Engineering Paper - 4

1. The graph between $\sqrt{E}$ and $1/p$ is (E = kinetic energy and p = momentum)

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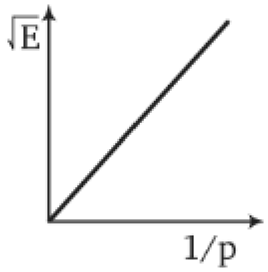

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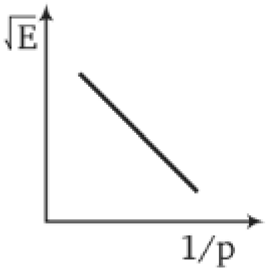

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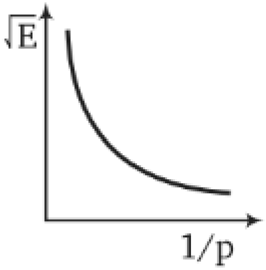

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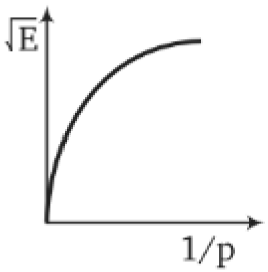


Answer: c

Solution

$P = \sqrt{2mE}$ it is clear that $P \propto \sqrt{E}$

So the graph between P and $\sqrt{E}$ will be straight line but graph between $\frac{1}{P}$ and $\sqrt{E}$ will be hyperbola.

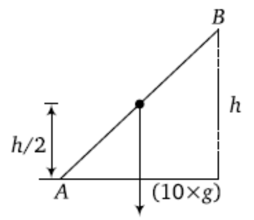
2. A rod AB of mass 10 kg and length 4 m rests on a horizontal floor with end A fixed so as to rotate it in vertical plane about perpendicular axis passing through A. If the work done on the rod is 100 J, the height to which the end B be raised vertically above the floor is

(a) 1.5 m
(b) 2.0 m
(c) 1.0 m
(d) 2.5 m

Answer: b

Solution

Work done $= mg(h/2)$



$$100 = \frac{10 \times 10 \times h}{2}$$

$\Rightarrow$ $h = 2.0\ \text{m}$

3. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?

(a) 7.2 J
(b) 3.6 J
(c) 120 J
(d) 1200 J

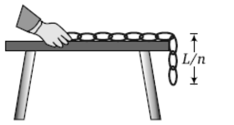
Answer: b

Solution

Fraction of length of the chain hanging from the table

$$= \frac{1}{n} = \frac{60\ \text{cm}}{200\ \text{cm}} = \frac{3}{10} \Rightarrow n = \frac{10}{3}$$

Work done in pulling the chain on the table



$$W = \frac{mgL}{2n^2} = \frac{4 \times 10 \times 2}{2 \times (10/3)^2} = 3.6\ \text{J}$$

4. If the kinetic energy of a body becomes four times of its initial value, then new momentum will

(a) Becomes twice its initial value
(b) Becomes three times its initial value
(c) Become four times its initial value
(d) Remains constant

Answer: a

Solution

$P = \sqrt{2mE} \therefore P \propto \sqrt{E}$ *i. e.*, if kinetic energy becomes four times then new momentum will become twice.

5. The potential energy of a 1 kg particle free to move along the x-axis is given by

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right) J$$

The total mechanical energy of the particle is 2J. Then, the maximum speed (in m/s) is

(a) $\sqrt{2}$
(b) $1/\sqrt{2}$
(c) 2
(d) $3/\sqrt{2}$

Answer: d

Solution

Potential energy $V = \frac{x^4}{4} - \frac{x^2}{2}$

For maximum kinetic energy, potential energy of a particle should be minimum.

For minimum value of V, $\frac{dV}{dx} = 0$ and $\frac{d^2V}{dx^2} > 0$

Force $F = -\left(\frac{dV}{dx}\right) = \frac{4x^3}{4} - \frac{2x}{2} = 0 \Rightarrow x^3 - x = 0$

$\Rightarrow$ $x(x^2 - 1) = 0$

i.e., at $x = 0$, $x = +1$ and $x = -1$ force on the particle will be zero.

Now, $\frac{d^2V}{dx^2} = 3x^2 - 1$

For $x = +1$ and $x = -1$ $\frac{d^2V}{dx^2} > 1$

It means the potential energy of the particle will be minimum at $x = 1$ and $x = -1$

Now, substituting these values in expression of potential energy

$$V_{\text{min}} = \left[\frac{(1)^4}{4} - \frac{(1)^2}{2}\right] J = \left[\frac{1}{4} - \frac{1}{2}\right] J = -\frac{1}{4} J$$

(Kinetic energy)$_{\text{max}}$

$= \text{Total energy} - (\text{Potential energy})_{\text{min}}$

$= 2 - \left(-\frac{1}{4}\right)$

$\frac{1}{2} mv^2_{\text{max}} = \frac{9}{4} \Rightarrow v^2_{\text{max}} = \frac{9}{2} \Rightarrow v_{\text{max}} = \frac{3}{\sqrt{2}}$ m/sec

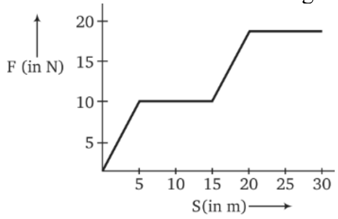
6. **Assertion** A light body and heavy body have same momentum. Then they also have same kinetic energy.
Reason Kinetic energy does not depend on mass of the body.

(a) If both Assertion and Reason are true and the Reason is the correct explanation of the Assertion.
(b) If both Assertion and Reason are true and the Reason is not the correct explanation of the Assertion.
(c) If Assertion is true but Reason is false.
(d) If the Assertion and Reason both are false
(e) If Assertion is false but Reason is true.

