$; $\Delta H = -22.0\ kcal$

∴ Enthalpy of formation of $HCl\ gas = -22.0\ Kcal$

34. The heat evolved in the combustion of benzene is given by
$C_6H_6 + 7\frac{1}{2}O_2 \rightarrow 6CO_2(g) + 3H_2O(l)$; $\Delta H = -3264.6KJ$
Which of the following quantities of heat energy will be evolved when $39C_6H_6$ are bumt

(a) 816.15 KJ
(b) 1632.3 KJ
(c) 6528.2 KJ
(d) 2448.45 KJ

Answer: b
Solution

78 g of benzene on combustion produces heat = −3264.6 KJ

$\therefore 39g\ will\ produce = \frac{-3264.6}{2} = -1632.3\ KJ$

35. The heat of combustion of solid benzoic acid at constant volume is – 321.30 kJ at 27°C. The heat of combustion at constant pressure is

(a) –321.30 – 300 R
(b) –321.30 + 300 R
(c) –321.30 – 150 R
(d) –321.30 + 900 R

Answer: c

Solution

$C_6H_5COOH(s) + \frac{15}{2}O_2(g) \rightarrow 7CO_2(g) + 3H_2O(l)$

$\Delta n_g = n_p - n_r = 7 - \frac{15}{2} = \frac{-1}{2}$

$q_p = q_v + \Delta n_g RT = -321.30 + \left(-\frac{1}{2}\right)300R$

$= -321.30 - 150\ R$

36. The heat evolved in the combustion of glucose ($C_6H_{12}O_6$) is given by the equation $(C_6H_{12}O_6)(s) + 6O_2(g) \rightarrow 6CO_2(g) + 6H_2O(g), \Delta H = -680$ kcal. The weight of $CO_2(g)$ produced when 170 kcal of heat is evolved in the combustion of glucose is

(a) 264 g
(b) 66 g
(c) 11g
(d) 44g

Answer: b

Solution

Evolution of 680 kcal is accompanied by

$CO_2 = 6 \times 44 = 264\ g$

Evolution of 170 kcal will be accompanied by

$CO_2 = \frac{264}{680} \times 170 = 66g$

37. If $S + O_2 \rightarrow SO_2, \Delta H = -298.2\ kJ$

$SO_2 + \frac{1}{2}O_2 \rightarrow SO_3, \Delta H = -98.7\ kJ$

$SO_3 + H_2O \rightarrow H_2SO_4, \Delta H = -130.2\ kJ$

$H_2 + \frac{1}{2}O_2 \rightarrow H_2O, \Delta H = -287.3\ kJ$

Then the enthalpy of formation of H_2SO_4 at 298 K will be

(a) – 814.4 kJ
(b) +320.5 kJ
(c) – 650. 3 kJ
(d) – 933.7 kJ.

Answer: a

Solution

Aim : $H_2 + S + 2O_2 \rightarrow H_2SO_4$

Operate Eqn. (iv) + Eqn. (i) + Eqn. (ii) + Eqn. (iii).

we get $\Delta H = -287.3 + (-298.2) + (-98.7)$

$+(-130.2) = -814.4\ kJ$

38. If $C(s) + O_2(g) \rightarrow CO_2(g)$; $\Delta H = r$ and $CO(g) + \frac{1}{2}O_2 \rightarrow CO_2(g)$; $\Delta H = s$, then the heat of formation of CO is

(a) r + s
(b) r - s
(c) s - r
(d) rs

Answer: b

Solution

Given,

$C(s) + O_2(g) \rightarrow CO_2(g); \Delta H = r$ ...(1)

$CO(g) + \frac{1}{2}O_2 \rightarrow CO_2(g); \Delta H = s$ ...(2)

Then formation of CO is given as

$C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g); \Delta H = ?$ ...(3)

If equation (1) and (2), we get

$C(s) + \frac{1}{2}O_2 \rightarrow CO(g); \Delta H = r - s$ ....(4)

Hence from equation (4) and (3), the heat of formation of CO is $\Delta H = r - s$.

