Engineering Paper 2 - Chapter 3 and 4

1. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 Non the gun. How many bullets can he fire per second at the most?

(a) One
(b) Four
(c) Two
(d) Three

Answer: d

Solution

u = velocity of bullet

$\frac{dm}{dt}$ = Mass of bullet fired per second by the gun

$\frac{dm}{dt}$ = Mass of one bullet $(m_B) \times$ Bullets fired per sec (N)

Maximum force that man can exert $F = u\left(\frac{dm}{dt}\right)$

$\therefore \quad F = u \times m_B \times N$

$\Rightarrow \quad N = \frac{F}{m_B \times u} = \frac{144}{40 \times 10^{-3} \times 1200} = 3$

2. A rocket of mass 5700 kg ejects mass at a constant rate of 15 kg/s with constant speed of 12 km/s. The acceleration of the rocket 1 minute after the blast is (g = $10m/s^2$)

(a) 34.9 ms^2
(b) 27.5 ms^2
(c) 3.50 ms^2
(d) 13.5 ms^2

Answer: b

Solution

$F = \frac{vdm}{dt} - mg$ ...(i)

where m is mass of the rocket after 1 minute

So m = [5700 – 15 (60)]

= 4800 kg

in (i)

$F = (12 \times 10^3)(15) - (4800)g$

$= (12000)(15) - 48000$

$a = \frac{F}{m} = \frac{12000(15) - 48000}{4800}$

$= 27.5 \text{ m/s}^2$

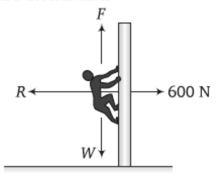
3. A fireman of mass 60 kg slides down a pole. He is pressing the pole with a force of 600 N. The coefficient of friction between the hands and the pole is 0.5, with what acceleration will the fireman slide down(g = 10 m/s^2)

(a) 1 m/s^2
(b) 2.5 m/s^2
(c) 10 m/s^2
(d) 5 m/s^2

Answer: d

Solution

Net downward acceleration



$= \frac{\text{Weight} - \text{Friction force}}{\text{Mass}}$

$= \frac{(mg - \mu R)}{m}$

$= \frac{60 \times 10 - 0.5 \times 600}{60} = \frac{300}{60} = 5 \text{ m/s}^2$

4. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them √3. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

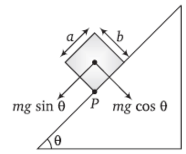
(a) At θ = 30°, the block will start sliding down the plane
(b) The block will remain at rest on the plane up to certain θ and then it will topple
(c) At θ = 60°, the block will start sliding down the plane and continue to do so at higher angles
(d) At θ = 60°, the block will start sliding down the plane and on further increasing 0, it will topple at certain 0

Answer: b

Solution

For rotational equilibrium about point 'P',

$mg \sin\theta\left(\frac{b}{2}\right) = mg\cos\theta\left(\frac{a}{2}\right)$



$\Rightarrow \quad \tan\theta = \frac{a}{b} = \frac{10}{15} = \frac{2}{3}$

$\Rightarrow \quad \theta = 33.69°$

i. e., toppling starts at θ = 33.69°

and angle of repose = $\tan^{-1}(\mu)\tan^{-1}(\sqrt{3}) = 60°$

It mean the block will remain at rest on the plane up to certain angle θ and then it will topple.

5. The acceleration of system of two bodies over thewedge as shown in

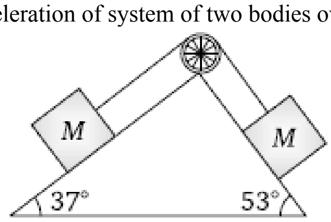
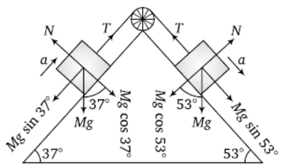


figure is

(a) 1 ms^{-2}
(b) 2 ms^{-2}
(c) 0.5 ms^{-2}
(d) 10 ms^{-2}

Answer: a

Solution



Let T be the tension in the string.

Let a be acceleration of the system.

The equations of motion are

$Ma = Mg\sin 53° - T$ ...(i)

and $Ma = T - Mg\sin 37°$ ...(ii)

Adding Eqs. (i) and (ii), we get

$a = \frac{Mg(\sin 53° - \sin 37°)}{2M}$

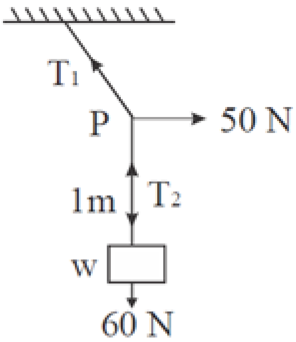
$= g\cos 45° \sin 8°$

$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$= 10 \times \frac{1}{\sqrt{2}} \times 0.139$

$= 0.98 \text{ ms}^{-2} \approx 1 \text{ ms}^{-2}$

6. A mass of 6 kg is suspended by a rope of length 2 m form the ceiling. A force of 50 N in the horizontal direction is applied at the mid Point P of the rope- as shown in figure. what is the angle the rope makes with the vertical in equilibrium ? (g = 10 ms^{-2}) Neglect



mass of the rope.