Answer: d
Solution
When two bodies have same momentum then lighter body possess more kinetic energy because $E = \frac{P^2}{2m}$

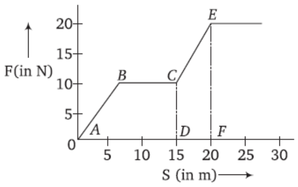
$\therefore \quad E \propto \frac{1}{m}$ when P = constant

7. The work done by a force acting on a body is as shown in the graph. The total work done in covering an initial distance of 20 m is



(a) 225 J
(b) 200 J
(c) 400 J
(d) 175 J

Answer: b
Solution



Work done W = Area under $F - S$ graph
= area of trapezium $ABCD$ + area of trapezium $CEFD$
$= \frac{1}{2} \times (10+15) \times 10 + \frac{1}{2} \times (10+20) \times 5$
$= 125 + 75 = 200$ J

8. A force of 5 N acts on a 15 kg body initially at rest. The work done by the force during the first second of motion of the body is

(a) 5 J
(b) $\frac{5}{6}$ J
(c) 6 J
(d) 75 J

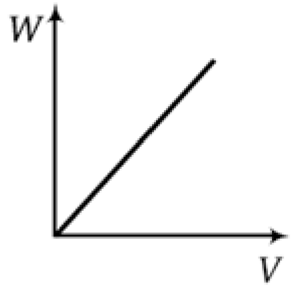
Answer: b
Solution
$W = Fs = F \times \frac{1}{2}at^2 \left[\text{from } s = ut + \frac{1}{2}at^2\right]$
$\Rightarrow W = F\left[\frac{1}{2}\left(\frac{F}{m}\right)t^2\right] = \frac{F^2t^2}{2m} = \frac{25 \times (1)^2}{2 \times 15} = \frac{25}{30} = \frac{5}{6}$ J

9. Two bodies having same mass 40 kg are moving in opposite direction, one with a velocity of 10 m/s and the other with 7 m/s. If they collide and move as one body, the velocity of the combination is

(a) 10 m/s
(b) 7 m/s
(c) 3 m/s
(d) 1.5 m/s

Answer: d
Solution
By the conservation of momentum
$40 \times 10 + (40) \times (-7) = 80 \times v \Rightarrow v = 1.5$ m/s

10. A particle, initially at rest on a frictionless horizontal surface, is acted upon by a horizontal force which is constant in size and direction. A graph is plotted between the work done (W) on the particle, against the speed of the particle, (v). If there are no other horizontal force acting on the particle, the graph would look like

(a)


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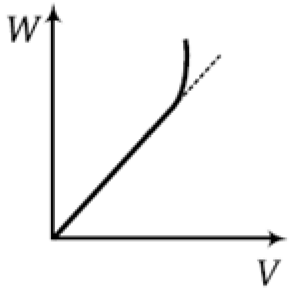

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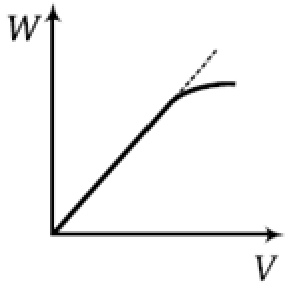

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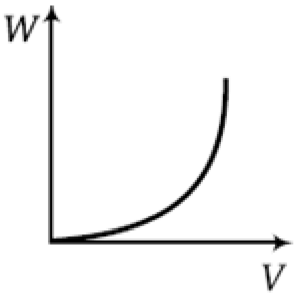


Answer: d
Solution
Work done = Change in kinetic energy
$W = \frac{1}{2} mv^2$
$\therefore \quad W \propto v^2$ graph will be parabolic in nature

11. For spheres each of mass M and radius R are placed with their centres on the four corners A , B, C and D of a square of side b. The spheres A and B are hollow and C and D are solids. The moment of inertia of the system about side AD of square is

(a) $\frac{8}{3} MR^2 + 2Mb^2$

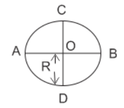
(b) $\frac{8}{5} MR^2 + 2Mb^2$

(c) $\frac{32}{15} MR^2 + 2Mb^2$

(d) $32\, MR^2 + 4Mb^2$

Answer: c

Solution



$I_A = \frac{2}{3} MR^2 \to (1)$

$I_B = \frac{2}{5} MR^2 \to (2)$

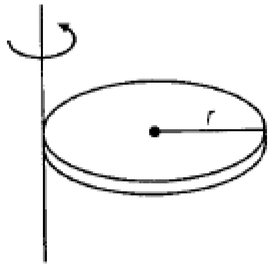
∴ Moment of Inertia of whole system about side.

$AD = I_A + I_D + I_B + I_C$

$AD = \frac{2}{3}MR^2 + \frac{2}{5}MR^2 + \left(Mb^2 + \frac{2}{3}MR^2\right) + \left(Mb^2 + \frac{2}{5}MR^2\right)$

$AD = \frac{32}{15}MR^2 + 2Mb^2$

12. A solid sphere of radius R has moment of inertia I about its geometrical axis. If it is melted into a disc of radius r and thickness t. If its moment of inertia about the tangential axis (which is perpendicular to plane of the disc), is also equal to I, then the value of r is equal to



(a) $\frac{2}{\sqrt{15}} R$

(b) $\frac{2}{\sqrt{5}} R$

(c) $\frac{3}{\sqrt{15}} R$

(d) $\frac{\sqrt{3}}{\sqrt{15}} R$

Answer: a

Solution

From perpendicular axis theorem

$$I_z = I_x + I_y$$

$$\frac{2}{5} MR^2 = \frac{1}{2} MR^2 + MR^2$$

$$\frac{2}{5} MR^2 = \frac{3}{2} MR^2$$

$$\therefore \quad r = \frac{4}{\sqrt{15}} R$$

13. ~~A sphere of mass m and radius r rolls on a horizontal plane without~~ slipping with the speed u. Now, if it rolls up vertically then maximum height, it would attain will be