39. A gas is allowed to expand under adiabatic conditions. What is zero for such a process ?

(a) ΔG
(b) ΔT
(c) ΔS
(d) none of these

Answer: c

Solution

For adiabatic expansion, $q = 0, \Delta S = \frac{q}{T} = 0,$

$\Delta T \neq \theta, \Delta G = RT\ \text{in}\ \frac{V_1}{V_2} \neq 0$

40. The difference between ΔH and ΔE on a molar basis for the combustion of n–octane at 25°C would be : 25°C

(a) – 13.6 kJ
(b) – 1.14 kJ
(c) – 11.15 kJ
(d) + 11.15 kJ

Answer: c

Solution

$\Delta H - \Delta E = -4.5 \times 8.315 \times 298\ J$

$= -11.15\ KJ$

41. Given that n is odd, the number of ways in which three numbers in A.P. can be selected from 1, 2, 3,....., n is

(a) $\frac{(n+1)^2}{4}$

(b) $\frac{(n-1)^2}{4}$

(c) $\frac{(n+1)^2}{2}$

(d) $\frac{(n-1)^2}{2}$

Answer: b

Solution

Let $n = 2m + 1$

For the three nos. in A.P., we have the following pattern.

Common difference	Nos.	Ways
1	(1, 2, 3), (2, 3, 4)... (n – 2, n – 1, n)	(n – 2)
2	(1, 3, 5), (2, 4, 6),... (n – 4, n – 2, n)	(n – 4)
3	(1, 4, 7), (2, 5, 8) (n – 6, n – 3, n)	(n – 6)
...	......	....
...	......	....
m	(1, m + 1, 2m + 1)	

Favourable no. of ways

$= \frac{(n-2)+(n-4)+(n-6)+\cdots+3+1}{(m\ \text{terms})}$

$= \frac{m}{2}(n-2+1) = \frac{m(n-1)}{2}$

$= \left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}\right) \quad \left[\because m = \frac{n-1}{2}\right]$

$= \frac{(n-1)^2}{4}$

42. ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7 =$

(a) 1
(b) 2
(c) 0
(d) None of these

Answer: c

Solution

${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7 = 0. \quad [\because {}^nC_r = {}^nC_{n-r}]$

43. A car will hold 2 in the front seat and 1 in the rear seat. If among 6 persons 2 can drive, than no. of ways in which the car can be filled is _______

(a) 10
(b) 20
(c) 30
(d) 40

Answer: b

Solution

Number of ways of section of drivers $= {}^2C_1 = 2$

Number of 2 seats from remaining 5 seats $= {}^5C_2 = 10$

$\therefore$ Total number of selection $= 2 \times 10 = 20$

44. From 12 books, the difference between number of ways a selection of 5 books when one specified book is always included is

(a) 64
(b) 118
(c) 132
(d) 330
(e) 462

Answer: d

Solution

$$^{11}C_4 = \frac{11.10.9.8}{1.2.3.4} = 330.$$

45. How many numbers of 6 digits can be formed from the digits of the number 112233 ?

(a) 30
(b) 60
(c) 90
(d) 120

Answer: c

Solution

$$\frac{6!}{2! \times 2! \times 2!} = 90$$

46. 12 Persons are to be arranged to a round table, If two particular persons among them are not to be side by side, the total no,of arrangements is : _______

(a) 9(10!)
(b) 2(10)!
(c) 45(8!)
(d) 10 !

Answer: a

Solution

The number of ways of 12 persons are to be arrangment around round table = 11!
The arrangement in which two particular persons are side by side = 10! (2!)
∴ Required arrangements
$= 11! - 10!(2!) = 10!(11-2)$
$= 9(10)!$

47. The set $S = \{1,2,3,\ldots\ldots,12\}$ is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S, A \cap B = B \cap C = A \cap C = \phi$. Then number of ways to partition is

(a) $\frac{12!}{3!(4!)^3}$

(b) $\frac{12!}{3!(3!)^4}$

(c) $\frac{12!}{(4!)^3}$

(d) $\frac{12!}{(3!)^4}$

Answer: c

Solution

Number of ways is $^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4 = \frac{12!}{(4!)^3}$.