(a) 40^o
(b) 30^o
(c) 35^o
(d) 45^o

Answer: a
Solution

$T_1 \sin\theta = 50$ N and $T_1 \cos\theta = 60$ N

$\therefore \tan\theta = 5/4 \ \therefore \theta = 40^0$

7. A box is lying on a inclined plane what is the coefficient of static friction, if the box starts sliding when an angle of inclination is 60°

(a) 1.173
(b) 1.732
(c) 2.732
(d) 1.677

Answer: b
Solution

$\mu = \tan$ (Angle of repose) $= \tan 60° = 1.732$

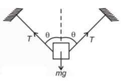
8. A weight Mg is suspended from the middle of a rope whose ends are at the same level. The rope is no longer horizontal. The minimum tension required to completely straighten the rope is

(a) $\frac{Mg}{2}$
(b) $Mg\cos\theta$
(c) $2Mg\cos\theta$
(d) Infinitely large

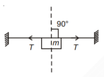
Answer: d
Solution

$2T\cos\theta = mg$

$T = \frac{mg}{2\cos\theta}$ ...(i)



To make this string completely straight



$\theta = 90°$

in (i) put $\theta = 90°$

$T = \frac{mg}{2\cos 90°} \approx \infty$

9. A block of mass 50 kg slides over a horizontal distance of 1 m. If the coefficient of friction between their surfaces is 0.2, then work done against friction is

(a) 98 J
(b) 72 J
(c) 56 J
(d) 34 J

Answer: a
Solution

$W = \mu mgS = 0.2 \times 50 \times 9.8 \times 1 = 98$ J

10. Three solids of masses m1, m2 and m3 are connected with weightless string in succession and are placed on a frictionless table. If the mass m3 is dragged with a force T, the tension in the string between m2 and m3 is

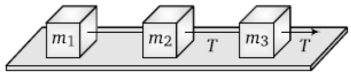
(a) $\frac{m_2}{m_1+m_2+m_3}T$

(b) $\frac{m_3}{m_1+m_2+m_3}T'$

(c) $\frac{m_1+m_2}{m_1+m_2+m_3}T$

(d) $\frac{m_2+m_3}{m_1+m_2+m_3}T$

Answer: c
Solution



$T' = (m_1 + m_2) \times \frac{T}{m_1 + m_2 + m_3}$

11. A particle of mass m moving with velocity V_o strikes a simple pendulum of mass m and sticks to it. The maximum height attained by the pendulum will be

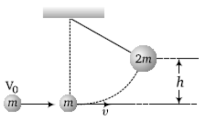
(a) $h = \frac{V_0^2}{8g}$

(b) $\sqrt{V_0 g}$

(c) $2\sqrt{\frac{V_0}{g}}$

(d) $\frac{V_0^2}{4g}$

Answer: a
Solution



Initial momentum of particle $= mV_0$

Final momentum of system (particle + pendulum) $= 2mv$

By the law of conservation of momentum.

$\Rightarrow \quad mV_0 = 2\,mv$

$\Rightarrow$ Initial velocity of system $v = \frac{V_0}{2}$

$\therefore$ Initial K.E. of the system

$= \frac{1}{2}(2m)v^2 = \frac{1}{2}(2m)\left(\frac{V_0}{2}\right)^2$

If the system rises up to height h, then P.E. $= 2mgh$

By the law of conservation of energy

$\frac{1}{2}(2m)\left(\frac{V_0}{2}\right)^2 = 2mgh \Rightarrow h = \frac{V_0^2}{8g}$

12. A rod AB of mass 10 kg and length 4 m rests on a horizontal floor with end A fixed so, as to rotate it in vertical plane about perpendicular axis passing through A. If the work done on the rod is 100 J, the height to which the end B be raised vertically above the floor is

(a) 1.5 m
(b) 2 m
(c) 1 m
(d) 2.5 m
(e) 3 m

Answer: b

Solution

$$w = 100J$$

$$m = 10kg$$

$$\text{work done} = \frac{mgh}{2}$$

$$100 = \frac{10 \times 10 \times h}{2}$$

$$\Rightarrow h = 2m$$

13. A nucleus at rest splits into two nuclear parts having same density and radii in the ratio 1: 2. Their velocities are in the ratio

(a) 2 : 1
(b) 4 : 1
(c) 6 : 1
(d) 8 : 1

Answer: d

Solution

Let a nucleus of mass M splits into two nuclear parts having masses M_1 and M_2 and radii R_1 and R_2 and densities ρ_1 and ρ_2.