(a) $\frac{3u^2}{4g}$

(b) $\frac{5u^2}{2g}$

(c) $\frac{7u^2}{10g}$

(d) $\frac{u^2}{2g}$

(e) $\frac{11u^2}{9g}$

Answer: c

Solution

By conservation of energy

$$\Rightarrow mgh = \frac{1}{2}mu^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow mgh = \frac{1}{2}mu^2 + \frac{1}{2}(\frac{2}{5}mr^2)$$

$$\Rightarrow gh = \frac{7}{10}u^2$$

$$\Rightarrow h = \frac{7u^2}{10g}$$

14. A constant torque of 1500 Nm turns a wheel of moment of inertia 300 kgm^2 about an axis passing through its centre the angular velocity of the wheel after 3 sec will be.......... rad/sec

(a) 5
(b) 10
(c) 15
(d) 20

Answer: c

Solution

$$\tau = I\alpha = I\frac{d\omega}{dt}$$

15. If rotational kinetic energy is 50% of translational kinetic energy, then the body is

(a) Ring
(b) Cylinder
(c) Hollow sphere
(d) Solid sphere

Answer: b

Solution

$K_R = 50\% \, K_T$

$$\frac{1}{2}mv^2\frac{K^2}{R^2} = \frac{50}{100} \times \frac{1}{2}mv^2$$

$$\Rightarrow \therefore \quad \frac{K^2}{R^2} = \frac{1}{2}$$

i.e., body will be solid cylinder.

16. ~~One solid sphere A and another hollow sphere B are of same mass~~ and same outer radius. Their moments of inertia about their diameters are respectively I_A and I_B such that

Note: d_A and d_B are their densities.

(a) $I_A = I_B$
(b) $I_A > I_B$
(c) $I_A < I_B$
(d) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

Answer: c

Solution

The moment of inertia of solid sphere A,

$$I_A = \frac{2}{5}MR^2 \rightarrow (1)$$

The moment of inertia of hallow sphere B,

$$I_B = \frac{2}{3}MR^2 \rightarrow (2)$$

From equn (1) and (2) we get

$$I_A < I_B$$

17. The moment of inertia of a solid sphere about an axis passing through centre of gravity is $\frac{2}{5}MR^2$, then its radius of gyration about a parallel axis at a distance 2R from first axis is

(a) 5 R

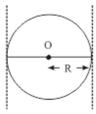
(b) $\sqrt{\frac{22}{5}}\,R$

(c) $\frac{5}{2}\,R$

(d) $\sqrt{\frac{12}{5}}\,R$

Answer: b

Solution



According to parallel axis theorem

$$I_0 = I_{CM} + MR^2 \quad ...(i)$$

$$I_{CM} = \frac{2}{5}MR^2$$

Moment of inertia about a diameter

$$I = \frac{2}{5}MR^2$$

$$\therefore R = 2R$$

$\therefore$ Substituting values in equation (i) we get,

$$I = I_{CM} + MR^2$$

$$I = \frac{2}{5}MR^2 + 4MR^2$$

$$I = \frac{22}{5}MR^2$$

Moment of inertia $I = MK^2$

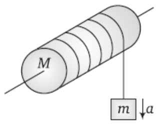
$$MK^2 = \frac{22}{5}MR^2$$

$$K = \sqrt{\frac{22}{5}}R$$

18. A reel of thread unrolls itself falling down under gravity. Neglecting mass of the thread, the acceleration of the reel is

(a) g
(b) g/2
(c) 2g/3
(d) 4g/3

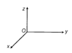
Answer: c
Solution



$$a = \frac{g}{1 + \frac{K^2}{R^2}} \qquad \text{[For solid cylinder } \frac{K^2}{R^2} = \frac{1}{2}]$$

$$\therefore \quad a = \frac{g}{1 + \frac{1}{2}} = \frac{2}{3}g.$$

19. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1, -1) is



(a) $F(\hat{\imath} - \hat{\jmath})$

(b) $-F(\hat{\imath} + \hat{\jmath})$

(c) $F(\hat{\imath} + \hat{\jmath})$

(d) $-F(\hat{\imath} - \hat{\jmath})$

Answer: c
Solution

$$\tau = r \times F$$

$$= (\hat{\imath} - \hat{\jmath}) \times (-F\hat{k})$$

$$= F[(-\hat{\imath} \times \hat{k}) + (\hat{\jmath} \times \hat{k})]$$

$$= F(\hat{\jmath} + \hat{\imath}) = F(\hat{\imath} + \hat{\jmath})$$

20. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω, about the same axis. The final angular velocity of the combination of discs is

(a) $\dfrac{I_2\omega}{I_1 + I_2}$

(b) ω

(c) $\dfrac{I_1\omega}{I_1 + I_2}$

(d) $\dfrac{(I_1 + I_2)\omega}{I_1}$

Answer: c
Solution

A disc with moment of inertia I_1

has angular momentum $\alpha_1 = I_1\omega$

When another disc of moment

of inertia I_2 is placed on first disc

$\alpha_2 = (I_1 + I_2)\omega'$

So, by conservation of angular momentum

$\alpha_1 = \alpha_2$

$$\Rightarrow I_1\omega = (I_1 + I_2)\omega' \Rightarrow \omega' = \frac{I_1\omega}{I_1 + I_2}$$

21. If $S + O_2 \rightarrow SO_2, \Delta H = -298.2$ kJ
$SO_2 + \frac{1}{2}O_2 \rightarrow SO_3, \Delta H = -98.7$ kJ
$SO_3 + H_2O \rightarrow H_2SO_4, \Delta H = -130.2$ kJ
$H_2 + \frac{1}{2}O_2 \rightarrow H_2O, \Delta H = -287.3$ kJ
Then the enthalpy of formation of H_2SO_4 at 298 K will be

(a) – 814.4 kJ
(b) +320.5 kJ
(c) – 650. 3 kJ
(d) – 933.7 kJ.