48. The total number of ways in which 5 balls of different colors can be distributed among 3 persons so that each person gets atleast one ball, is

(a) 75
(b) 150
(c) 210
(d) 243

Answer: b

Solution

Number of ways to distribute 5 *balls are*

$$\left({}^5C_1 \cdot {}^4C_1 \cdot {}^3C_3 \times \frac{3!}{2!}\right) + \left({}^5C_1 \cdot {}^4C_2 \cdot {}^2C_2 \times \frac{3!}{2!}\right)$$

$= 90 + 60 = 150$

49. The sum $\sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-i}$, (where $\binom{p}{q} = 0$ if $p < q$), is maximum when m is

(a) 5
(b) 15
(c) 10
(d) 20

Answer: b

Solution

$^{10}C_0 \cdot {}^{20}C_m + {}^{10}C_1 \cdot {}^{20}C_{m-1} + {}^{10}C_2 \cdot {}^{20}C_{m-2}$
$+ \ldots {}^{10}C_m \cdot {}^{20}C_0$

Greatest coefficient is of middle term. So, value for m must be

$m = \frac{30}{2} = 15$

50. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent

(a) $6 \cdot 7 \cdot {}^8C_4$

(b) $6 \cdot 8 \cdot {}^7C_4$

(c) $7 \cdot {}^6C_4 \cdot {}^8C_4$

(d) $8 \cdot {}^6C_4 \cdot {}^7C_4$

Answer: c

Solution

×.×.×.×.×.×.×.×

Total letters = 11, No, of S's = 4

First place 7 letters at dot place other than S $= \frac{7!}{4!\,2!}$

Now place S at × places such that no two S are adjacent $= \frac{^8C_4}{4!} = {}^8C_4$

∴ Total no. of arrangements $= \frac{7!}{4!\,2!} \times {}^8C_4$

$= 7 \cdot {}^6C_4 \times {}^8C_4.$

51. First term of a G. P. of 2n terms is a, and the last term is l The product of all the terms of the G. P. is ______

(a) $(al)^{n/2}$
(b) $(al)^{n-1}$
(c) $(al)^{n}$
(d) $(al)^{2n}$

Answer: c

Solution

If r be the common ratio, then the product

of first 2n terms $= P = a(ar)(ar^2)\ldots\frac{1}{r}, 1 \to (1)$

$and\ also\ P = 1\left(\frac{1}{r}\right)\left(\frac{1}{r^2}\right)\ldots(ar)a \to (2)$

$(1) \times (2) \Rightarrow P^2 = (al)(al)(al)\ldots up\ to\ 2n\ factors$
$= (al)^{2n}$

$\Rightarrow P = (al)^n$

52. $\sum_{k=1}^{\infty}\frac{1}{k!}\left(\sum_{n=1}^{k}2^{n-1}\right)$ is equal to

(a) e
(b) $e^2 + e$
(c) e^2
(d) $e^2 - e$

Answer: d

Solution

$$\sum_{k=1}^{\infty}\frac{1}{k!}\left(\sum_{n=1}^{k}2^{n-1}\right) = \sum_{k=1}^{\infty}\frac{2^k}{k!} - \sum_{k=1}^{\infty}\frac{1}{k!}$$
$$= e^2 - 1 - (e-1)$$
$$= e^2 - e$$

53. The value of χ which satisfies $8^{1+\cos\chi+\cos^2\chi+\cdots} = 64$ in [-π, π] is

(a) ± π/2 , ± π/3
(b) ± π/3
(c) ± π/2 , ± π/6
(d) ± π/6 , ± π/3

Answer: b

Solution

$$8^{1+cosx+cosx^2+\cdots} = 8^2$$

$$\Rightarrow 8^{\frac{1}{1-cosx}} = 8^2$$

$$\Rightarrow \frac{1}{1-cosx} = 2$$

$$\Rightarrow cosx = \frac{1}{2} \Rightarrow x = \pm\frac{\pi}{3}$$

54. The first term of an A.P is 2 and common difference is 4.The sum of its 40 terms will be

(a) 3200
(b) 1600
(c) 200
(d) 2800

Answer: a

Solution

First terms $a = 2$ and common difference $d = 4$ and $n = 40$.

Then $S_n = \frac{n}{2}[2a + (n-1)d]$

$= 20[4 + 39 \times 4] = 20[4 + 156] = 160 \times 20 = 3200$

55. if a^2, b^2, c^2 are in AP, then which of the following is also an AP?