$\therefore \quad M_1 = \rho_1 \frac{4}{3}\pi R_1^3$ and $M_2 = \rho_2 \frac{4}{3}\pi R_2^3$

Given : $\quad \rho_1 = \rho_2$

$\therefore \quad \frac{M_1}{M_2} = \left(\frac{R_1}{R_2}\right)^3$

According to law conservation of linear momentum,

$M \times 0 = M_1 v_1 + M_2 v_2$

or $\quad \frac{M_1}{M_2} = -\frac{v_2}{v_1}$

–ve sign show that both the parts are move in opposite direction in order to conserve the linear momentum

$\therefore \quad \frac{v_1}{v_2} = \frac{M_2}{M_1}$ or $\frac{v_1}{v_2} = \left(\frac{R_2}{R_1}\right)^3$

$\frac{v_1}{v_2} = \left(\frac{2}{1}\right)^3 = \frac{8}{1} \qquad \left[\text{Given } \frac{R_1}{R_2} = \frac{1}{2}\right]$

14. A body of mass `M\' collides against a wall with a velocity v and retraces its path with the same speed. The change in momentum is (take initial direction of velocity as positive)

(a) zero
(b) 2Mv
(c) Mv
(d) –2Mv

Answer: d

Solution

Change in momentum

$$= m\vec{v}_2 - m\vec{v}_1 = -mv - mv = -2mv$$

15. A position dependent force F =7 – 2x + 3x^2 N acts on a small body of mass 2 kg and displaces it from x = 0 to x = 5 m. The work done in joules is

(a) 70
(b) 270
(c) 35
(d) 135

Answer: d

Solution

$W = \int_0^5 F dx = \int_0^5 (7 - 2x + 3x^2)\, dx = (7x - x^2 + x^3)_0^5$

$= 35 - 25 + 125 = 135\text{ J}$

16. Two trolleys of mass m and 3 m are connected by a spring. They were compressed and released once, they move off in opposite direction and comes to rest after covering distances S_1 and S_2 respectively. Assuming the coefficient of friction to be uniform, the ratio of distances $S_1 : S_2$ is

(a) 1 : 9
(b) 1 : 3
(c) 3 : 1
(d) 9 : 1

Answer: d

Solution

When trolley are released then they possess same linear momentum but in opposite direction. Kinetic energy acquired by any trolley will dissipate against friction.

$\therefore \quad \mu\, mg\, s = \frac{p^2}{2m} \Rightarrow s \propto \frac{1}{m^2}$

[As P and u are constants]

$\Rightarrow \quad \frac{s_1}{s_2} = \left(\frac{m_2}{m_1}\right)^2 = \left(\frac{3}{1}\right)^2 = \frac{9}{1}$

17. A ball dropped from a height of 2 m rebounds to a height of 1.5 m after hitting the ground. Then, the percentage of energy lost is

(a) 25
(b) 30
(c) 50
(d) 100

Answer: a

Solution

$$\%\text{ of energy loss} = \frac{mg(2-1.5)}{mgh} \times 100$$

$$= \frac{mg(0.5)}{mg \times 2} \times 100 = 25\%$$

18. A metal ball of mass 2 kg moving with a velocity of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after the collision, the two balls move togather, the loss in kinetic energy due to collision is

(a) 40 J
(b) 60 J
(c) 100 J
(d) 140 J

Answer: b

Solution

By low of conservation of momentum

$$2 \times 10 = (2+3)\text{v} \therefore \text{v} = 4\frac{\text{m}}{\text{s}}$$

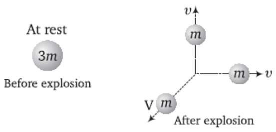
$$\text{Loss in K.E} = \frac{1}{2}(2)(10)^2 - \frac{1}{2}(5)(4)^2$$

19. An object of mass 3m splits into three equal fragments. Two fragments have velocities $v\hat{j}$ *and* $v\hat{i}$. The velocity of the third fragment is

(a) $v(\hat{j} - \hat{i})$
(b) $v(\hat{i} - \hat{j})$
(c) $-v(\hat{i} + \hat{j})$
(d) $\frac{v(\hat{i} + \hat{j})}{\sqrt{2}}$

Answer: c

Solution



Initial momentum of 3m mass = 0 ...(i)

Due to explosion this mass splits into three fragments of equal masses.

Final momentum of system $= m\vec{V} + mv\hat{i} + mv\hat{j}$ ...(ii)

By the law of conservation of linear momentum

$m\vec{V} + mv\hat{i} + m\vec{v}\,\hat{j} = 0 \Rightarrow \vec{V} = -v(\hat{i} + \hat{j})$

20. A shell of mass m moving with a velocity v breakes up suddenly into two pieces. The part having mass $\frac{m}{3}$ remains stationary. The velocity of the other part will be

(a) v
(b) 2 v
(c) $\frac{2}{3}v$
(d) $\frac{3}{2}v$

Answer: d
Solution

Momentum will be conserved

$P_i = P_f$

$mv = \frac{m}{3}(0) + \frac{2m}{3}(v^1)$

$v^1 = \frac{3}{2}v$

21. The correct order of size of iodine species is

(a) $I > I^- > I^+$
(b) $I^- > I > I^+$
(c) $I^+ > I > I^-$
(d) $I^- > I^+ > I$

Answer: b
Solution
Order of size of iodine species

$I^- > I > I^+$

Size of anion is larger than its neutral atom and size of cation is smaller than neutral atom.