Answer: a
Solution

Aim : $H_2 + S + 2O_2 \rightarrow H_2SO_4$
Operate Eqn. (iv) + Eqn. (i) + Eqn. (ii) + Eqn. (iii).
we get $\Delta H = -287.3 + (-298.2) + (-98.7)$
$+(-130.2) = -814.4$ kJ

22. ΔG for the reaction $Ag_2O \rightarrow 2Ag + \frac{1}{2} O_2$ at a certain temperature is found to be -10.0 kJ mol^{-1} , which one of the following statements is correct at this temperature ?

(a) Silver oxide decomposes to give silver and oxygen
(b) Silver and oxygen combine to form silver oxide
(c) The reaction is in equilibrium
(d) The reaction can neither occur in the forward direction nor in the backward direction

Answer: a
Solution

Given, $Ag_2O \rightarrow 2Ag + \frac{1}{2} O_2$ and $\Delta G = -10.0$ kJ mol^{-1}

ΔG is negative , when the reaction proceeds

spontaneously in the forward direction.

In the spontaneous process decomposition of

compound takes places.

Therefore silverdioxide decomposes to gives

silver and oxygen.

23. Match list I with List II and select the correct answer using the codes given below the lists:

List I	(A) $\left(\frac{\partial G}{\partial p}\right)_T$	(B) $\left(\frac{\partial G}{\partial T}\right)_p$	(C) $\left(\frac{\partial H}{\partial S}\right)_p$	(D) $\left(\frac{\partial T}{\partial p}\right)_H$	
List II	1. μJT	2.T	3. –S	4. P	5. V

(a) A = 5, B = 1, C = 2, D = 4
(b) A = 5, B = 3, C = 2, D = 4
(c) A = 3, B = 5, C = 2, D = 1
(d) A = 5, B = 3, C = 2, D = 1

Answer: d

Solution

From thermo dynamics, $dG = Vdp - SdT$

At constant T, $dT = 0$ so that $\left(\frac{\partial G}{\partial P}\right)_T = V$

At constant P, $dP = 0$ so that $\left(\frac{\partial G}{\partial T}\right)_p = -S$

Also $\mu_{JT} = \left(\frac{\partial T}{\partial P}\right)_H$

24. When one mole of an ideal gas is compressed to half of its initial volume and simultaneously heated to twice its initial temperature, the change in entropy (AS) is

(a) $C_V \ln 2$
(b) $C_p \ln 2$
(c) $R \ln 2$
(d) $(C_V - R) \ln 2$

Answer: d

Solution

$$\Delta S = C_V \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1} = C_V \ln 2 + R\ln\frac{1}{2}$$
$$= C_V \ln 2 - R\ln 2$$
$$= (C_V - R)\ln 2$$

25. Ka for CH3COOH at 25ºC is 1.754×10^{-5} . At 50ºC, Ka is 1.633×10^{-5} What will be value of ΔSº for the ionisation of CH_3COOH?

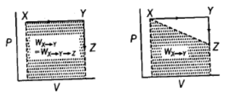
(a) –94.44 J/mole K
(b) –96.66 J/mole K
(c) –96.44 J/mole K
(d) –90.44 J/mole K

Answer: c

Solution

$$(\Delta G^o)_{298} = -2.303RT\log K$$
$$= -2.303 \times 8.314 \times \log(1.754 \times 10^{-5})$$
$$= 27194\,J$$
$$\Delta G^o = \Delta H^o - T\Delta S^o$$
$$29605 = \Delta H^o - 298\,\Delta S^o$$
$$\Delta S^o = -96.44 J/mol.K$$

26. For an ideal gas, consider only p-V work in going from an initial state X to the final state Z. The final state Z can be reached by either of the two paths shown in the figure.
Which of the following choice (s) is (are) correct ?



(Take ΔS as change in entropy and W as work done.)

(a) $\Delta S_{X\to Z} = \Delta S_{X\to Y} + \Delta S_{Y\to Z}$
(b) $W_{X\to Z} = W_{X\to Y} + W_{Y\to Z}$
(c) $W_{X\to Y\to Z} = W_{X\to Y}$
(d) $\Delta S_{X\to Y\to Z} = \Delta S_{X\to Y}$

Answer: a

Solution

$$\therefore \Delta S_{X\to Y} + \Delta S_{Y\to Z} + \Delta S_{Z\to X} = 0$$
$$\Rightarrow \Delta S_{X\to Y} + \Delta S_{Y\to Z} = -\Delta S_{Z\to X}$$
$$\Rightarrow \Delta S_{X\to Z} = \Delta S_{X\to Y} + \Delta S_{Y\to Z}$$

27. What is the entropy change (in $JK^{-1}\,mol^{-1}$) when one mole of ice is converted into water at 0^oC ? (The enthalpy change for the conversion of ice to liquid water is 6.0 kJ mol^{-1} at 0^oC)

(a) 20.13
(b) 2.013
(c) 2.198
(d) 21.98

Answer: d

Solution

$$\Delta S = \frac{\Delta H_f}{T_f} = \frac{6000\,J}{273\,K} = 21.98\,JK^{-1}\,mol^{-1}$$

28. A gas expands isothermally against a constant external pressure of 1 atm from a volume of 10 dm^3 to a volume of 20 dm^3. It absorbs 800 J of thermal energy from its surroundings. The ΔU is

(a) - 312 J
(b) + 123 J
(c) - 213 J
(d) + 231 J

Answer: c

Solution

From first law of thermodynamics

$$\Delta U = q + w$$
$$= q + \left[-P(V_2 - V_1) \times \frac{8.314}{0.08211}\right]$$
$$= 800 + \left[-1(20 - 10) \times \frac{8.314}{0.08211}\right]$$
$$= 800 - 1013J$$
$$\Delta U = -213J$$

29. Given that $C + O_2 \to CO_2\,\Delta H^o = -xkj$
$2CO + O_2 \to 2CO_2\,\Delta H^o = -ykj$
the enthalpy of formation of carbon monoxide will be

(a) $\frac{2x - y}{2}$
(b) $\frac{y - 2x}{2}$
(c) $2x - y$
(d) $y = 2x$

Answer: b

Solution

For $C + \frac{1}{2}O_2 \to CO$, operate eqn,. (i) $-\frac{1}{2}$ eqn.