(a) sin A, sin B , sin C
(b) tan A, tan B, tan C
(c) cot A , cot B , cot C
(d) None of these

Answer: c

Solution

$sin^2B - sin^2A = sin^2C - sin^2B$

$\Rightarrow$ sin(B-A)sin(B+A)= sin(C-B)sin(C+B)

$\Rightarrow$sinC[sinBcosA-cosBsinA]

=sinA(sinCcosB-cosCsinB)

$\Rightarrow$2cotB =cot A+cotC

56. The sum of $1^3 + 2^3 + 3^3 + 4^3 + \cdots + 15^3$, is

(a) 22000
(b) 10000
(c) 14400
(d) 15000

Answer: c

Solution

Sum of cubes of 'n'natural number

$$= \frac{n^2(n+1)^2}{4} = \frac{15^2(16)^2}{4} = 14,400$$

57. $1 + \frac{3}{2!} + \frac{7}{3!} + \frac{15}{4!} + \cdots\infty =$

(a) $e(e+1)$
(b) $e(1-e)$
(c) $e(e-1)$
(d) $3e$

Answer: c

Solution

$T_n = \frac{2^n - 1}{n!} = \frac{2^n}{n!} - \frac{1}{n!}$

$T_1 = \frac{2^1}{1!} - \frac{1}{1!}$

$T_2 = \frac{2^2}{2!} - \frac{1}{2!}$

....

$\therefore T_1 + T_2 + T_3 + \cdots$

$= \left(\frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \cdots\right) - \left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\right)$

$= \left(1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \cdots\right) - \left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\right)$

$= e^2 - e$

$= e(e-1)$

58. If A_1, A_2 be two arithmetic means between $\frac{1}{3}$ and $\frac{1}{24}$ then their values are

(a) $\frac{7}{72}, \frac{5}{36}$
(b) $\frac{17}{72}, \frac{5}{36}$
(c) $\frac{7}{36}, \frac{5}{72}$
(d) $\frac{5}{72}, \frac{17}{72}$

Answer: b

Solution

Here $\frac{1}{3}, A_1, A_2, \frac{1}{24}$ will be in A.P.

then $A_1 - \frac{1}{3} = \frac{1}{24} - A_2 \Rightarrow A_{12} + A_2 = \frac{3}{8}$ ...(i)

Now, A_1 is a arithmetic mean of $\frac{1}{3}$ and A_2, we have

$2A_1 = \frac{1}{3} + A_2 \Rightarrow 2A_1 - A_2 = \frac{1}{3}$ ...(ii)

From (i) and (ii), we get $A_1 = \frac{17}{72}$ and $A_2 = \frac{5}{36}$

Aliter : As we have formula $A_m = a + \frac{m(b-a)}{n+1}$

where $n = 2, a = \frac{1}{3}, b = \frac{1}{24}$

$\therefore$ $A_1 = \frac{1}{3} + \frac{-7/24}{3} = \frac{17}{72}$

$A_2 = \frac{1}{3} + \frac{-14/24}{3} = \frac{10}{72} = \frac{5}{36}$

59. For an A. P...$S_{100} = 3\ S_{50}$ The value of $S_{150} : S_{50}$ =_______

(a) 8
(b) 3
(c) 6
(d) 10

Answer: c

Solution

$For\ an\ A.P.S_{100} = 3\ S_{50}$

$\Rightarrow \frac{100}{2}[2a + 99d] = 3.\frac{50}{2}[2a + 49d]$

$\Rightarrow 2a = 51d \rightarrow (1)$

$\therefore \frac{S_{150}}{S_{50}} = \frac{\frac{150}{2}[51d + 149d]}{\frac{50}{2}[51d + 49d]} = \frac{3(200d)}{100d} = 6$

60. The interior angles of a polygon are in A.P. If the smallest angle be 120^o and the common difference be 5°, then the number of sides of the polygon

(a) 8
(b) 10
(c) 9
(d) 6

Answer: c

Solution

Let the number of sides of the polygon be n.
Then the sum of interior angles of the polygon

$$= (2n-4)\frac{\pi}{2} = (n-2)\,\pi = (n-2)\,180°$$

Since the angles are in A.P. and $a = 120°$, $d = 5$

therefore $\frac{n}{2}[2 \times 120 + (n-1)\,5] = (n-2)\,180$

$\Rightarrow \quad n^2 - 25n + 144 = 0$

$\Rightarrow \quad (n-9)(n-16) = 0$

$\Rightarrow \quad n = 9, 16$

But $n = 16$ gives $T_{16} = a + 15d = 120° + 15 \cdot 5° = 195°$ which is impossible as interior angles cannot be greater than 180°. Hence $n = 9$.