22. The ionization of hydrogen atom would give rise to

(a) Hydride ion
(b) Hydronium ion
(c) Proton
(d) Hydroxylion

Answer: c
Solution

$$H(g) \xrightarrow{IE} H^+(g) + e^-$$

23. The ionization enthalpies of Li and sodium are 520 kJ mol^{-1} and 495 kJ mol^{-1} respectively. The energy required to convert all the atoms present in 7 mg of Li vapours and 23 mg of sodium vapours to their respective gaseous cations respectively are

(a) 52 J, 49.5 J
(b) 520J, 495 J
(c) 49.5 J, 52 J
(d) 495 J, 520 J

Answer: b
Solution

No. of moles of Li $= \frac{7}{1000 \times 7} = 10^{-3}$

No. of moles of Na $= \frac{23}{1000 \times 23} = 10^{-3}$

∴ The amounts of energies required for 10^{-3} mole each of Li and Na are 520 kJ × 10^{-3} and 495 kJ × 10^{-3} or 520 J and 495J respectively.

24. Which of the following is an amphoteric oxide?

(a) SO_3
(b) MgO
(c) Al_2O_3
(d) P_4O_{10}

Answer: c
Solution

$Al_2O_3 + 6HCl \rightarrow 2AlCl_3 + 3H_2O$

$Al_2O_3 + 2NaOH \rightarrow 2NaAlO_2 + H_2O$

Al_2O_3 acts as amphoteric oxide.

25. The electronic configurations of four elements are given below. Arrange these elements in the correct order of the magnitude (without sign) of their electron affinity.

I. $2s^2 2p^5$
II. $3s^2 3p^5$
III. $2s^2 2p^4$
IV. $3s^2 3p^4$

Select the correct answer using the codes given below

(a) I < II < IV < III
(b) II < I < IV < III
(c) I < III < IV < II
(d) III < IV< I < II

Answer: d
Solution
Order of electron affinity is

$2s^2 2p^4 < 3s^2 3p^4 < 2s^2, 2p^5 < 3s^2 3p^5$

Electron affinity decreases with increases in size of atom since nuclear attraction decreases down a group. Electron affinity increases as we move along period.

26. Which of the following statements is wrong ?

(a) The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table
(b) Nitrogen cannot form dπ—pπ bond
(c) Single N—N bond is weaker than the single P—P bond
(d) N_2O_4 has two resonance structure

Answer: a
Solution
Stability of hydrides decreases on moving top to bottom in periodic table.

$BiH_3 < SbH_3 < AsH_3 PH_3 < NH_3$

$BiH_3 \rightarrow$ least stable

$NH_3 \rightarrow$ most stable.

27. Which of the following oxides doesn't react with both of an acid and alkali, is?

(a) ZnO
(b) SnO_2
(c) Al_2O_3
(d) BeO

Answer: d
Solution
BeO is basic oxide. ZnO, SnO_2 , Al_2O_3 are amphoteric oxides.

28. One mole of magnesium in the vapour state absorbed 1200 $kJmo1^{-1}$ of energy. If the first and second ionisation energies of Mg are 750 and 1450 $kJmo1^{-1}$ respectively, the final composition of the mixture is

(a) 31% Mg^+ + 69% Mg^{2+}
(b) 69% Mg^+ + 31% Mg^{2+}
(c) 86% Mg^+ + 14% Mg^{2+}
(d) 14% Mg^+ + 86% Mg^{2+}
(e) 13% Mg^+ + 87% Mg^{2+}

Answer: b
Solution
$Mg \rightarrow Mg^+$, E = 750KJ

Remaining energy = 1200-750

= 450KJ

Energy needed to convert 1 mole of Mg^+ to Mg^{2+} =1450

The percentage of number of moles of Mg^{2+} produced

$= \frac{1}{1450} \times 450 = 31\%$

∴Percentage no. of moles of Mg^{2+}

= 1-0.31 = 0.69

= 69%

29. The electron affinity values (in kJ mol^{-1}) of three halogens X, Y and Z are respectively -349, - 333 and -325. Then, X, Y and Z respectively, are

(a) F_2 , Cl_2 and Br_2
(b) Cl_2 , F_2 and Br_2
(c) Cl_2 , Br_2 and F_2
(d) Br_2 , Cl_2 and F_2

Answer: b
Solution
Order of electron affinity

$Cl_2 > F_2 > Br_2$

Chlorine has highest electron affinity.

30. The element with atomic number 117 if discovered would be placed in

(a) noble gas family
(b) alkali family
(c) alkaline earth family
(d) halogen family

Answer: d
Solution
$117 = [Rn]5f^{14}6d^{10}7s^27p^5$

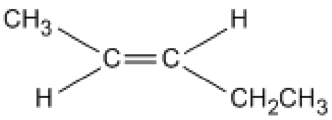
Group number = 5+2 = 7

So element will be placed in halogen family.

31. Which of the following molecule possesses dipole movement ?

(a) trans - 1, 2 - dichloro ethene
(b) trans pent - 2 - ene
(c) trans pent - 2 - ene
(d) 2, 2, 3, 3- tetra methyl butane

Answer: b
Solution



trans-pent-2-ene
possess magnetic moment

32. Which of the following is not paramagnetic?

(a) N_2^+
(b) CO
(c) O_2^-
(d) NO

Answer: b
Solution
N_2^+ (13), O_2^- (17). NO (15), since they

have odd number of electrons.