(ii) $\Delta H = -x - \frac{1}{2}(-y) = -x + \frac{y}{2} = \frac{y - 2x}{2}$

30. The enthalpy of dissolution of $BaCl_2$(s) and $BaCl_2$. 2 H_2O (s) are -20.6 and 8.8 kJ mol^{-1} respectively. The enthalpy of hydration for $BaCl_2(s) + 2\,H_2O \to BaCl_2.2H_2O$ (s) is

(a) 29.4 kJ
(b) -29.4 kJ
(c) -11.8 kJ
(d) 38.2 kJ

Answer: b

Solution

Given (i) $BaCl_2(s) + aq \to BaCl_2(aq), \Delta H = -20.6\,kJ$
(ii) $BaCl_2.2H_2O(s) + aq \to BaCl_2(aq), \Delta H = +8.8\,kJ$
Eqn. (i) can be split in two steps as
$BaCl_2(s) + 2H_2O(l) \to BaCl_2.2H_2O(s), \Delta H = \Delta H_1$
$BaCl_2.2H_2O(s) + aq \to BaCl_2(aq). \Delta H = \Delta H_2$
$\Delta H_1 + \Delta H_2 = -20.6$ or $\Delta H_1 + 8.8 = -20.6$ or
$$\Delta H_1 = -29.4\,kJ$$

31. On adding which of the following, the pH of 20 mL of 0.1 N HCl will not alter ?

(a) 20 mL of distilled water
(b) 1 mL of 0.1 N NaOH
(c) 500 mL of HCL of pH = 1
(d) 1 mL of 1 N HCL

Answer: c

Solution

Given ,for HCl, $N_1 = 0.1N$,

$V_1 = 20\text{ mL} \Rightarrow N_1V_1 = 20 \times 0.1 = 2$

1) 20 mL of distilled water

pH = 7 for distilled water.

$\therefore [H^+] = 10^{-7}$ M

∴ (for HCl normality = Molarity)

$\therefore N_1 = 10^{-7}$ and $V_1 = 20$ mL

$\Rightarrow N_1 \times V_1 = 10^{-7} \times 20 = 2 \times 10^{-6}$

2) 1mL of NaOH $\Rightarrow 0.1 \times 1 = 0.1$

3) 500 mL of HCl of pH = 1

$\therefore [H^+] = 10^{-1} = 0.1$ M

$\therefore N_1V_1 = 0.1 \times 500 = 50$

4) 1mL of 1N HCl $= 1 \times 1 = 1$

Since milli equivalents (N_1V_1) of 500 mL of HCl of pH = 1, is more than that of 20 mL of 0.1 NHCl , therefore, on adding this, pH of hydrochloric acid will not alter.

32. The ionzation constant of formic acid is 7.8×10^{-4} . Calculate ratio of sodium formate & formic acid in a buffer of P^H 4.25

(a) 9.63
(b) 3.24
(c) 6.48
(d) 3.97

Answer: b

Solution

$$pka = -\log(1.8 \times 10^{-4}) = 3.74$$

$$\log\frac{[\text{Salt}]}{[\text{Acid}]} = P^H - pka = 4.25 - 3.74 = 0.51$$

$$\therefore \frac{[\text{Salt}]}{[\text{Acid}]} = \text{antilog of } 0.51 = 3.24$$

33. For the following reaction in gaseous phase

$CO + \frac{1}{2}O_2 \rightleftharpoons CO_2$

K_C/K_P is

(a) $(RT)^{\frac{1}{2}}$
(b) $(RT)^{-\frac{1}{2}}$
(c) RT
(d) $(RT)^{-1}$

Answer: a

Solution

Formula :

$$Kp = Kc(RT)^{\Delta n_{(g)}}$$

$$\Delta n_{(g)} = n_p - n_r$$

34. Which of the following is not a conjugate acid-base pair?

(a) HPO_3^{2-}, PO_3^{3-}
(b) $H_2PO_4^-$, HPO_4^{2-}
(c) H_3PO_4. $H_2PO_4^-$
(d) $H_2PO_4^-$, PO_3^{3-}

Answer: d

Solution

1. $HPO_3^{2-} \underset{+H^+}{\overset{-H^+}{\rightleftharpoons}} PO_3^{3-}$

2. $H_2PO_4^- \underset{+H^+}{\overset{-H^+}{\rightleftharpoons}} HPO_4^{2-}$

3. $H_3PO_4 \underset{+H^+}{\overset{-H^+}{\rightleftharpoons}} H_2PO_4^-$

4. $H_2PO_4^- \underset{+H^+}{\overset{-H^+}{\rightleftharpoons}} HPO_4^{2-} \underset{+H^+}{\overset{-H^+}{\rightleftharpoons}} PO_4^{3-}$

Hence $H_2PO_4^-$ and PO_3^{3-} is not a conjugate acid – base pair.

35. The concentration of oxalic acid is *x* mol L^{-1}.40 mL of this solution reacts with 16 mL of 0.05 M acidified $KMnO_4$. What is the pH of *x* M oxalic acid solution? (assume that oxalic acid dissociates completely)

(a) 1.3
(b) 1.699
(c) 1
(d) 2

Answer: a

Solution

Given for oxalic acid,

$V_1 = 40$mL

$M_1 = x \text{ mol } L^{-1}$

For $KMnO_4$,

$M_2 = 0.05$M

$V_2 = 16$mL

p^H of oxalic acid =?

we know that

$M_1V_1 = M_2V_2$

$x \times 40 = 0.05 \times 16$

$\Rightarrow x = \frac{0.05 \times 16}{40}$

$\Rightarrow x = 0.02$M

Now, Normality $= \frac{\text{Molarity} \times \text{molecular weight of axalic acid}}{\text{Eq. wt of oxalic acid}}$

$= \frac{0.02 \times 90}{45}$

Normality = 0.04 N(for oxalic acid)

Here Normality ≈ $[H^+]$ions = 0.04

$p^H = -\log[H^+]$

$= -\log[0.04]$

$p^H = 1.39$ for 0.02 M oxalic acid.