33. What type of hybridisation takes place in the N - atom Of NH_3 ?

(a) sp^2
(b) sp^3
(c) dsp^2
(d) sp

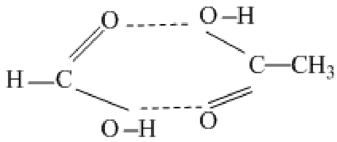
Answer: b
Solution

$N = 1s^2 2s^2 2P_x^1 2P_y^1 2P_z^1$

34. The pair of molecules forming strongest hydrogen bonds are

(a) SiH_4 and SiF_6
(b) $CH_3-C(=O)-CH_3$ abd $CHCl_3$
(c) $H-C-OH$ and $CH_3-C(=O)-OH$
(d) H_2O and H_2

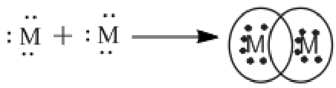
Answer: c
Solution



35. The electronic configurations of four elements are given in brackets
L ($1s^22s^2 p^1$), M ($1s^22s^2 2p^5$)
Q ($1s^22s^2 2 p^6 3s^1$), R ($1s^22s^2 2p^2$)
diatomic molecule is

(a) Q
(b) M
(c) R
(d) L

Answer: b
Solution



36. The type of bonding in HCl molecule is

(a) Pure covalent
(b) Polar covalent
(c) Highly polar
(d) Hydrogen bonding

Answer: b
Solution
δ+ δ–
H – Cl has polar covalent bond

37. Which of the following molecule has lowest bond space angle ?

(a) NH_3
(b) SO_2
(c) H_2O
(d) H_2S

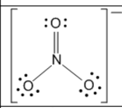
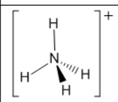
Answer: d
Solution

Compound	SO_2	H_2O	H_2S	NH_3
Bond angle	119.5°	104.5°	92.5°	106.5°

38. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively

(a) sp, sp^2 , sp^3
(b) sp^2 ,sp, sp^3
(c) sp, sp^3 , sp^2
(d) sp^2 , sp^3 , sp

Answer: b
Solution

Compound	Structure	Hybridization of N atom
NO_3^-		SP^2
NO_2^+	$\ddot{O}=\overset{+}{N}=\ddot{O}$	SP
NH_4^+		SP^3

∴ The hybridization of orbitals of N – atom in

NO_3^-, NO_2^+ and NH_4^+ are SP^2 , SP and SP^3

respectively.

39. Bond angle of 109°28' is found in

(a) NH_3
(b) H_2O
(c) $\overset{\oplus}{CH_5}$
(d) $\overset{\oplus}{NH_4}$

Answer: d
Solution

Compound	Bond angle
NH_3	107°
H_2O	104.5°
CH_5^+	≃ 107.8° (unstable)
NH_4^+	109°28′

40. The hybridization of S atom in SO_2 is

(a) sp
(b) sp^2
(c) sp^3
(d) sp^3d

Answer: b
Solution

Total no. of valence electrons is

$SO_2 = 6 + 2 \times 6 = 18$

Now $18 \div 8 = 2(Q_1) + 2(R_1)$ and $2 \div 2 = 1(Q_2) + 0(R_2)$

Thus, sum of $Q_1 + Q_2 + R_2 = 2 + 1 + 0 = 3$

and hence type of hybridization in sp^2

41. Let complex numbers a and 1/a lie on circles $(x — x_0)^2 + (y — y_0)^2 = r^2$ and $(x - x_0)^2 (y - y_0)^2 4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then |a|is equal to

(a) 1/√2
(b) 01-Feb
(c) 1/√7
(d) 01-Mar

Answer: c
Solution

$(x - x_o)^2 + (y - y_o)^2 = r^2$

$(x - x_o)^2 + (y - y_o)^2 = 4r^2$

$|z - Z_o|^2 = r^2 \text{ and } |z - Z_o| = 4r^2$

$\alpha \text{ and } \frac{1}{\alpha} \text{ lies on first and second circles respectively}$

$\therefore |\alpha - Z_o|^2 = r^2;$

$\left|\frac{1}{\alpha} - Z_o\right|^2 = 4r^2$

$(\alpha - Z_o)(\bar{\alpha} - \overline{Z_o}) = r^2$

$|\alpha|^2 - \overline{Z_o}\bar{\alpha} - \overline{Z_o}\alpha + |Z_o|^2 = r^2 \rightarrow (1)$

$\left|\frac{1}{\alpha} - Z_o\right|^2 = 4r^2$

$\left(\frac{1}{\alpha} - Z_o\right)\left(\frac{1}{\alpha} - \overline{Z_o}\right) = 4r^2$

$\therefore \frac{1}{|\alpha|^2} - \frac{Z_o}{\alpha} - \frac{\overline{Z_o}}{\alpha} + |Z_o|^2 = 4r^2|\alpha| \rightarrow (2)$

$(1) - (2); we\ get$

$\therefore (|\alpha| - 1) + |Z_o|^2(1 - |\alpha|^2) = r^2(1 - 4|\alpha|^2)$

$(|\alpha|^2 - 1)\left[1 - \frac{r^2 = 2}{2}\right] = r^2(1 - 4|\alpha|^2)$

$\therefore (|\alpha|^2 - 1) = -2 + 8|\alpha|^2$

$\therefore 7|\alpha|^2 = 1$

$\therefore |\alpha| = \frac{1}{\sqrt{7}}$

42. The imaginary part of $\cosh(\alpha + i\beta)$ is

(a) $\cosh\alpha \cos\beta$
(b) $\sinh\alpha \sin\beta$
(c) $\cos\alpha \cosh\beta$
(d) $\cos\alpha \cos\beta$

Answer: b
Solution

$\cosh(\alpha + i\beta) = \cosh\alpha \cosh(i\beta) + \sinh\alpha \sinh(i\beta)$

Imaginary part $= \sinh\alpha \sin\beta$

43. The value of $\sum_{k=1}^{10}(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11})$ is

(a) 1
(b) -1
(c) #NAME?
(d) i

Answer: c
Solution

$$\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$$

$$i\sum_{k=1}^{10}\left(\cos\frac{2k\pi}{11} - i\sin\frac{2k\pi}{11}\right)$$

$$[\because \sin\theta + i\cos\theta = i(\cos\theta - i\sin\theta)]$$

$$i\sum_{k=1}^{10}\left(e^{-\frac{2k\pi i}{11}}\right)$$

$$= i\sum_{k=1}^{10} r^k; where\ r = e^{-\frac{2\pi i}{11}}$$

$$i(r + r^2 + r^3 + \cdots + r^{10})$$

$$= \frac{i.r(r^{10} - 1)}{r - 1}$$

$$\left[sin = a\left(\frac{r^n - 1}{r - 1}\right)\right]$$

$$\frac{i(r^{11} - r)}{r - 1} = \frac{i(1 - r)}{r - 1}$$

$$\left[\therefore r^{11} = \left(e^{-\frac{2\pi i}{11}}\right)^{11} = e^{-2\pi i} = 1\right]$$

$$= -i$$

44. If ω is a complex cube root of unity, then $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8)$ is equal to

(a) 0
(b) 1
(c) −1
(d) 9

Answer: d
Solution

$$(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8)$$

$$= (1 - \omega)(1 - \omega^2)(1 - \omega)(1 - \omega^2)$$

$$= (1 - \omega)^2(1 - \omega^2)^2$$

$$= (-3\omega)(-3\omega^2) = 9\omega^3 = 9$$

45. If the imaginary part of $\frac{2z-3}{iz+1}$ is -2 then the locus of the point representing Z in the complex plane is

(a) A circle
(b) A straight line
(c) A parabola
(d) An ellipse

Answer: b

Solution

$$\frac{2z-3}{iz+1} = \frac{2(x+iy)-3}{i(x+iy)+1}$$

$$= \frac{(2x-3)+2yi}{(1-y)+ix}$$

$$= \frac{[(2x-3)+i\,2y][(1-y)-ix]}{(1-y)^2+x^2}$$

$$\therefore \text{Im}\left(\frac{2z-3}{iz+1}\right) = \frac{2y(1-y)-x(2x-3)}{x^2+(1-y)^2} = -2$$

$$\therefore 3x-2y+2=0$$

Which is represent equation of line.

46. Let z be a non−real complex number lying on the circle | z | = 1. Then z is equal to

(a) $\dfrac{1-i\tan\left(\frac{\arg z}{2}\right)}{1+i\tan\left(\frac{\arg z}{2}\right)}$

(b) $\dfrac{1+i\tan\left(\frac{\arg z}{2}\right)}{1-i\tan\left(\frac{\arg z}{2}\right)}$

(c) $\dfrac{2-i\tan\left(\frac{\arg z}{2}\right)}{2+i\tan\left(\frac{\arg z}{2}\right)}$

(d) none of these

Answer: b

Solution

$Since\ |z| = 1 \therefore z = \cos\alpha + i\sin\alpha$

$Where \arg z = \alpha$

$$Now\ z = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} + i.\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

$$= \frac{1-\tan^2\frac{\alpha}{2}+2i\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{\left(1+i\tan\frac{\alpha}{2}\right)^2}{\left(1+i\tan\frac{\alpha}{2}\right)\left(1-i\tan\frac{\alpha}{2}\right)}$$

$$= \frac{1+i\tan\frac{\alpha}{2}}{1-i\tan\frac{\alpha}{2}} = \frac{1+i\tan\left(\frac{\arg z}{2}\right)}{1-i\tan\left(\frac{\arg z}{2}\right)}$$

47. If z is a complex number, then the minimum value of $|z|+|z-1|$ is

(a) 1
(b) 0
(c) 01-Feb
(d) None of these

Answer: a

Solution

First note that $|-z| = |z|$ and $|z_1+z_2| \le |z_1|+|z_2|$

Now $|z|+|z-1| = |z|+|1-z| \ge |z+(1-z)|$

$= |1| = 1$

Hence, minimum value of $|z|+|z-1|$ is 1.