36. 5 mL of 0.4 N NaOH is mixed with 20 mL of 0.1 N HCl. The pH of the resulting solution will be

(a) 7
(b) 8
(c) 5
(d) 6

Answer: a

Solution

Given, For NaOH

$V_1 = 5$ml, $N_1 = 0.4$N

For HCl, $V_2 = 20$mL, $N_2 = 0.1$N

p^H =?

Before neutralization

millimoles of HCl $= 20 \times 0.1 = 2$

millimoles of NaOH $= 5 \times 0.4 = 2$

After neutralization,

Millimoles of NaOH = millimoles of HCl = 0

[By concept of limiting reagent]

Hence the resulting solution will be neutral

∴ The pH will be 7 for neutral solution

37. An equilibrium mixture for the reaction $2H_2S(g) \rightleftharpoons 2H_2(g) + S_2$ had one mole of hydrogen sulphide,0.2 mole of H_2 and 0.8 mole of S2 in a 2 litre vessel. The value of K_c in mole $litre^{-1}$ is

(a) 0..004
(b) 0.016
(c) 0.08
(d) 0.16

Answer: b

Solution

$$K = \frac{(0.2/2)^2(0.8/2)}{(1/2)^2} = \frac{(0.1)^2(0.4)}{(0.5)^2} = 0.016$$

38. The P^H of neutral water is 6.8 .Then the temperature of H_2O

(a) is 25^0 C
(b) is more than 25^0 C
(c) is less than 25^0 C
(d) can not be predicted.

Answer: b

Solution

At 25°C temp p^H of $H_2O = 7[H^+] = 10^{-7}$ M

$P^H = 6.8$ means $P^H < 7 \therefore [H^+]$is more than 10^{-7} M

Self ionisation of H_2O is endothermic so by increasing temp $[H^+]$ion increases.

39. The pK_a of a weak acid is equal to

(a) $\log K_a$
(b) $\dfrac{1}{\log k_a}$
(c) $-\log\dfrac{1}{K_a}$
(d) $\log\dfrac{1}{K_a}$

Answer: d

Solution

$$pK_a = -kog\ K_a = \log(1/K_a)$$

40. The equilibrium constant for the reaction $N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$ at temperature 300k is 4×10^{-4}. The value of Kc for the reaction $NO_{(g)} \rightleftharpoons \frac{1}{2}N_{2(g)} + \frac{1}{2}O_2(g)$ at the same temperature is :

(a) 2.5×10^2
(b) 4×10^{-4}
(c) 0.02
(d) 50

Answer: d

Solution

$N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$

$$K_p = \frac{P_{NO}^2}{P_{N_2} \times P_{O_2}}$$

For reaction ,

$NO(g) \rightleftharpoons \frac{1}{2}O_{2(g)} + \frac{1}{2}N_2$

$$K'_p = \frac{P_{O_2}^{\frac{1}{2}} \times P_{N_2}^{\frac{1}{2}}}{P_{NO}} = \frac{1}{(Kp)^{1/2}} = \frac{1}{(4 \times 10^{-4})^{1/2}} = 50$$

41. The value of 7 $\log_2$ 16/15 + 5 log $_2$25/24 + 3 log $_2$ 81/80 is

(a) 1
(b) $\log_2(105)$
(c) $\log_2(9/80$
(d) $\log_2(8/9)$

Answer: a

Solution

$7\ log_2\frac{16}{15} + 5\ log_2\frac{25}{24} + 3log_2\frac{81}{80}$

$\Rightarrow 7log_2 16 - 7log_2 15 + 5log_2 25 - 5log_2 24$
$\quad +3log_2 81 - 3log_2$

$\Rightarrow 28log_2 2 - 7log_2 5 - 7log_2 3 + 10log_2 5 - 5log_2 8$
$\quad -5log_2 3 + 12log_2 3 - 3log_2 16 - 3log_2 5$

$\Rightarrow 28log_2 2 - 15log_2 2 - 12log_2 2 - 7log_2 5$
$\quad +10log_2 5 - 3log_2 5 - 7log_2 3 - 5log_2 3 + 12log_2 3$

$log_2 2 + 0 + 0 = 1$

42. The coefficients of χ in the quadratic equation $\chi^2 + b\chi + c = 0$ was taken as 17 in place of 13, its roots were found to be - 2 and -15. The correct roots of the original equation are

(a) - 10, - 3
(b) - 9, - 4
(c) - 8, - 5
(d) - 7, - 6

Answer: a

Solution

$Given\ equation\ is\ x^2 + bx + c = 0\ and\ b = 17$

$i.e\ x^2 + bx + c = 0\ and\ \ b = 17\ \ \ldots\ldots(1)$

$Since\ roots\ of\ this\ equation\ are - 2\ and - 15$

$\therefore (x+2)(x+15) = x^2 + 17x + 30\ \ \ldots\ldots(2)$

$from\ (1)\ and\ (2), we\ get\ c = 30$

$If\ b = 13, then\ \ x^2 + 13x + c = 0$

$\Rightarrow x^2 + 13x + 30 = 0$

$\Rightarrow x = -3, -10.$

43. A value of K for which the quadratic equation χ^2 - 2χ (1 + 3K) + 7(2K + 3) = 0 has equal roots, is

(a) 1
(b) 2
(c) 3
(d) 4

Answer: b

Solution

$Given\ \ x^2 - 2x(1+3k) + 7(2k+3) = 0\ has\ equal\ roots$

$i.e\ \ [2(1+3k)]^2 - 4[7(2k+3)] = 0$

$\Rightarrow (1+3k)^2 - 7(2k+3) = 0$

$\Rightarrow 1 + 9k^2 + 6k - 14k - 21 = 0$

$\Rightarrow 9k^2 - 8k - 20 = 0$

$\Rightarrow k = 2, \frac{-10}{9}$

44. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater then -2 but less than 4 lie in interval