48. If $iz^4 + 1 = 0$, then z can take the value

(a) $\dfrac{1+i}{\sqrt{2}}$
(b) $\cos\pi/8 + i\sin\pi/8$
(c) 1/4i
(d) i

Answer: b

Solution

$$iZ^4 + 1 = 0$$

$$\therefore iZ^4 = -1$$

$$Z^4 = -\frac{1}{i} = i$$

$$\therefore Z = e^{\frac{1}{4}}$$

$$Z = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{\frac{1}{4}}$$

$$Z = \cos\left(\frac{\pi}{4}\times\frac{1}{4}\right) + i\sin\left(\frac{\pi}{2}\times\frac{1}{2}\right)$$

$$Z = \cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$$

49. Let z_1 and z_2 be two complex numbers with α and β as their principal arguments such that $\alpha+\beta > \pi$, then principal $\arg(z_1z_2)$ is given by

(a) $\alpha+\beta+\pi$

(b) $\alpha+\beta-\pi$

(c) $\alpha+\beta-2\pi$

(d) $\alpha+\beta$

Answer: c

Solution

We know that principal arguments of a complex number lie between $-\pi$ and π, but $\alpha+\beta > \pi$, therefore principal $\arg(z_1z_2) = \arg z_1 + \arg z_2$ $= \alpha+\beta$, is given by $\alpha+\beta-2\pi$.

50. If 1, ω and ω^2 are the cube roots of unity, $(3 + \omega^2 + \omega^4)^6$ is equal to

(a) 64
(b) 729
(c) 2
(d) 0

Answer: a

Solution

$$(3+w^2+w)^6$$

$$= (3-1)^6$$

$$= 64$$

51. If α, β are the roots of the equation $x^2 + ax + b = 0$ then the value of $\alpha^3 + \beta^3$ is equal to

(a) $-(a^3 + 3ab)$

(b) $a^3 + 3ab$

(c) $-a^3 + 3ab$

(d) $a^3 - 3ab$

Answer: c

Solution

Sum of root $\alpha + \beta = -a$ and product of roots $\alpha\beta = b$

So, $\alpha^3 + \beta^2 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta] = -aa(a^2 - 3b)$

$= -a^3 + 3ab.$

52. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$

(a) Is always positive
(b) Is always negative
(c) Does not exist
(d) None of these

Answer: c

Solution

Note that for $t = R, t^2x^2 + |x| + 9 \geq 9$ an hence

the given equation can not have real roots.

53. The only value of x satisfying the equation $\sqrt{\frac{x}{x+2}} - \sqrt{\frac{x+2}{x}} = \frac{3}{2}$ is ________

(a) $\frac{8}{3}$

(b) $\frac{-8}{3}$

(c) -4

(d) 4

Answer: b

Solution

$$\sqrt{\frac{x}{x+2}} - \sqrt{\frac{x+2}{2}} = \frac{3}{2} = 2 - \frac{1}{2}$$

$$\therefore \sqrt{\frac{x}{x+2}} = 2 \ \therefore x = \frac{-8}{3}$$

54. If the roots of the equation $b\chi^2 + c\chi + a = 0$ is imaginary, then for all real values of χ, the expression $3b^2\chi^2 + 6bc\chi + 2c^2$ is

(a) greater than 4ab
(b) less than 4ab
(c) greater than – 4ab
(d) less than – 4ab

Answer: c

Solution

$Given\ bx^2 + cx + a = 0\ has\ imaginary\ roots$

$i.e\ \ c^2 - 4ab < 0 \Rightarrow c^2 < 4ab$

$Let\ \ f(x) = 3b^2x^2 + 6bcx + 2c^2$

$The\ given\ expression\ has\ a\ minimum\ value$

$Since\ 3b^2 > 0$

$\therefore Minimum\ value\ is \dfrac{4(3b^2)(2c^2) - 36\,b^2c^2}{4(3b^2)}$

$Since\ \ f(x) = ax^2 + bx + c\ with\ a > 0\ , has$

$minimum\ value\ \dfrac{4ac - b^2}{4a}$

$\Rightarrow \dfrac{-12b^2c^2}{12b^2} = -c^2 > -4ab$

55. $a^{\log_a (x^2 - 4x + 5)} = 3x - 5$ then the solution set is ________

(a) {5, -2}
(b) {-5, 2}
(c) {-5, -2}
(d) {5, 2}

Answer: d

Solution

$a^{\log_a (x^2 - 4x - 5)} = 3x - 5\ a \in R^+ - \{1\}$

$\therefore x^2 - 4x - 5 = 3x - 5$

$\therefore x = 5, x = 2$

Solution set : $\{5, 2\}$

56. If p, q and r are positive and are in AP, then the roots of the quadratic equation $p\chi^2 + q\chi + r = 0$ are complex for

(a) $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$

(b) $\left|\frac{p}{r} - 7\right| < 4\sqrt{3}$

(c) all p and r
(d) no p and r

Answer: b

Solution

$Given\ 2q = p + r$

$and\ q^2 - 4pr < 0$

$$\Rightarrow \left(\frac{p + r}{2}\right)^2 - 4pr < 0$$

$$\Rightarrow \frac{1}{4}(p^2 + r^2 + 2pr) - 4pr < 0$$

$$\Rightarrow p^2 + r^2 + 2pr - 16pr < 0$$

$$\Rightarrow p^2 + r^2 - 14pr < 0$$

$$\Rightarrow \left(\frac{p}{r}\right)^2 + 1 - 14\left(\frac{p}{r}\right) < 0$$

$$\Rightarrow \left(\frac{p}{r} - 7\right)^2 + 1 - 49 < 0$$

$$\Rightarrow \left(\frac{p}{r} - 7\right)^2 < 48$$

$$\Rightarrow \left|\frac{p}{r} - 7\right| < 4\sqrt{3}$$

57. If (1 - p) is a root of quadratic equation $\chi^2 + p\chi + (1 - p) = 0$, then its roots are

(a) 0, 1
(b) - 1, 1
(c) 0, - 1
(d) - 1, 2

Answer: c

Solution

$Since\ (1 - p) in\ the\ root\ of\ the\ equation$

$x^2 + px + (1 - p) = 0, so\ it\ will\ satisfy\ the$

$given\ equation$

$i.e\ \ (1 - p)^2 + p(1 - p) + (1 - p) = 0$

$\Rightarrow 1 - p = 0 \quad or \quad 1 - p + p + 1 = 0$

$\Rightarrow p = 1$

$\therefore On\ substituting\ in\ the\ given\ equation$

$we\ get\ x^2 + x = 0$

$x = 0\ , x = -1$

58. If the equation $(a + 1)\chi^2 - (a + 2)\chi + (a + 3) = 0$ has roots equal in magnitude but opposite in sign, then the roots of the equation are

(a) $\pm a$
(b) $\pm (1/2)\, a$
(c) $\pm (3/2)a$
(d) $\pm 2a$

Answer: b

Solution

Let α, −α be the roots of equation

$$(a+1)x^2-(a+2)x+(a+3)=0$$

$$i.e\ \alpha-\alpha=\frac{a+2}{a+1}\ and\ -\alpha^2=\frac{a+3}{a+1}$$

$$\Rightarrow a+2=0\ and\ -\alpha^2=\frac{a+3}{a+1}$$

$$\Rightarrow -\alpha^2=\frac{1}{-1}$$

$$\Rightarrow \alpha^2=1$$

$$\Rightarrow \alpha=\pm 1$$

$$\Rightarrow \alpha=\pm\frac{1}{2}a$$

59. If every pair from among the equations $x^2+px+qr+qx+rp=0$ and $x^2+rx+pq=0$ have a common root, then $\left(\frac{\text{sum of roots}}{\text{product of roots}}\right)$ is

(a) $\frac{\Sigma p}{pqr}$

(b) $\frac{i}{pqr}$

(c) $(p+q+r)^2$

(d) None of these

Answer: a

Solution

Let α,β be the roots of $x^2+px+qr=0$ ...(1)

β,γ be the roots of $x^2+qx+rp=0$ ....(2)

and γ,α be the roots of $x^2+rx+pq=0$ ...(3)

Since β is a common root of (1), (2)

$\therefore \beta^2+p\beta+qr=0$

and $\beta^2+q\beta+rp=0$

$\therefore (p-q)\beta+(q-p)r=0$

$\Rightarrow \beta-r=0 \Rightarrow \beta=r$

Again $\alpha\beta=qr$ [From (1)]

$\Rightarrow \alpha r=qr$

$\Rightarrow \alpha=q$

Similarly from (2)and, we get

$\gamma=p$ and from (3)and (1), $\beta=r$

$\therefore \alpha+\beta+\gamma=q+r+p=p+q+r$

Again $(\alpha\beta)(\beta\gamma)(\gamma\alpha)=(qr)(rp)(pq)$

$\Rightarrow (\alpha\beta\gamma)^2=(pqr)^2 \Rightarrow \alpha\beta\gamma=pqr$

$$\therefore \frac{\text{sum of the roots}}{\text{product of roots}}=\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}$$

$$=\frac{p+q+r}{pqr}=\frac{\Sigma p}{pqr}$$

60. For what value of λ from the following sum of the squares of the roots of $x^2+(2+\lambda)x-\frac{1}{2}(1+\lambda)=0$ is minimum

(a) $\frac{3}{2}$

(b) 1

(c) $-\frac{5}{2}$

(d) $\frac{11}{4}$

Answer: c

Solution

Given equation is $x^2+(2+\lambda)x-\frac{1}{2}(1+\lambda)=0$

So, $\alpha+\beta=-(2+\lambda)=0$ and $\alpha\beta=-\left(\frac{1+\lambda}{2}\right)$

Now, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow \alpha^2+\beta^2=[-(2+\lambda)]^2+2\frac{(1+\lambda)}{2}$

$\Rightarrow \alpha^2+\beta^2=\lambda^2+4+4\lambda+1+\lambda=\lambda^2+5\lambda+5$

Which is minimum for $\lambda=-5/2$