(a) $m > 3$
(b) $-1 < m < 3$
(c) $1 < m < 4$
(d) $-2 < m < o$

Answer: b

Solution

$x^2 - 2mx + m^2 - 1 = 0$

$\Delta = 4$

$\therefore \alpha = m + 1, \beta = m - 1$

$\alpha < 4$ and $\beta > -2$

$m + 1 < 4, m - 1 > -2\ \therefore -1 < m < 3$

45. The roots of the equation $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$ are

(a) 2018/01/04
(b) 1, –4
(c) 1, –8
(d) 2018/01/08

Answer: c

Solution

$$x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$$

$$\Rightarrow x^{\frac{2}{3}} + 1\left(x^{\frac{1}{3}}\right) - 2 = 0$$

Let $a = x^{\frac{1}{3}}$

then $a^2 = a - 2 \Rightarrow a = 1, -2$

hence $x = 1, -8\left(\text{by } a = x^{\frac{1}{3}}\right)$

46. If $a = \log_2 3$, $b = \log_2 5$ and $c = \log_7 2$, then $\log_{140} 63$ in terms of a,b and c is equal to

(a) $\frac{2ac+1}{2c+abc+1}$

(b) $\frac{2ac+1}{2a+c+a}$

(c) $\frac{2ac+1}{2c+ab+a}$

(d) None of these

Answer: d

Solution

Given

$a = \log_2 3, b \log_2 5, and\ c = \log_7 2$

$Now \log_{140} 63 = \frac{\log_2 63}{\log_2 140}$

$\Rightarrow \frac{\log_2 7 + 2\log_2 3}{log_2 7 + 2\log_2 2 + \log_2 5}$

$\Rightarrow \frac{\frac{1}{c} + 2a}{\frac{1}{c} + 2 + b}$

$\Rightarrow \frac{1 + 2ac}{1 + 2c + bc}$

47. For all X ϵ R, the value of expression $\frac{x^2+2x+1}{x^2+2x+7}$ lies in ______

(a) [2, 3]
(b) [0, 1]
(c) [1, 2]
(d) [0, 2]

Answer: b

Solution

$\text{Suppose } \frac{x^2+2x+1}{x^2+2x+7} = m$

$\therefore (1-m)x^2 + 2(1-m)x + 1 - 7m = 0$

$\Delta = 4(1-m)^2 - 4(1-m)(1-7m)$

x is real number. $\therefore \Delta \geq 0$

$\therefore (1-m)(6m) \geq 0$

$\therefore 0 \leq m \leq 1$

$\therefore m(m-1) \leq 0$

$\therefore 0 \leq m \leq 1, m \in [0,1]$

48. The number of roots of $3^{|x||2-x||=1}$ is

(a) 1
(b) 2
(c) 3
(d) 4

Answer: c

Solution

$3^{|x||2-|x||} = 1 = 3^o$

$\Rightarrow |x|\big|2 - |x|\big| = 0$

$\Rightarrow$ either $|x| = 0$ or $|x| = 2$

$\Rightarrow x = 0$ or $x = \pm 2$

$\therefore$ no. of roots $= 3$.

49. The sequence $\log a, \log\frac{a^2}{b}, \log\frac{a^3}{b^2}, \ldots$ is

(a) a GP
(b) an AP
(c) a HP
(d) both a GP and a HP

Answer: b

Solution

Given

$\log a, \log\frac{a^2}{b}, \log\frac{a^3}{b^2}, \ldots$

$T_2 - T_1 = \log\frac{a^2}{b} - \log a = \log\frac{\frac{a^2}{b}}{a} = \log\frac{a}{b}$

$T_3 - T_2 = \log\frac{a^3}{b^2} - \log\frac{a^2}{b} = \log\left(\frac{\frac{a^3}{b^2}}{\frac{a^2}{b}}\right) = \log\left(\frac{a}{b}\right)$

$\therefore$ *The given sequence is in A.P*

50. The number of solutions of $\log_4(x - 1) = \log_2(x - 3)$

(a) 3
(b) 1
(c) 2
(d) 0

Answer: b

Solution

Given

$log_4(x-1) = log_2(x-3)$

$\Rightarrow log_{2^2}(x-1) = log_2(x-3)$

$\Rightarrow log_2\sqrt{(x-1)} = log_2(x-3)$

$\Rightarrow \sqrt{x-1} = (x-3)$

On squaring both sides

$(x-1) = (x-3)^2$

$\Rightarrow x - 1 = x^2 - 6x + 9$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x-5)(x-2) = 0$

[*Since* $log_2 x - 3$ *is not defined when* $x = 2$]

$\therefore x = 5$

51. If ${}^{n+1}C_3 = 2\ {}^{n}C_2$, then n =

(a) 3
(b) 4
(c) 5
(d) 6

Answer: c

Solution

${}^{n+1}C_3 = 2 \cdot {}^{n}C_2$

$\Rightarrow \frac{(n+1)!}{3!(n-2)!} = 2 \cdot \frac{n!}{2!(n-2)} \Rightarrow \frac{n+1}{3.2!} = \frac{2}{2!}$

$\Rightarrow \quad n + 1 = 6$

$\Rightarrow \quad n = 5.$

52. The number of integer a,b,c,d, such that a+b+c+d = 20 and a,b,c,d ≥ 0 is ______

(a) ${}^{24}C_3$

(b) ${}^{25}C_3$

(c) ${}^{26}C_3$

(d) ${}^{27}C_3$

Answer: d

Solution

$a = x - 1, b = y - 1, c = z - 1, d = w - 1$
Here, $x, y, z, w \geq 0$ and
$x - 1 + y - 1 + z - 1 + w - 1 = 20$
$\therefore x + y + z + w = 24$
$\therefore$ Non zero integer solutions of the equation are
$= {}^{24+4-1}C_{4-1} = {}^{27}C_3$

53. The number of ways in which four letters of the word \'MATHEMATICS\' can be arranged is given by

(a) 136
(b) 192
(c) 1680
(d) 2454

Answer: d

Solution

Word 'MATHEMATICS' has 2M, 2T, 2A, H, E, I, C, S.

Therefore 4 letters can be chosen in the following ways.

Case I : 2 alike of one kind and 2 alike of second kind

i.e., ${}^3C_2 \Rightarrow$ No. of words $= {}^3C_2 \frac{4!}{2!2!} = 18$

Case II : 2 alike of one kind and 2 different

i.e., ${}^3C_1 \times {}^7C_2 \Rightarrow$ No. of words $= {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$

Case III : All are different

i.e., ${}^8C_4 \Rightarrow$ No. of words $= {}^8C_4 \times 4! = 1680.$

Hence total number of words are 2454.

54. A watch of meritorius students of the coaching institute is to be formed. For this a student must obtain at least 80% aggregate marks in a test of Physics, Chemistry and Math. (M.M. = 100 each).
The number of ways in which a student can qualify for the batch of meritorius students is

(a) $(101)^3$
(b) $(101)^2$
(c) $(101)^3 - {}^{242}C_{239}$
(d) none of these

Answer: c

Solution

$p + c + m \geq 240 \ldots (1)$

$p + c + m < 240 \ldots (2)$

where p, c, m denote the physics, chemistry and mathematics resp.

$(2) \Rightarrow p + c + m + t = 240 (t \neq 0) \ldots (3)$

Positive integral solution of (3)

$=$ number of integral solution of inequality (2)

$=$ co $-$ eff. of α^{240} in $(1 + \alpha + \alpha^2 + \cdots)^3 (\alpha + \alpha^2 + \cdots)$

$=$ co $-$ eff. of α^{239} in $(1 + \alpha + \alpha^2 + \cdots)^4$ i.e. $(1 - \alpha)^{-4}$

$= 239 + 4 - 1 = {}^{242}C_{239}$

Total number of C_{239} ways of scoring maths

$= 101 \times 101 \times 101 = (101)^3$

[$\because$ marks can be 0, 1, 2, ... 100]

$\therefore$ number of ways $= (101)^3 - {}^{242}C_{239}$

55. How many different words canbe formed by the letters of the word MISSISSIPPI in which no two S are adjacent ?

(a) $8\,{}^6C_4\,{}^7C_4$
(b) $2 \times 7 \times {}^8C_4$
(c) $6 \times 8 \times {}^7C_4$
(d) $7\,{}^6C_4\,{}^8C_4$

Answer: d

Solution

$\rightarrow$ arrangement of seven letters MIIIIPP $= \frac{7!}{4!\,2!}$

$\rightarrow$ arrangement of four S between 8 places of 7 letters $= {}^8C_4$

$\therefore$ Total required arrangement $= \frac{7!}{4!\,2!}\,{}^8C_4 = 7\,{}^6C_4\,{}^8C_4$

56. If r,s,t are prime numbers and p,q are the positive integers such that the LCM of p,q is $r^2\,t^4\,s^2$, then the number of ordered pair (p,q) is

(a) 252
(b) 254
(c) 225
(d) 224

Answer: d

Solution

Required number of ordered pair (p, q) is

$(2 \times 3 - 1)(2 \times 5 - 1)(2 \times 3 - 1) - 1 = 224.$

57. Let E={1,2,3,4}, F = {a,b} then the no. of onto function from E to F is _______

(a) 14
(b) 16
(c) 12
(d) 32

Answer: a

Solution

Number of total functions $= 2^4 = 16$
Number of constant functions $= f_1(x) = a \,\forall\, x \in E$
$f_2(x) = b \,\forall\, x \in E$
which are not onto.
$\therefore$ Number of such functions $= 16 - 2 = 14$

58. If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57/16$, then n is equal to

(a) 19
(b) 2
(c) 20
(d) 5

Answer: a

Solution

Given,

${}^{n+2}C_8 : {}^{n-2}P_4 = \frac{57}{16}$

$\Rightarrow \frac{{}^{n+2}C_8}{{}^{n-2}P_4} = \frac{57}{16}$

$\Rightarrow \frac{(n+2)(n+1)n(n-1)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8} = \frac{57}{16}$

$\Rightarrow (n+2)(n+1)n(n-1) = 57 \times 3 \times 4 \times 5 \times 6 \times 7$

$\Rightarrow (n+2)(n+1)n(n-1) = 21 \times 20 \times 19 \times 18$

$\therefore n = 19$

59. How many words can be formed with the letters of the word MATHEMATICS by rearranging them

(a) $\frac{11!}{2!\,2!}$
(b) $\frac{11!}{2!}$
(c) $\frac{11!}{2!\,2!\,2!}$
(d) 11!

Answer: c

Solution

Since there are 2 M's 2A's and 2Ts.

$\therefore$ Required number of ways are $\frac{11!}{2!\,2!\,2!}$.

60. The number of ways in which a committee of 3 women and 4 men be chosen from 8 women and 7 men is formed if mr.A refuses to serve on the commitee if mr. B is a member of the committee is _______

(a) 420
(b) 840
(c) 1540
(d) None of these

Answer: d

Solution

women can be selected in ${}^8C_3$
ways, men can be selected in $= {}^7C_4 - {}^5C_2$ ways